

# STUDENT SOLUTIONS MANUAL

KEVIN BODDEN    RANDY GALLAHER

*Algebra & Trigonometry*<sup>8</sup>

MICHAEL SULLIVAN



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## Table of Contents

### Preface

### Chapter R Review

R.1	Real Numbers .....	1
R.2	Algebra Essentials .....	3
R.3	Geometry Essentials .....	6
R.4	Polynomials .....	8
R.5	Factoring Polynomials .....	11
R.6	Synthetic Division .....	14
R.7	Rational expressions .....	15
R.8	$n$ th Roots; Rational Exponents .....	20
	Chapter Review .....	24
	Chapter Test .....	28

### Chapter 1 Equations and Inequalities

1.1	Linear Equations .....	30
1.2	Quadratic Equations .....	39
1.3	Complex Numbers; Quadratic Equations in the Complex Number System .....	48
1.4	Radical Equations; Equations Quadratic in Form; Factorable Equations .....	50
1.5	Solving Inequalities .....	59
1.6	Equations and Inequalities Involving Absolute Value .....	65
1.7	Problem Solving: Interest, Mixture, Uniform Motion, and Constant Rate Job Applications .....	69
	Chapter Review .....	73
	Chapter Test .....	80

### Chapter 2 Graphs

2.1	The Distance and Midpoint Formulas .....	82
2.2	Graphs of Equations in Two Variables; Intercepts; Symmetry .....	87
2.3	Lines .....	93
2.4	Circles .....	101
2.5	Variation .....	106
	Chapter Review .....	108
	Chapter Test .....	114
	Cumulative Review .....	116

### Chapter 3 Functions and Their Graphs

3.1	Functions .....	118
3.2	The Graph of a Function .....	125
3.3	Properties of Functions .....	129
3.4	Library of Functions; Piecewise-defined Functions .....	136
3.5	Graphing Techniques: Transformations .....	142
3.6	Mathematical Models: Building Functions .....	150
	Chapter Review .....	153
	Chapter Test .....	159
	Cumulative Review .....	163



**Chapter 4 Linear and Quadratic Functions**

4.1 Linear Functions and Their Properties.....	165
4.2 Building Linear Functions from Data.....	170
4.3 Linear and Quadratic Functions.....	172
4.4 Properties of Quadratic Functions .....	182
4.5 Inequalities Involving Quadratic Functions.....	185
Chapter Review.....	195
Chapter Test.....	201
Cumulative Review.....	203

**Chapter 5 Polynomial and Rational Functions**

5.1 Polynomial Functions and Models.....	205
5.2 Properties of Rational Functions.....	215
5.3 The Graph of a Rational Function .....	219
5.4 Polynomial and Rational Inequalities .....	242
5.5 The Real Zeros of a Polynomial Function .....	249
5.6 Complex Zeros; Fundamental Theorem of Algebra .....	270
Chapter Review.....	273
Chapter Test.....	289
Cumulative Review.....	293

**Chapter 6 Exponential and Logarithmic Functions**

6.1 Composite Functions .....	296
6.2 One-to-One Functions; Inverse Functions .....	303
6.3 Exponential Functions .....	313
6.4 Logarithmic Functions.....	321
6.5 Properties of Logarithms .....	329
6.6 Logarithmic and Exponential Equations.....	333
6.7 Compound Interest.....	341
6.8 Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models .....	345
6.9 Building Exponential, Logarithmic, and Logistic Models from Data.....	348
Chapter Review.....	351
Chapter Test.....	361
Cumulative Review.....	364

**Chapter 7 Trigonometric Functions**

7.1 Angles and Their Measure.....	367
7.2 Right Triangle Trigonometry.....	371
7.3 Computing the Values of Trigonometric Functions of Acute Angles .....	378
7.4 Trigonometric Functions of General Angles .....	384
7.5 Unit Circle Approach: Properties of the Trigonometric Functions.....	391
7.6 Graphs of the Sine and Cosine Functions .....	394
7.7 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions.....	404
7.8 Phase Shift; Sinusoidal Curve Fitting.....	407
Chapter Review.....	413
Chapter Test.....	421
Cumulative Review.....	425

**Chapter 8 Analytic Trigonometry**

8.1	The Inverse Sine, Cosine, and Tangent Functions.....	428
8.2	The Inverse Trigonometric Functions (continued) .....	433
8.3	Trigonometric Identities .....	439
8.4	Sum and Difference Formulas .....	445
8.5	Double-Angle and Half-Angle Formulas.....	455
8.6	Product-to-Sum and Sum-to-Product Formulas.....	466
8.7	Trigonometric Equations I.....	469
8.8	Trigonometric Equations II.....	476
	Chapter Review.....	481
	Chapter Test.....	494
	Cumulative Review.....	499

**Chapter 9 Applications of Trigonometric Functions**

9.1	Applications Involving Right Triangles.....	503
9.2	The Law of Sines .....	506
9.3	The Law of Cosines .....	513
9.4	Area of a Triangle .....	517
9.5	Simple Harmonic Motion; Damped Motion; Combining Waves .....	521
	Chapter Review .....	524
	Chapter Test.....	529
	Cumulative Review.....	532

