Student Solutions Manual

KEVIN BODDEN RANDY GALLAHER

Algebra & Trigonometry^8

MICHAEL SULLIVAN

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LEWIS AND CLARK COMMUNITY COLLEGE

Algebra & Trigonometry 8

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Preface

This solution manual accompanies *Algebra & Trigonometry, 8e* by Michael Sullivan. The Instructor Solutions Manual (ISM) contains detailed solutions to all exercises in the text and the chapter projects (both in the text and those posted on the internet). The Student Solutions Manual (SSM) contains detailed solutions to all odd exercises in the text and all solutions to chapter tests. In both manuals, some TI-84 Plus graphing calculator screenshots have been included to demonstrate how technology can be used to solve problems and check solutions. A concerted effort has been made to make this manual as user-friendly and error free as possible. Please feel free to send us any suggestions or corrections.

We would like to extend our thanks to Dawn Murrin, Christine Whitlock and Bob Walters from Prentice Hall for all their help with manuscript pages and logistics. Thanks for everything!

We would also like to thank our wives (Angie and Karen) and our children (Annie, Ben, Ethan, Logan, Payton, and Shawn) for their patient support and for enduring many late evenings.

Kevin Bodden and Randy Gallaher
Department of Mathematics
Lewis and Clark Community College
5800 Godfrey Road
Godfrey, IL 62035
kbodden@lc.edu rgallahe@lc.edu

Chapter R
Review

Section R.1

1. rational

3. Distributive

5. True

7. False; 6 is the Greatest Common Factor of 12 and 18. The Least Common Multiple is the smallest value that both numbers will divide evenly. The LCM for 12 and 18 is 36.

9. $A \cup B = \{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

11. $A \cap B = \{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\} = \{4\}$

13. $(A \cup B) \cap C$
$= (\{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}) \cap \{1, 3, 4, 6\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 3, 4, 6\}$
$= \{1, 3, 4, 6\}$

15. $\overline{A} = \{0, 2, 6, 7, 8\}$

17. $\overline{A \cap B} = \overline{\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}}$
$= \overline{\{4\}} = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$

19. $\overline{A} \cup \overline{B} = \{0, 2, 6, 7, 8\} \cup \{0, 1, 3, 5, 9\}$
$= \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$

21. **a.** $\{2, 5\}$

b. $\{-6, 2, 5\}$

c. $\left\{-6, \frac{1}{2}, -1.333\ldots = -1.\overline{3}, 2, 5\right\}$

d. $\{\pi\}$.

e. $\left\{-6, \frac{1}{2}, -1.333\ldots = -1.\overline{3}, \pi, 2, 5\right\}$

23. **a.** $\{1\}$

b. $\{0, 1\}$

c. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

d. None

e. $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

25. **a.** None

b. None

c. None

d. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$

e. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$

27. **a.** 18.953 **b.** 18.952

29. **a.** 28.653 **b.** 28.653

31. **a.** 0.063 **b.** 0.062

33. **a.** 9.999 **b.** 9.998

35. **a.** 0.429 **b.** 0.428

37. **a.** 34.733 **b.** 34.733

39. $3 + 2 = 5$

41. $x + 2 = 3 \cdot 4$

43. $3y = 1 + 2$

45. $x - 2 = 6$

47. $\frac{x}{2} = 6$

49. $9 - 4 + 2 = 5 + 2 = 7$

51. $-6 + 4 \cdot 3 = -6 + 12 = 6$

53. $4 + 5 - 8 = 9 - 8 = 1$

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x+4$

$y^2 = x+4$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x+4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x+4$

$y^2 = -x+4$ different

Therefore, the graph will have x-axis symmetry.

57. $y = \sqrt[3]{x}$

x-intercepts:

$0 = \sqrt[3]{x}$

$0 = x$

y-intercepts:

$y = \sqrt[3]{0} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[3]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[3]{-x} = -\sqrt[3]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$

$y = \sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

59. $x^2 + y - 9 = 0$

x-intercepts:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

y-intercepts:

$0^2 + y - 9 = 0$

$y = 9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - y - 9 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + y - 9 = 0$

$x^2 + y - 9 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - y - 9 = 0$

$x^2 - y - 9 = 0$ different

Therefore, the graph will have y-axis symmetry.

