

SOLUTIONS MANUAL

Logic and Discrete Mathematics

A Concise Introduction

WILLEM CONRADIE
VALENTIN GORANKO
CLAUDETTE ROBINSON

WILEY

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LOGIC AND DISCRETE MATHEMATICS

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LOGIC AND DISCRETE MATHEMATICS

A CONCISE INTRODUCTION, SOLUTIONS MANUAL

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Preface

This manual contains answers and solutions to roughly three quarters of the exercises in *Logic and Discrete Mathematics: A Concise Introduction* by Willem Conradie and Valentin Goranko. Most solutions are worked out in full detail. In deciding which solutions to include we were guided by two principles: fundamental exercises were given preference above the more esoteric ones intended mainly for enrichment; where a number of very similar exercises occur in succession, complete solutions were given for a few while the others were omitted or provided with answers only. We trust that these solutions will be a very valuable resource to students and instructors using *Logic and Discrete Mathematics*.

Full file at
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About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/conradie/logic

The website includes:

- Lecture Slides
- Quizzes

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Preliminaries

1.1. Sets

$$(1a) A = \{-1, 4\}$$

$$(1b) B = \{-5, -3, -\frac{3}{2}, 1\}$$

$$(1c) C = \{-3, -2, -1, 0, 1, 2\}$$

$$(2a) B = \{x \in \mathbb{R} \mid x^2 - 3 = 0\}$$

$$(2b) A = \{x \mid x = 2k, \text{ where } k \in \mathbb{Z} \text{ and } 1 \leq k \leq 4\}$$

$$(2c) A = \{x \mid x = 2k, \text{ where } k \in \mathbb{Z}^+\}$$

$$(2d) A = \{x \mid x = 3k + 2, \text{ where } k \in \mathbb{Z}\}$$

$$(2e) A = \{x \mid x = n^3, \text{ where } n \in \mathbb{N}\}$$

$$(2f) B = \{x \in \mathbb{Z} \mid -3 \leq x \leq 6\}$$

$$(2g) B = \{x \in \mathbb{Z} \mid x \leq -3 \vee x \geq 5\}$$

(3) Only a, d, e, g, h and j are true.

$$(5a) A \cap B = \{2, 7\}$$

$$(5b) A \cup B = \{0, 1, 2, 4, 5, 6, 7, 9\}$$

$$(5c) A - B = \{1, 4\}$$

$$(5d) B - A = \{0, 5, 6, 9\}$$

$$(5e) A - (A - B) = \{2, 7\}$$

$$(5f) B - (A - B) = B$$

$$(6a) A \cap B = \{0\}$$

$$(6b) A \cup B = \{0, A\}$$

$$(6c) B - A = \{A\}$$

$$(6d) A \cap (B \cup C) = \{0\}$$

$$(1d) D = \{-1\}$$

$$(1e) E = \{-3, -2, 0, 1, 2\}$$

$$(1f) F = \{1, 3, 5, 7, 9\}$$

(4) Only a, g, j and l are true.

$$(5g) B' = \{1, 3, 4, 8\}$$

$$(5h) (A \cap B)' = \{0, 1, 3, 4, 5, 6, 8, 9\}$$

$$(5i) A \cap B' = \{1, 4\}$$

$$(5j) (A \cup B)' = \{3, 8\}$$

$$(5k) (A \cup B)' - (B - A)' = \emptyset$$

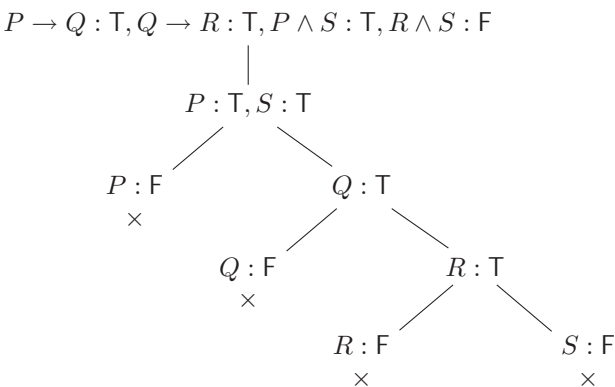
$$(6e) (B \cup C) - A = \{A, \{0\}, \{A\}\}$$