**Chapter 10 Polar Coordinates; Vectors**

10.1	Polar Coordinates.....	535
10.2	Polar Equations and Graphs.....	538
10.3	The Complex Plane; DeMoivre's Theorem .....	550
10.4	Vectors.....	555
10.5	The Dot Product.....	558
	Chapter Review.....	562
	Chapter Test.....	569
	Cumulative Review.....	572

**Chapter 11 Analytic Geometry**

11.2	The Parabola .....	574
11.3	The Ellipse .....	581
11.4	The Hyperbola .....	589
11.5	Rotation of Axes; General Form of a Conic .....	597
11.6	Polar Equations of Conics.....	603
11.7	Plane Curves and Parametric Equations .....	606
	Chapter Review .....	612
	Chapter Test.....	622
	Cumulative Review.....	626

**Chapter 12 Systems of Equations and Inequalities**

12.1 Systems of Linear Equations: Substitution and Elimination.....	628
12.2 Systems of Linear Equations: Matrices .....	637
12.3 Systems of Linear Equations: Determinants .....	649
12.4 Matrix Algebra.....	656
12.5 Partial Fraction Decomposition .....	663
12.6 Systems of Nonlinear Equations.....	669
12.7 Systems of Inequalities .....	682
12.8 Linear Programming.....	689
Chapter Review.....	695
Chapter Test.....	709
Cumulative Review.....	717

**Chapter 13 Sequences; Induction; the Binomial Theorem**

13.1 Sequences .....	720
13.2 Arithmetic Sequences .....	723
13.3 Geometric Sequences; Geometric Series .....	726
13.4 Mathematical Induction.....	731
13.5 The Binomial Theorem.....	734
Chapter Review.....	737
Chapter Test.....	741
Cumulative Review.....	744

**Chapter 14 Counting and Probability**

14.1 Sets and Counting.....	745
14.2 Permutations and Combinations .....	746
14.3 Probability.....	747
Chapter Review.....	750
Chapter Test.....	751
Cumulative Review.....	753

**Appendix Graphing Utilities**

Section 1 The Viewing Rectangle.....	755
Section 2 Using a Graphing Utility to Graph Equations.....	755
Section 3 Using a Graphing Utility to Locate Intercepts and Check for Symmetry .....	759
Section 5 Square Screens .....	760



## Preface

This solution manual accompanies *Algebra & Trigonometry, 8e* by Michael Sullivan. The Instructor Solutions Manual (ISM) contains detailed solutions to all exercises in the text and the chapter projects (both in the text and those posted on the internet). The Student Solutions Manual (SSM) contains detailed solutions to all odd exercises in the text and all solutions to chapter tests. In both manuals, some TI-84 Plus graphing calculator screenshots have been included to demonstrate how technology can be used to solve problems and check solutions. A concerted effort has been made to make this manual as user-friendly and error free as possible. Please feel free to send us any suggestions or corrections.

We would like to extend our thanks to Dawn Murrin, Christine Whitlock and Bob Walters from Prentice Hall for all their help with manuscript pages and logistics. Thanks for everything!

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## Chapter R

### Review

#### Section R.1

1. rational
3. Distributive
5. True
7. False; 6 is the Greatest Common Factor of 12 and 18. The Least Common Multiple is the smallest value that both numbers will divide evenly. The LCM for 12 and 18 is 36.
9.  $A \cup B = \{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}$   
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
11.  $A \cap B = \{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\} = \{4\}$
13.  $(A \cup B) \cap C$   
 $= (\{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}) \cap \{1, 3, 4, 6\}$   
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 3, 4, 6\}$   
 $= \{1, 3, 4, 6\}$
15.  $\overline{A} = \{0, 2, 6, 7, 8\}$
17.  $\overline{A \cap B} = \overline{\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}}$   
 $= \overline{\{4\}} = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
19.  $\overline{A} \cup \overline{B} = \{0, 2, 6, 7, 8\} \cup \{0, 1, 3, 5, 9\}$   
 $= \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
21. a.  $\{2, 5\}$   
b.  $\{-6, 2, 5\}$   
c.  $\left\{-6, \frac{1}{2}, -1.333\dots = -1.\overline{3}, 2, 5\right\}$   
d.  $\{\pi\}$   
e.  $\left\{-6, \frac{1}{2}, -1.333\dots = -1.\overline{3}, \pi, 2, 5\right\}$
23. a.  $\{1\}$   
b.  $\{0, 1\}$   
c.  $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$   
d. None  
e.  $\left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
25. a. None  
b. None  
c. None  
d.  $\left\{\sqrt{2}, \pi, \sqrt{2} + 1, \pi + \frac{1}{2}\right\}$   
e.  $\left\{\sqrt{2}, \pi, \sqrt{2} + 1, \pi + \frac{1}{2}\right\}$
27. a. 18.953                      b. 18.952
29. a. 28.653                      b. 28.653
31. a. 0.063                        b. 0.062
33. a. 9.999                        b. 9.998
35. a. 0.429                        b. 0.428
37. a. 34.733                        b. 34.733
39.  $3 + 2 = 5$
41.  $x + 2 = 3 \cdot 4$
43.  $3y = 1 + 2$
45.  $x - 2 = 6$
47.  $\frac{x}{2} = 6$
49.  $9 - 4 + 2 = 5 + 2 = 7$
51.  $-6 + 4 \cdot 3 = -6 + 12 = 6$
53.  $4 + 5 - 8 = 9 - 8 = 1$