61. $9x^2 + 4y^2 = 36$

x-intercepts:

$9x^2 + 4(0)^2 = 36$

$9x^2 = 36$

$x^2 = 4$

$x = \pm 2$

y-intercepts:

$9(0)^2 + 4y^2 = 36$

$4y^2 = 36$

$y^2 = 9$

$y = \pm 3$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-3)$, and $(0,3)$.

Test x-axis symmetry: Let $y = -y$

$9x^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test y-axis symmetry: Let $x = -x$

$9(-x)^2 + 4y^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$9(-x)^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

63. $y = x^3 - 27$

x-intercepts:

$0 = x^3 - 27$

$x^3 = 27$

$x = 3$

y-intercepts:

$y = 0^3 - 27$

$y = -27$

The intercepts are $(3,0)$ and $(0,-27)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^3 - 27$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^3 - 27$

$y = -x^3 - 27$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^3 - 27$

$y = x^3 + 27$ different

Therefore, the graph has none of the indicated symmetries.

c. $f(x) = g(x)$

$-x^2 + 1 = 4x + 1$

$0 = x^2 + 4x$

$0 = x(x+4)$

$x = 0; x - 4$

Solution set: $\{-4, 0\}$.

d. $f(x) > 0$

We graph the function $f(x) = -x^2 + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $-x^2 + 1 = 0$

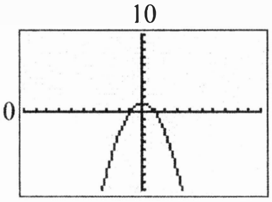
$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1; x = 1$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(-1)} = 0$. Since $f(0) = 1$, the vertex is $(0, 1)$.

10

−10  10

−10

The graph is above the x-axis when $-1 < x < 1$. Since the inequality is strict, the solution set is $\{x \mid -1 < x < 1\}$ or, using interval notation, $(-1, 1)$.

e. $g(x) \le 0$

$4x + 1 \le 0$

$4x \le -1$

$x \le -\dfrac{1}{4}$

The solution set is $\left\{x \,\middle|\, x \le -\dfrac{1}{4}\right\}$ or, using interval notation, $\left(-\infty, -\dfrac{1}{4}\right]$.

f. $f(x) > g(x)$

$-x^2 + 1 > 4x + 1$

$-x^2 - 4x > 0$

We graph the function $p(x) = -x^2 - 4x$.

The intercepts of p are

y-intercept: $p(0) = 0$

x-intercepts: $-x^2 - 4x = 0$

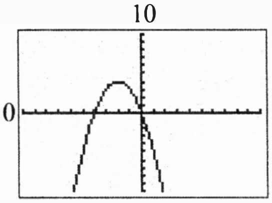
$-x(x+4) = 0$

$x = 0; x = -4$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(-1)} = \dfrac{4}{-2} = -2$.

Since $p(-2) = 4$, the vertex is $(-2, 4)$.

10

−10  10

−10

The graph of p is above the x-axis when $-4 < x < 0$. Since the inequality is strict, the solution set is $\{x \mid -4 < x < 0\}$ or, using interval notation, $(-4, 0)$.

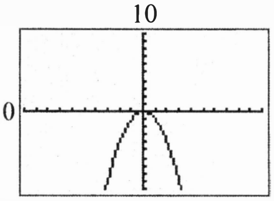
g. $f(x) \ge 1$

$-x^2 + 1 \ge 1$

$-x^2 \ge 0$

We graph the function $p(x) = -x^2$. The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(-1)} = 0$. Since $p(0) = 0$, the vertex is $(0, 0)$. Since $a = -1 < 0$, the parabola opens downward.

10

−10  10

−10

The graph of p is never above the x-axis, but it does touch the x-axis at $x = 0$. Since the inequality is not strict, the solution set is $\{0\}$.