**Chapter 2: Graphs**Test x-axis symmetry: Let  $y = -y$ 

$$(-y)^2 = x + 4$$

$$y^2 = x + 4 \text{ same}$$

Test y-axis symmetry: Let  $x = -x$ 

$$y^2 = -x + 4 \text{ different}$$

Test origin symmetry: Let  $x = -x$  and  $y = -y$ .

$$(-y)^2 = -x + 4$$

$$y^2 = -x + 4 \text{ different}$$

Therefore, the graph will have x-axis symmetry.

**57.**  $y = \sqrt[3]{x}$

x-intercepts: y-intercepts:

$$0 = \sqrt[3]{x}$$

$$y = \sqrt[3]{0} = 0$$

$$0 = x$$

The only intercept is  $(0, 0)$ .Test x-axis symmetry: Let  $y = -y$ 

$$-y = \sqrt[3]{x} \text{ different}$$

Test y-axis symmetry: Let  $x = -x$ 

$$y = \sqrt[3]{-x} = -\sqrt[3]{x} \text{ different}$$

Test origin symmetry: Let  $x = -x$  and  $y = -y$ 

$$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$$

$$y = \sqrt[3]{x} \text{ same}$$

Therefore, the graph will have origin symmetry.

**59.**  $x^2 + y - 9 = 0$

x-intercepts: y-intercepts:

$$x^2 - 9 = 0$$

$$0^2 + y - 9 = 0$$

$$x^2 = 9$$

$$y = 9$$

$$x = \pm 3$$

The intercepts are  $(-3, 0)$ ,  $(3, 0)$ , and  $(0, 9)$ .Test x-axis symmetry: Let  $y = -y$ 

$$x^2 - y - 9 = 0 \text{ different}$$

Test y-axis symmetry: Let  $x = -x$ 

$$(-x)^2 + y - 9 = 0$$

$$x^2 + y - 9 = 0 \text{ same}$$

Test origin symmetry: Let  $x = -x$  and  $y = -y$ 

$$(-x)^2 - y - 9 = 0$$

$$x^2 - y - 9 = 0 \text{ different}$$

Therefore, the graph will have y-axis symmetry.

**61.**  $9x^2 + 4y^2 = 36$

x-intercepts:y-intercepts:

$$9x^2 + 4(0)^2 = 36$$

$$9(0)^2 + 4y^2 = 36$$

$$9x^2 = 36$$

$$4y^2 = 36$$

$$x^2 = 4$$

$$y^2 = 9$$

$$x = \pm 2$$

$$y = \pm 3$$

The intercepts are  $(-2, 0)$ ,  $(2, 0)$ ,  $(0, -3)$ , and  $(0, 3)$ .Test x-axis symmetry: Let  $y = -y$ 

$$9x^2 + 4(-y)^2 = 36$$

$$9x^2 + 4y^2 = 36 \text{ same}$$

Test y-axis symmetry: Let  $x = -x$ 

$$9(-x)^2 + 4y^2 = 36$$

$$9x^2 + 4y^2 = 36 \text{ same}$$

Test origin symmetry: Let  $x = -x$  and  $y = -y$ 

$$9(-x)^2 + 4(-y)^2 = 36$$

$$9x^2 + 4y^2 = 36 \text{ same}$$

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

**63.**  $y = x^3 - 27$

x-intercepts:y-intercepts:

$$0 = x^3 - 27$$

$$y = 0^3 - 27$$

$$x^3 = 27$$

$$y = -27$$

$$x = 3$$

The intercepts are  $(3, 0)$  and  $(0, -27)$ .Test x-axis symmetry: Let  $y = -y$ 

$$-y = x^3 - 27 \text{ different}$$

Test y-axis symmetry: Let  $x = -x$ 

$$y = (-x)^3 - 27$$

$$y = -x^3 - 27 \text{ different}$$

Test origin symmetry: Let  $x = -x$  and  $y = -y$ 

$$-y = (-x)^3 - 27$$

$$y = x^3 + 27 \text{ different}$$

Therefore, the graph has none of the indicated symmetries.



**Chapter 4: Linear and Quadratic Functions**

c.  $f(x) = g(x)$

$$-x^2 + 1 = 4x + 1$$

$$0 = x^2 + 4x$$

$$0 = x(x + 4)$$

$$x = 0; x = -4$$

Solution set:  $\{-4, 0\}$ .

d.  $f(x) > 0$

We graph the function  $f(x) = -x^2 + 1$ .

y-intercept:  $f(0) = 1$

x-intercepts:  $-x^2 + 1 = 0$

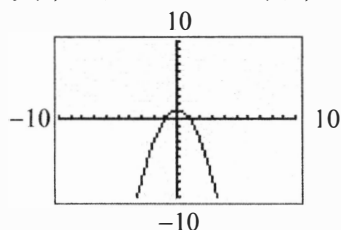
$$x^2 - 1 = 0$$

$$(x + 1)(x - 1) = 0$$

$$x = -1; x = 1$$

The vertex is at  $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$ . Since

$f(0) = 1$ , the vertex is  $(0, 1)$ .



The graph is above the  $x$ -axis when  $-1 < x < 1$ . Since the inequality is strict, the solution set is  $\{x \mid -1 < x < 1\}$  or, using interval notation,  $(-1, 1)$ .

e.  $g(x) \leq 0$

$$4x + 1 \leq 0$$

$$4x \leq -1$$

$$x \leq -\frac{1}{4}$$

The solution set is  $\left\{x \mid x \leq -\frac{1}{4}\right\}$  or, using

interval notation,  $\left(-\infty, -\frac{1}{4}\right]$ .

f.  $f(x) > g(x)$

$$-x^2 + 1 > 4x + 1$$

$$-x^2 - 4x > 0$$

We graph the function  $p(x) = -x^2 - 4x$ .

The intercepts of  $p$  are

y-intercept:  $p(0) = 0$

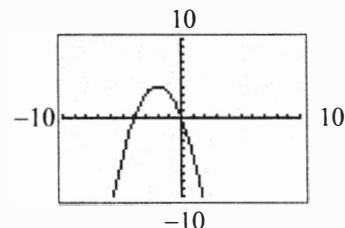
x-intercepts:  $-x^2 - 4x = 0$

$$-x(x + 4) = 0$$

$$x = 0; x = -4$$

The vertex is at  $x = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$ .

Since  $p(-2) = 4$ , the vertex is  $(-2, 4)$ .



The graph of  $p$  is above the  $x$ -axis when  $-4 < x < 0$ . Since the inequality is strict, the solution set is  $\{x \mid -4 < x < 0\}$  or, using interval notation,  $(-4, 0)$ .

g.  $f(x) \geq 1$

$$-x^2 + 1 \geq 1$$

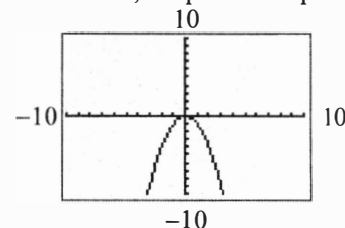
$$-x^2 \geq 0$$

We graph the function  $p(x) = -x^2$ . The

vertex is at  $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$ . Since

$p(0) = 0$ , the vertex is  $(0, 0)$ . Since

$a = -1 < 0$ , the parabola opens downward.



The graph of  $p$  is never above the  $x$ -axis, but it does touch the  $x$ -axis at  $x = 0$ . Since the inequality is not strict, the solution set is  $\{0\}$ .

29.  $f(x) = x^2 - 4$ ;  $g(x) = -x^2 + 4$

a.  $f(x) = 0$

$$x^2 - 4 = 0$$

$$(x - 2)(x + 2) = 0$$

$$x = 2; x = -2$$

Solution set:  $\{-2, 2\}$ .

**Chapter 5: Polynomial and Rational Functions**

$$g(x) = (x-3)(2x+1)(x+5)$$

To find the remaining zeros of  $g$ , we set the last two factors equal to 0 and solve.

$$2x+1=0 \quad x+5=0$$

$$2x=-1 \quad x=-5$$

$$x = -\frac{1}{2}$$

Therefore, the zeros are  $-5$ ,  $-\frac{1}{2}$ , and  $3$ .

Notice how these rational zeros were all in the list of potential rational zeros.

- e. The  $x$ -intercepts of a graph are the same as the zeros of the function. In the previous part, we found the zeros to be  $-5$ ,  $-\frac{1}{2}$ , and

$3$ . Therefore, the  $x$ -intercepts are  $-5$ ,  $-\frac{1}{2}$ , and  $3$ .

To find the  $y$ -intercept, we simply find  $g(0)$ .

$$g(0) = 2(0)^3 + 5(0)^2 - 28(0) - 15 = -15$$

So, the  $y$ -intercept is  $-15$ .

- f. Whether the graph crosses or touches at an  $x$ -intercept is determined by the multiplicity. Each factor of the polynomial occurs once, so the multiplicity of each zero is 1. For odd multiplicity, the graph will cross the  $x$ -axis at the zero. Thus, the graph crosses the  $x$ -axis at each of the three  $x$ -intercepts.
- g. The power function that the graph of  $g$  resembles for large values of  $|x|$  is given by the term with the highest power of  $x$ . In this case, the power function is  $y = 2x^3$ . So, the graph of  $g$  will resemble the graph of  $y = 2x^3$  for large values of  $|x|$ .