29. $f(x) = x^2 - 4; \quad g(x) = -x^2 + 4$

a. $f(x) = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = 2; x = -2$

Solution set: $\{-2, 2\}$.

$g(x)=(x-3)(2x+1)(x+5)$

To find the remaining zeros of g, we set the last two factors equal to 0 and solve.

$2x+1=0 \qquad x+5=0$

$2x=-1 \qquad x=-5$

$x=-\frac{1}{2}$

Therefore, the zeros are -5, $-\frac{1}{2}$, and 3.

Notice how these rational zeros were all in the list of potential rational zeros.

e. The x-intercepts of a graph are the same as the zeros of the function. In the previous part, we found the zeros to be -5, $-\frac{1}{2}$, and 3. Therefore, the x-intercepts are -5, $-\frac{1}{2}$, and 3.

To find the y-intercept, we simply find $g(0)$.

$g(0)=2(0)^3+5(0)^2-28(0)-15=-15$

So, the y-intercept is -15.

f. Whether the graph crosses or touches at an x-intercept is determined by the multiplicity. Each factor of the polynomial occurs once, so the multiplicity of each zero is 1. For odd multiplicity, the graph will cross the x-axis at the zero. Thus, the graph crosses the x-axis at each of the three x-intercepts.

g. The power function that the graph of g resembles for large values of $|x|$ is given by the term with the highest power of x. In this case, the power function is $y=2x^3$. So, the graph of g will resemble the graph of $y=2x^3$ for large values of $|x|$.

h. The three intercepts are -5, $-\frac{1}{2}$, and 3.

Near -5:

$$g(x)=(x-3)(2x+1)(x+5)$$
$$\approx -8(-9)(x+5)=72(x+5)$$

(a line with slope 72)

Near $-\frac{1}{2}$:

$$g(x)=(x-3)(2x+1)(x+5)$$
$$\approx\left(-\frac{7}{2}\right)(2x+1)\left(\frac{9}{2}\right)$$
$$=-\frac{63}{4}(2x+1)=-\frac{63}{2}x-\frac{63}{4}$$

(a line with slope $-\frac{63}{2}$ or -31.5)

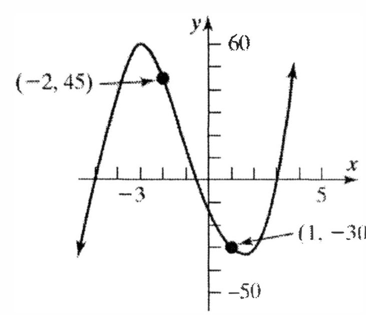
Near 3:

$$g(x)=(x-3)(2x+1)(x+5)$$
$$=(x-3)(7)(8)=56(x-3)$$

(a line with slope 56)

i. We could first evaluate the function at several values for x to help determine the scale.

Putting all this information together, we obtain the following graph:

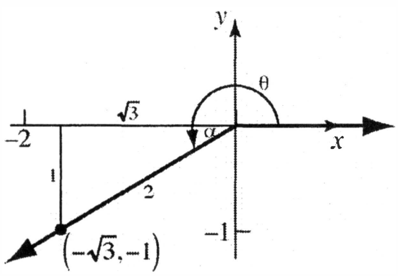


3. $x^3-4x^2+25x-100=0$

$$x^2(x-4)+25(x-4)=0$$
$$(x-4)(x^2+25)=0$$
$$x-4=0 \quad\text{or}\quad x^2+25=0$$
$$x=4 \qquad x^2=-25$$
$$x=\pm\sqrt{-25}$$
$$x=\pm 5i$$

The solution set is $\{4,-5i,5i\}$.

105. $\csc\theta = -2, \quad \tan\theta > 0 \Rightarrow \theta$ in quadrant III
Since θ is in quadrant III, $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$, while $\tan\theta > 0$ and $\cot\theta > 0$.
If α is the reference angle for θ, then $\csc\alpha = 2$. Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \dfrac{1}{2}$ $\qquad$ $\cos\alpha = \dfrac{\sqrt{3}}{2}$

$\tan\alpha = \dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ $\qquad$ $\sec\alpha = \dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\cot\alpha = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

Finally, assign the appropriate signs to the values of the other trigonometric functions of θ.