- h. The three intercepts are  $-5$ ,  $-\frac{1}{2}$ , and  $3$ .

Near  $-5$ :

$$g(x) = (x-3)(2x+1)(x+5) \\ \approx -8(-9)(x+5) = 72(x+5)$$

(a line with slope 72)

Near  $-\frac{1}{2}$ :

$$g(x) = (x-3)(2x+1)(x+5) \\ \approx \left(-\frac{7}{2}\right)(2x+1)\left(\frac{9}{2}\right) \\ = -\frac{63}{4}(2x+1) = -\frac{63}{2}x - \frac{63}{4}$$

(a line with slope  $-\frac{63}{2}$  or  $-31.5$ )

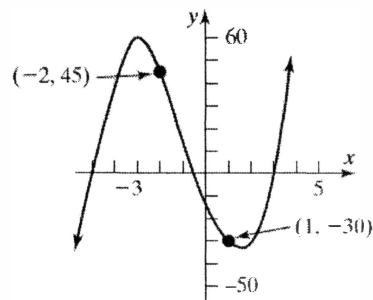
Near  $3$ :

$$g(x) = (x-3)(2x+1)(x+5) \\ = (x-3)(7)(8) = 56(x-3)$$

(a line with slope 56)

- i. We could first evaluate the function at several values for  $x$  to help determine the scale.

Putting all this information together, we obtain the following graph:

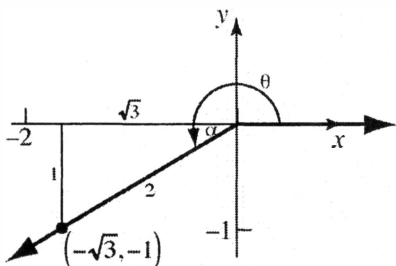


$$\begin{aligned} 3. \quad x^3 - 4x^2 + 25x - 100 &= 0 \\ x^2(x-4) + 25(x-4) &= 0 \\ (x-4)(x^2 + 25) &= 0 \\ x-4 = 0 \quad \text{or} \quad x^2 + 25 &= 0 \\ x = 4 \quad x^2 &= -25 \\ x &= \pm\sqrt{-25} \\ x &= \pm 5i \end{aligned}$$

The solution set is  $\{4, -5i, 5i\}$ .

**Chapter 7: Trigonometric Functions**

105.  $\csc \theta = -2$ ,  $\tan \theta > 0 \Rightarrow \theta$  in quadrant III  
 Since  $\theta$  is in quadrant III,  $\cos \theta < 0$ ,  $\sec \theta < 0$ ,  
 $\sin \theta < 0$  and  $\csc \theta < 0$ , while  $\tan \theta > 0$  and  
 $\cot \theta > 0$ .  
 If  $\alpha$  is the reference angle for  $\theta$ , then  $\csc \alpha = 2$ .  
 Now draw the appropriate triangle and use the  
 Pythagorean Theorem to find the values of the  
 other trigonometric functions of  $\alpha$ .



$$\begin{aligned}\sin \alpha &= \frac{1}{2} & \cos \alpha &= \frac{\sqrt{3}}{2} \\ \tan \alpha &= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} & \sec \alpha &= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \\ \cot \alpha &= \frac{\sqrt{3}}{1} = \sqrt{3}\end{aligned}$$

Finally, assign the appropriate signs to the values of the other trigonometric functions of  $\theta$ .

$$\begin{aligned}\sin \theta &= -\frac{1}{2} & \cos \theta &= -\frac{\sqrt{3}}{2} \\ \tan \theta &= \frac{\sqrt{3}}{3} & \sec \theta &= -\frac{2\sqrt{3}}{3} \\ \cot \theta &= \sqrt{3}\end{aligned}$$

107.  $\sin 40^\circ + \sin 130^\circ + \sin 220^\circ + \sin 310^\circ$   
 $= \sin 40^\circ + \sin(40^\circ + 90^\circ) + \sin(40^\circ + 180^\circ)$   
 $\quad + \sin(40^\circ + 270^\circ)$   
 $= \sin 40^\circ + \sin 40^\circ - \sin 40^\circ - \sin 40^\circ$   
 $= 0$
109. Since  $f(\theta) = \sin \theta = 0.2$  is positive,  $\theta$  must lie either in quadrant I or II. Therefore,  $\theta + \pi$  must lie either in quadrant III or IV. Thus,  
 $f(\theta + \pi) = \sin(\theta + \pi) = -0.2$
111. Since  $F(\theta) = \tan \theta = 3$  is positive,  $\theta$  must lie either in quadrant I or III. Therefore,  $\theta + \pi$  must also lie either in quadrant I or III. Thus,  
 $F(\theta + \pi) = \tan(\theta + \pi) = 3$ .

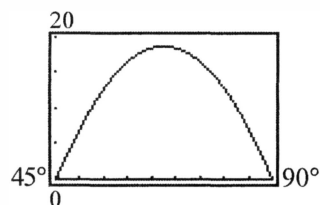
113. Given  $\sin \theta = \frac{1}{5}$ , then  $\csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{1}{5}} = 5$

Since  $\csc \theta > 0$ ,  $\theta$  must lie in quadrant I or II. This means that  $\csc(\theta + \pi)$  must lie in quadrant III or IV with the same reference angle as  $\theta$ . Since cosecant is negative in quadrants III and IV, we have  $\csc(\theta + \pi) = -5$ .

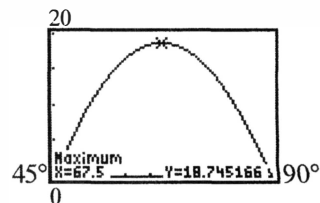
115.  $\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin 357^\circ$   
 $\quad + \sin 358^\circ + \sin 359^\circ$   
 $= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin(360^\circ - 3^\circ)$   
 $\quad + \sin(360^\circ - 2^\circ) + \sin(360^\circ - 1^\circ)$   
 $= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin(-3^\circ)$   
 $\quad + \sin(-2^\circ) + \sin(-1^\circ)$   
 $= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots - \sin 3^\circ - \sin 2^\circ - \sin 1^\circ$   
 $= \sin(180^\circ)$   
 $= 0$

117. a.  $R = \frac{32^2 \sqrt{2}}{32} [\sin(2(60^\circ)) - \cos(2(60^\circ)) - 1]$   
 $\approx 32\sqrt{2} (0.866 - (-0.5) - 1)$   
 $\approx 16.6 \text{ ft}$

b. Let  $Y_1 = \frac{32^2 \sqrt{2}}{32} [\sin(2x) - \cos(2x) - 1]$



c. Using the MAXIMUM feature, we find:



$R$  is largest when  $\theta = 67.5^\circ$ .

119. Answers will vary.

**Chapter 8: Analytic Trigonometry**

$$91. \cos\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{1}{2}\right)$$

Let  $\alpha = \sin^{-1}\frac{3}{5}$  and  $\beta = \cos^{-1}\frac{1}{2}$ .  $\alpha$  is in

quadrant I;  $\beta$  is in quadrant I. Then  $\sin \alpha = \frac{3}{5}$ ,

$$0 \leq \alpha \leq \frac{\pi}{2}, \text{ and } \cos \beta = \frac{1}{2}, \quad 0 \leq \beta \leq \frac{\pi}{2}.$$

$$\begin{aligned}\cos \alpha &= \sqrt{1 - \sin^2 \alpha} \\ &= \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\sin \beta &= \sqrt{1 - \cos^2 \beta} \\ &= \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\cos\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{1}{2}\right) &= \cos(\alpha - \beta) \\ &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \frac{4}{5} \cdot \frac{1}{2} + \frac{3}{5} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{4}{10} + \frac{3\sqrt{3}}{10} = \frac{4 + 3\sqrt{3}}{10}\end{aligned}$$

$$93. \tan\left[\sin^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\frac{3}{4}\right]$$

Let  $\alpha = \sin^{-1}\left(-\frac{1}{2}\right)$  and  $\beta = \tan^{-1}\frac{3}{4}$ .  $\alpha$  is in quadrant IV;  $\beta$  is in quadrant I. Then,

$$\sin \alpha = -\frac{1}{2}, \quad 0 \leq \alpha \leq \frac{\pi}{2}, \text{ and } \tan \beta = \frac{3}{4},$$

$$0 < \beta < \frac{\pi}{2}.$$

$$\begin{aligned}\cos \alpha &= \sqrt{1 - \sin^2 \alpha} \\ &= \sqrt{1 - \left(-\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}\end{aligned}$$

$$\tan \alpha = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right] = \tan(\alpha - \beta)$$

$$\begin{aligned}&= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ &= \frac{-\frac{\sqrt{3}}{3} - \frac{3}{4}}{1 + \left(-\frac{\sqrt{3}}{3}\right)\left(\frac{3}{4}\right)} \\ &= \frac{-\frac{4\sqrt{3} - 9}{12}}{1 - \frac{3\sqrt{3}}{4}} \\ &= \frac{-9 - 4\sqrt{3}}{12 - 3\sqrt{3}} \cdot \frac{12 + 3\sqrt{3}}{12 + 3\sqrt{3}} \\ &= \frac{-144 - 75\sqrt{3}}{117} \\ &= \frac{-48 - 25\sqrt{3}}{39} \\ &= -\frac{48 + 25\sqrt{3}}{39}\end{aligned}$$

$$95. \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$$

Let  $\alpha = \cos^{-1}\left(-\frac{3}{5}\right)$ .  $\alpha$  is in quadrant II. Then

$$\cos \alpha = -\frac{3}{5}, \quad \frac{\pi}{2} \leq \alpha \leq \pi.$$

$$\begin{aligned}\sin \alpha &= \sqrt{1 - \cos^2 \alpha} \\ &= \sqrt{1 - \left(-\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] &= \sin 2\alpha \\ &= 2\sin \alpha \cos \alpha \\ &= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}\end{aligned}$$