$\sin\theta = -\dfrac{1}{2}$ $\qquad$ $\cos\theta = -\dfrac{\sqrt{3}}{2}$

$\tan\theta = \dfrac{\sqrt{3}}{3}$ $\qquad$ $\sec\theta = -\dfrac{2\sqrt{3}}{3}$

$\cot\theta = \sqrt{3}$

107.
$$\begin{aligned}&\sin 40^\circ + \sin 130^\circ + \sin 220^\circ + \sin 310^\circ\\ &= \sin 40^\circ + \sin\left(40^\circ + 90^\circ\right) + \sin\left(40^\circ + 180^\circ\right)\\ &\qquad + \sin\left(40^\circ + 270^\circ\right)\\ &= \sin 40^\circ + \sin 40^\circ - \sin 40^\circ - \sin 40^\circ\\ &= 0\end{aligned}$$

109. Since $f(\theta) = \sin\theta = 0.2$ is positive, θ must lie either in quadrant I or II. Therefore, $\theta + \pi$ must lie either in quadrant III or IV. Thus, $f(\theta + \pi) = \sin(\theta + \pi) = -0.2$

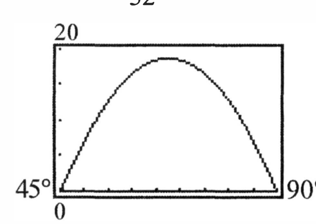
111. Since $F(\theta) = \tan\theta = 3$ is positive, θ must lie either in quadrant I or III. Therefore, $\theta + \pi$ must also lie either in quadrant I or III. Thus, $F(\theta + \pi) = \tan(\theta + \pi) = 3$.

113. Given $\sin\theta = \dfrac{1}{5}$, then $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{1}{5}} = 5$
Since $\csc\theta > 0$, θ must lie in quadrant I or II. This means that $\csc(\theta + \pi)$ must lie in quadrant III or IV with the same reference angle as θ. Since cosecant is negative in quadrants III and IV, we have $\csc(\theta + \pi) = -5$.

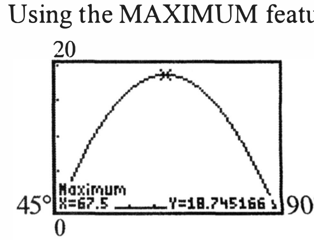
115.
$$\begin{aligned}&\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \ldots + \sin 357^\circ\\ &\qquad + \sin 358^\circ + \sin 359^\circ\\ &= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin(360^\circ - 3^\circ)\\ &\qquad + \sin(360^\circ - 2^\circ) + \sin(360^\circ - 1^\circ)\\ &= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin(-3^\circ)\\ &\qquad + \sin(-2^\circ) + \sin(-1^\circ)\\ &= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots - \sin 3^\circ - \sin 2^\circ - \sin 1^\circ\\ &= \sin(180^\circ)\\ &= 0\end{aligned}$$

117. a.
$$\begin{aligned}R &= \frac{32^2\sqrt{2}}{32}\left[\sin\left(2(60^\circ)\right) - \cos\left(2(60^\circ)\right) - 1\right]\\ &\approx 32\sqrt{2}\left(0.866 - (-0.5) - 1\right)\\ &\approx 16.6 \text{ ft}\end{aligned}$$

b. Let $Y_1 = \dfrac{32^2\sqrt{2}}{32}\left[\sin(2x) - \cos(2x) - 1\right]$



c. Using the MAXIMUM feature, we find:



R is largest when $\theta = 67.5^\circ$.