## Chapter 11: Analytic Geometry

23. Vertices:  $(0, -6)$ ,  $(0, 6)$ ; asymptote:  $y = 2x$ ;  
Center:  $(0, 0)$ ; Transverse axis is the  $y$ -axis;  
 $a = 6$ . Find the value of  $b$  using the slope of the

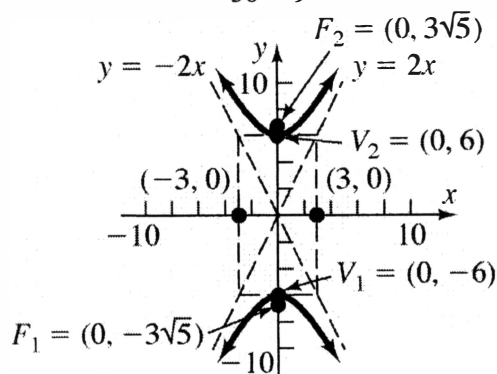
$$\text{asymptote: } \frac{a}{b} = \frac{6}{b} = 2 \Rightarrow 2b = 6 \Rightarrow b = 3$$

Find the value of  $c$ :

$$c^2 = a^2 + b^2 = 36 + 9 = 45$$

$$c = 3\sqrt{5}$$

$$\text{Write the equation: } \frac{y^2}{36} - \frac{x^2}{9} = 1.$$



25. Foci:  $(-4, 0)$ ,  $(4, 0)$ ; asymptote:  $y = -x$ ;  
Center:  $(0, 0)$ ; Transverse axis is the  $x$ -axis;  
 $c = 4$ . Using the slope of the asymptote:

$$-\frac{b}{a} = -1 \Rightarrow -b = -a \Rightarrow b = a.$$

Find the value of  $b$ :

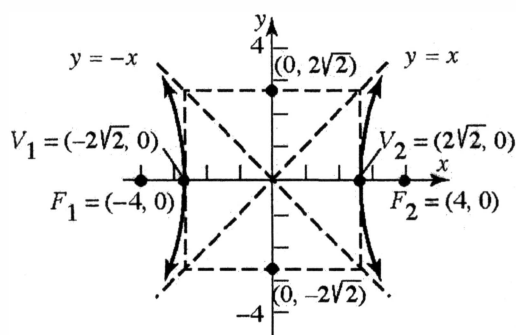
$$b^2 = c^2 - a^2 \Rightarrow a^2 + b^2 = c^2 \quad (c = 4)$$

$$b^2 + b^2 = 16 \Rightarrow 2b^2 = 16 \Rightarrow b^2 = 8$$

$$b = \sqrt{8} = 2\sqrt{2}$$

$$a = \sqrt{8} = 2\sqrt{2} \quad (a = b)$$

$$\text{Write the equation: } \frac{x^2}{8} - \frac{y^2}{8} = 1.$$



$$27. \frac{x^2}{25} - \frac{y^2}{9} = 1$$

The center of the hyperbola is at  $(0, 0)$ .

$a = 5$ ,  $b = 3$ . The vertices are  $(5, 0)$  and

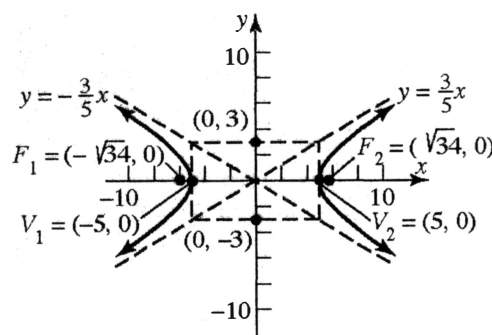
$(-5, 0)$ . Find the value of  $c$ :

$$c^2 = a^2 + b^2 = 25 + 9 = 34 \Rightarrow c = \sqrt{34}$$

The foci are  $(\sqrt{34}, 0)$  and  $(-\sqrt{34}, 0)$ .

The transverse axis is the  $x$ -axis. The asymptotes

$$\text{are } y = \frac{3}{5}x; y = -\frac{3}{5}x.$$



$$29. 4x^2 - y^2 = 16$$

Divide both sides by 16 to put in standard form:

$$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16} \Rightarrow \frac{x^2}{4} - \frac{y^2}{16} = 1$$

The center of the hyperbola is at  $(0, 0)$ .

$a = 2$ ,  $b = 4$ .

The vertices are  $(2, 0)$  and  $(-2, 0)$ .