119. Answers will vary.

91. $\cos\left(\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{1}{2}\right)$

Let $\alpha=\sin^{-1}\frac{3}{5}$ and $\beta=\cos^{-1}\frac{1}{2}$. α is in quadrant I; β is in quadrant I. Then $\sin\alpha=\frac{3}{5}$, $0\le\alpha\le\frac{\pi}{2}$, and $\cos\beta=\frac{1}{2}$, $0\le\beta\le\frac{\pi}{2}$.

$$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

$$\sin\beta=\sqrt{1-\cos^2\beta}=\sqrt{1-\left(\frac{1}{2}\right)^2}=\sqrt{1-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

$$\begin{aligned}\cos\left(\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{1}{2}\right)&=\cos(\alpha-\beta)\\&=\cos\alpha\cos\beta+\sin\alpha\sin\beta\\&=\frac{4}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot\frac{\sqrt{3}}{2}\\&=\frac{4}{10}+\frac{3\sqrt{3}}{10}=\frac{4+3\sqrt{3}}{10}\end{aligned}$$

93. $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)-\tan^{-1}\frac{3}{4}\right]$

Let $\alpha=\sin^{-1}\left(-\frac{1}{2}\right)$ and $\beta=\tan^{-1}\frac{3}{4}$. α is in quadrant IV; β is in quadrant I. Then, $\sin\alpha=-\frac{1}{2}$, $0\le\alpha\le\frac{\pi}{2}$, and $\tan\beta=\frac{3}{4}$, $0<\beta<\frac{\pi}{2}$.

$$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(-\frac{1}{2}\right)^2}=\sqrt{1-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

$$\tan\alpha=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

$$\begin{aligned}\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right]&=\tan(\alpha-\beta)\\&=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}\\&=\frac{-\frac{\sqrt{3}}{3}-\frac{3}{4}}{1+\left(-\frac{\sqrt{3}}{3}\right)\left(\frac{3}{4}\right)}\\&=\frac{\frac{-4\sqrt{3}-9}{12}}{1-\frac{3\sqrt{3}}{12}}\\&=\frac{-9-4\sqrt{3}}{12-3\sqrt{3}}\cdot\frac{12+3\sqrt{3}}{12+3\sqrt{3}}\\&=\frac{-144-75\sqrt{3}}{117}\\&=\frac{-48-25\sqrt{3}}{39}\\&=-\frac{48+25\sqrt{3}}{39}\end{aligned}$$

95. $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

Let $\alpha=\cos^{-1}\left(-\frac{3}{5}\right)$. α is in quadrant II. Then $\cos\alpha=-\frac{3}{5}$, $\frac{\pi}{2}\le\alpha\le\pi$.

$$\sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\left(-\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

$$\begin{aligned}\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]&=\sin 2\alpha\\&=2\sin\alpha\cos\alpha\\&=2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)=-\frac{24}{25}\end{aligned}$$

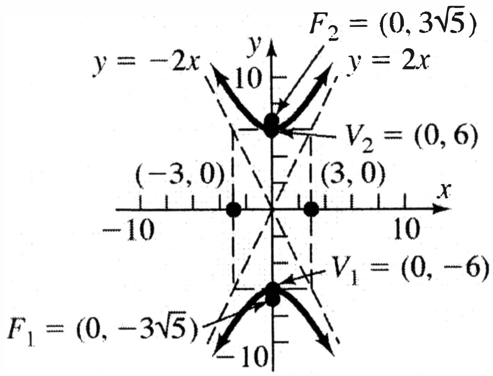
23. Vertices: $(0, -6)$, $(0, 6)$; asymptote: $y = 2x$;
Center: $(0, 0)$; Transverse axis is the y-axis;
$a = 6$. Find the value of b using the slope of the asymptote: $\frac{a}{b} = \frac{6}{b} = 2 \Rightarrow 2b = 6 \Rightarrow b = 3$

Find the value of c:

$c^2 = a^2 + b^2 = 36 + 9 = 45$

$c = 3\sqrt{5}$

Write the equation: $\frac{y^2}{36} - \frac{x^2}{9} = 1$.



25. Foci: $(-4, 0)$, $(4, 0)$; asymptote: $y = -x$;
Center: $(0, 0)$; Transverse axis is the x-axis;
$c = 4$. Using the slope of the asymptote:

$-\frac{b}{a} = -1 \Rightarrow -b = -a \Rightarrow b = a$.

Find the value of b:

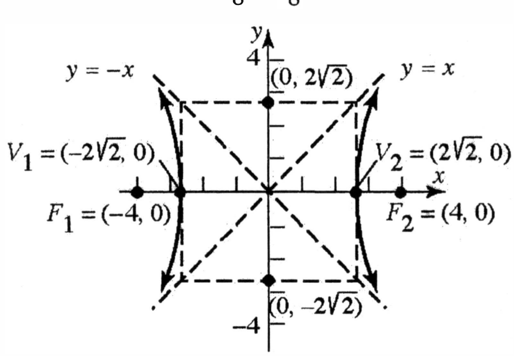
$b^2 = c^2 - a^2 \Rightarrow a^2 + b^2 = c^2 \quad (c = 4)$

$b^2 + b^2 = 16 \Rightarrow 2b^2 = 16 \Rightarrow b^2 = 8$

$b = \sqrt{8} = 2\sqrt{2}$

$a = \sqrt{8} = 2\sqrt{2} \quad (a = b)$

Write the equation: $\frac{x^2}{8} - \frac{y^2}{8} = 1$.



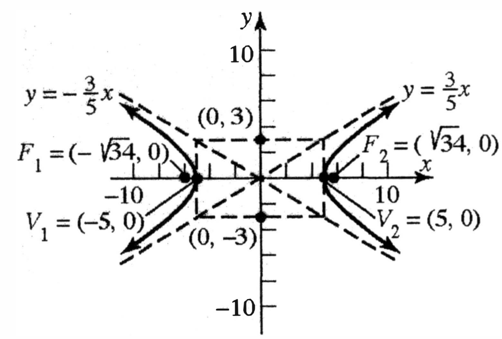
27. $\frac{x^2}{25} - \frac{y^2}{9} = 1$

The center of the hyperbola is at (0, 0).
$a = 5$, $b = 3$. The vertices are $(5, 0)$ and $(-5, 0)$. Find the value of c:

$c^2 = a^2 + b^2 = 25 + 9 = 34 \Rightarrow c = \sqrt{34}$

The foci are $(\sqrt{34}, 0)$ and $(-\sqrt{34}, 0)$.

The transverse axis is the x-axis. The asymptotes are $y = \frac{3}{5}x$; $y = -\frac{3}{5}x$.



29. $4x^2 - y^2 = 16$

Divide both sides by 16 to put in standard form:

$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16} \Rightarrow \frac{x^2}{4} - \frac{y^2}{16} = 1$

The center of the hyperbola is at (0, 0).
$a = 2$, $b = 4$.

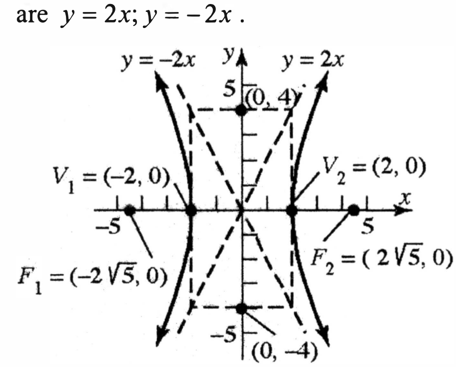
The vertices are (2, 0) and (−2, 0).
Find the value of c:

$c^2 = a^2 + b^2 = 4 + 16 = 20$

$c = \sqrt{20} = 2\sqrt{5}$

The foci are $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$.

The transverse axis is the x-axis. The asymptotes are $y = 2x$; $y = -2x$.