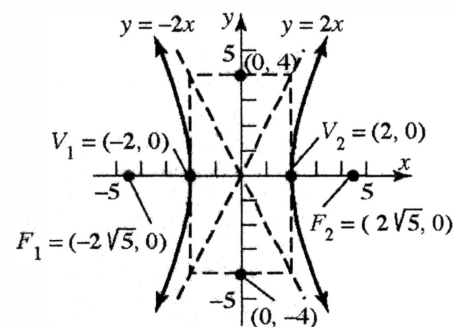
Find the value of  $c$ :

$$c^2 = a^2 + b^2 = 4 + 16 = 20$$

$$c = \sqrt{20} = 2\sqrt{5}$$

The foci are  $(2\sqrt{5}, 0)$  and  $(-2\sqrt{5}, 0)$ .

The transverse axis is the  $x$ -axis. The asymptotes are  $y = 2x$ ;  $y = -2x$ .



**Chapter 12: Systems of Equations and Inequalities**

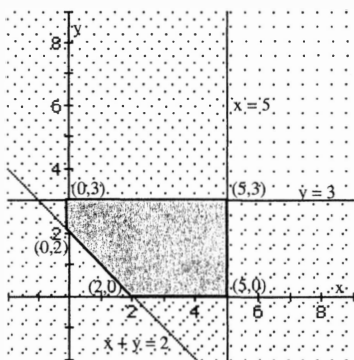
The corner points are (0, 1), (1, 0), (0, 6), (6, 0).

Evaluate the objective function:

Vertex	Value of $z = 2x + y$
(0, 1)	$z = 2(0) + 1 = 1$
(0, 6)	$z = 2(0) + 6 = 6$
(1, 0)	$z = 2(1) + 0 = 2$
(6, 0)	$z = 2(6) + 0 = 12$

The maximum value is 12 at (6, 0).

11. Minimize  $z = 2x + 5y$  subject to  $x \geq 0$ ,  $y \geq 0$ ,  $x + y \geq 2$ ,  $x \leq 5$ ,  $y \leq 3$ . Graph the constraints.

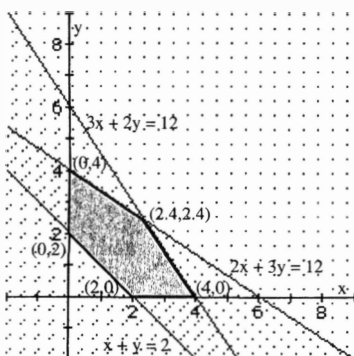


The corner points are (0, 2), (2, 0), (0, 3), (5, 0), (5, 3). Evaluate the objective function:

Vertex	Value of $z = 2x + 5y$
(0, 2)	$z = 2(0) + 5(2) = 10$
(0, 3)	$z = 2(0) + 5(3) = 15$
(2, 0)	$z = 2(2) + 5(0) = 4$
(5, 0)	$z = 2(5) + 5(0) = 10$
(5, 3)	$z = 2(5) + 5(3) = 25$

The minimum value is 4 at (2, 0).

13. Maximize  $z = 3x + 5y$  subject to  $x \geq 0$ ,  $y \geq 0$ ,  $x + y \geq 2$ ,  $2x + 3y \leq 12$ ,  $3x + 2y \leq 12$ . Graph the constraints.



To find the intersection of  $2x + 3y = 12$  and  $3x + 2y = 12$ , solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + 2y = 12 \end{cases}$$

Solve the second equation for  $y$ :  $y = 6 - \frac{3}{2}x$

Substitute and solve:

$$2x + 3\left(6 - \frac{3}{2}x\right) = 12$$

$$2x + 18 - \frac{9}{2}x = 12$$

$$-\frac{5}{2}x = -6$$

$$x = \frac{12}{5}$$

$$y = 6 - \frac{3}{2}\left(\frac{12}{5}\right) = 6 - \frac{18}{5} = \frac{12}{5}$$

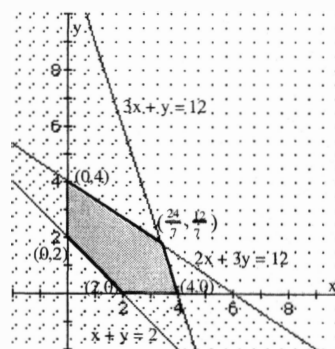
The point of intersection is  $(2.4, 2.4)$ .

The corner points are (0, 2), (2, 0), (0, 4), (4, 0),  $(2.4, 2.4)$ . Evaluate the objective function:

Vertex	Value of $z = 3x + 5y$
(0, 2)	$z = 3(0) + 5(2) = 10$
(0, 4)	$z = 3(0) + 5(4) = 20$
(2, 0)	$z = 3(2) + 5(0) = 6$
(4, 0)	$z = 3(4) + 5(0) = 12$
(2.4, 2.4)	$z = 3(2.4) + 5(2.4) = 19.2$

The maximum value is 20 at (0, 4).

15. Minimize  $z = 5x + 4y$  subject to  $x \geq 0$ ,  $y \geq 0$ ,  $x + y \geq 2$ ,  $2x + 3y \leq 12$ ,  $3x + y \leq 12$ . Graph the constraints.



To find the intersection of  $2x + 3y = 12$  and  $3x + y = 12$ , solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + y = 12 \end{cases}$$