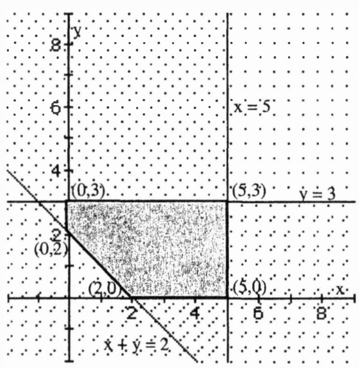


The corner points are (0, 1), (1, 0), (0, 6), (6, 0). Evaluate the objective function:

Vertex	Value of $z = 2x + y$
(0, 1)	$z = 2(0)+1 = 1$
(0, 6)	$z = 2(0)+6 = 6$
(1, 0)	$z = 2(1)+0 = 2$
(6, 0)	$z = 2(6)+0 = 12$

The maximum value is 12 at (6, 0).

11. Minimize $z = 2x + 5y$ subject to $x \ge 0$, $y \ge 0$, $x + y \ge 2$, $x \le 5$, $y \le 3$. Graph the constraints.

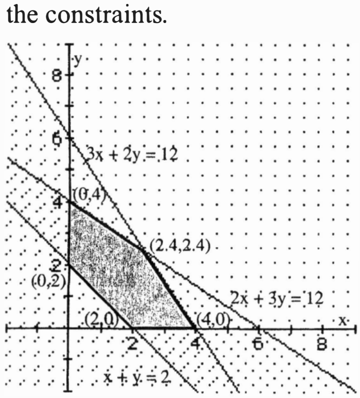


The corner points are (0, 2), (2, 0), (0, 3), (5, 0), (5, 3). Evaluate the objective function:

Vertex	Value of $z = 2x + 5y$
(0, 2)	$z = 2(0)+5(2) = 10$
(0, 3)	$z = 2(0)+5(3) = 15$
(2, 0)	$z = 2(2)+5(0) = 4$
(5, 0)	$z = 2(5)+5(0) = 10$
(5, 3)	$z = 2(5)+5(3) = 25$

The minimum value is 4 at (2, 0).

13. Maximize $z = 3x + 5y$ subject to $x \ge 0$, $y \ge 0$, $x + y \ge 2$, $2x + 3y \le 12$, $3x + 2y \le 12$. Graph the constraints.



To find the intersection of $2x + 3y = 12$ and $3x + 2y = 12$, solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + 2y = 12 \end{cases}$$

Solve the second equation for y: $y = 6 - \frac{3}{2}x$

Substitute and solve:

$$2x + 3\left(6 - \frac{3}{2}x\right) = 12$$
$$2x + 18 - \frac{9}{2}x = 12$$
$$-\frac{5}{2}x = -6$$
$$x = \frac{12}{5}$$
$$y = 6 - \frac{3}{2}\left(\frac{12}{5}\right) = 6 - \frac{18}{5} = \frac{12}{5}$$

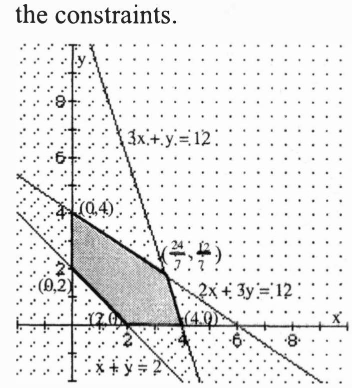
The point of intersection is $(2.4, 2.4)$.

The corner points are (0, 2), (2, 0), (0, 4), (4, 0), (2.4, 2.4). Evaluate the objective function:

Vertex	Value of $z = 3x + 5y$
(0, 2)	$z = 3(0)+5(2) = 10$
(0, 4)	$z = 3(0)+5(4) = 20$
(2, 0)	$z = 3(2)+5(0) = 6$
(4, 0)	$z = 3(4)+5(0) = 12$
(2.4, 2.4)	$z = 3(2.4)+5(2.4) = 19.2$

The maximum value is 20 at (0, 4).

15. Minimize $z = 5x + 4y$ subject to $x \ge 0$, $y \ge 0$, $x + y \ge 2$, $2x + 3y \le 12$, $3x + y \le 12$. Graph the constraints.



To find the intersection of $2x + 3y = 12$ and $3x + y = 12$, solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + y = 12 \end{cases}$$