$$(6f) (A \cap B) \cup (A \cap C) = \{0\}$$

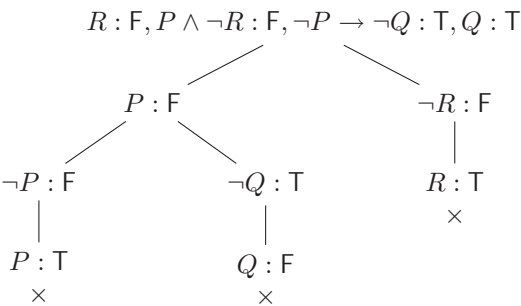
$$(6g) (C - B) - A = \{\{0\}, \{A\}\}$$

$$(6h) A \cap (C - A) = \emptyset$$

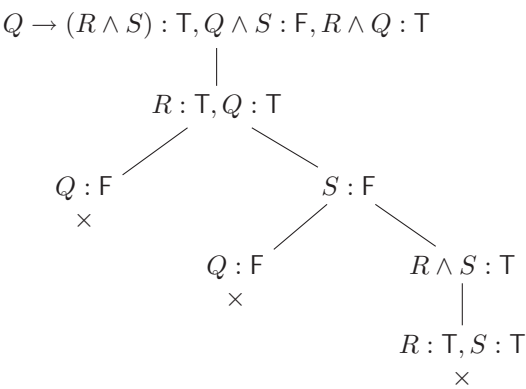
(3a) The tableau closes, so the consequence holds:



(3b) The tableau closes, so the consequence holds:



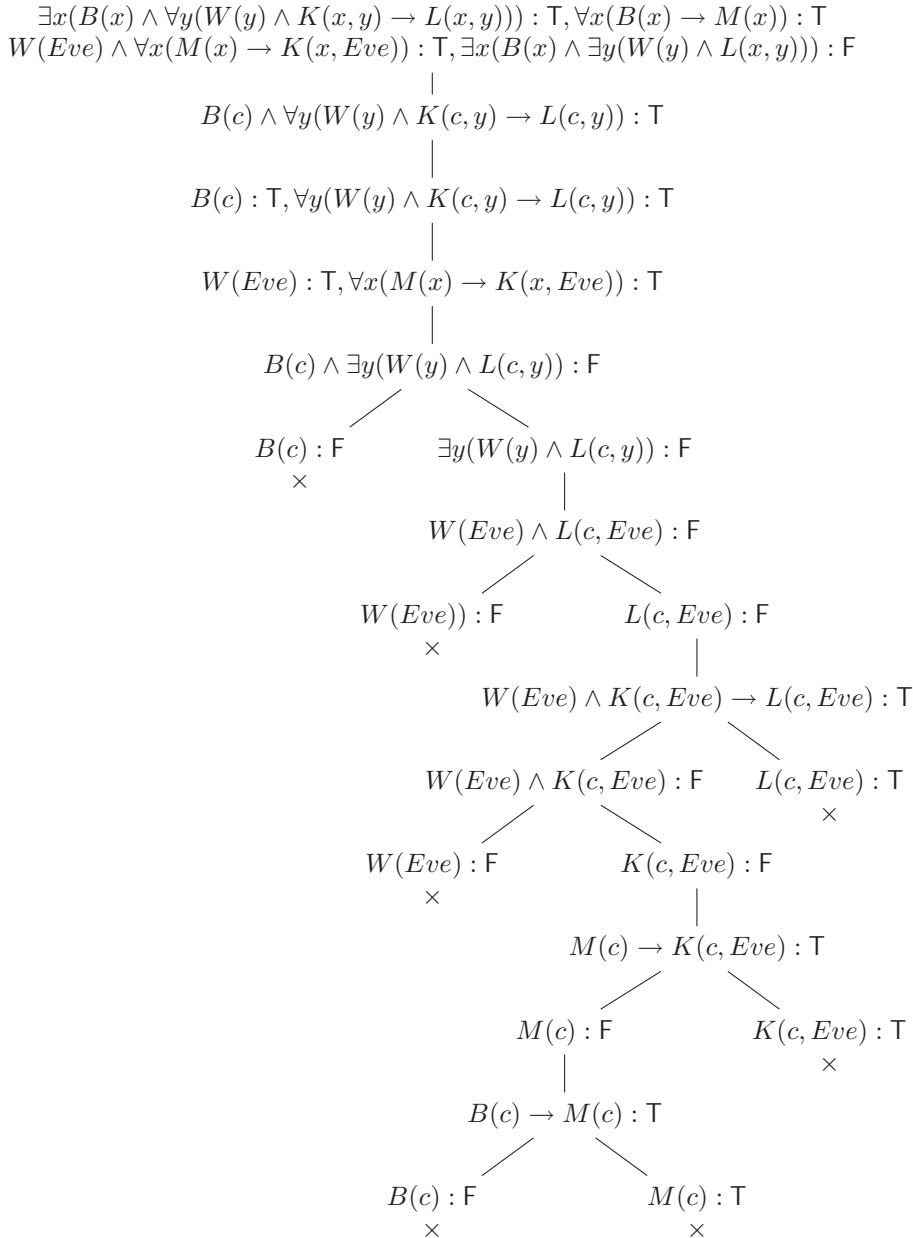
(3c) The tableau closes, so the consequence holds:



(7f) Using predicates $B(x)$ for “ x is a bachelor”, $M(x)$ for “ x is a man”, $W(x)$ for “ x is a woman”, $K(x, y)$ for “ x knows y ” and $L(x, y)$ for “ x loves y ”, we can formalize the argument as follows:

$$\frac{\begin{array}{l} \exists x(B(x) \wedge \forall y(W(y) \wedge K(x, y) \rightarrow L(x, y))) \\ \forall x(B(x) \rightarrow M(x)) \\ W(Eve) \wedge \forall x(M(x) \rightarrow K(x, Eve)) \end{array}}{\exists x(B(x) \wedge \exists y(W(y) \wedge L(x, y)))}$$

The argument is logically valid:



- (9) We have $606 > 605 = (5)(121)$, so, by the CPHP, if we divide the square into a 11×11 grid so that each of the smaller squares has a side $\frac{1}{11}$, at least 6 points will end up in the same small square. The farthest apart any two points in this square can be is the length of its diagonal; that is, $\sqrt{\left(\frac{1}{11}\right)^2 + \left(\frac{1}{11}\right)^2} = \sqrt{\frac{1}{121} + \frac{1}{121}} = \sqrt{\frac{2}{121}} = \frac{\sqrt{2}}{11}$. Inscribe the square in a circle with diameter $\frac{\sqrt{2}}{11}$, i.e. a radius of $\frac{\sqrt{2}}{22}$. However, $\frac{1}{15} > \frac{\sqrt{2}}{22}$, so we can cover the six points with a circle of radius $\frac{1}{15}$.
- (10a) Let P be the set of balls we select and H the set of different colours. To apply the CPHP, we need $|P| > (4)(4) = 16$, so choosing 17 or more balls will guarantee 5 balls of the same colour.
- (10b) The worst case scenario is selecting all the blue, red and yellow balls without selecting any green balls. In total, we can select $13 + 10 + 6 = 29$ balls without selecting a green ball. Hence, we have to select at least 32 balls in order to make sure that there will be at least 3 green balls.
- (10c) The worst case scenario is selecting all the blue, red and green balls without selecting a yellow ball. In total, we can select $13 + 10 + 8 = 31$ balls without selecting a yellow ball. Hence, we must select at least 32 balls to ensure we select at least one of each colour.
- (11) Choose any one of the six people, say a . Then take the remaining five people and group those who like a in the set L and those who hate a in the set H . By the CPHP, one of these sets, say L , contains at least three people. If these three people hate each other, then we are done; otherwise, there must be two of them who like each other and, together with a , we then have three people who like each other. Likewise if the set H contains at least three people.
- (14) Suppose there is a map from P to H that maps only finitely many elements of P to every element of H . Let m be the maximum number of elements of P mapped to any element of H . Then $|P| \leq m|H|$, which contradicts the fact that P is an infinite set.
- (15) There are two possibilities:

Case 1: there are m distinct sets each containing n of the same integers.

$mn + 1 > mn$, so, by the CPHP, with $|P| = mn + 1$, $|H| = m$ and $k = n$, there is one such set containing $n + 1$ equal integers.

Case 2: there are n equal sets containing m distinct integers.

$mn + 1 > mn$, so, by the CPHP (with $|P| = mn + 1$, $|H| = n$ and $k = m$), one of these sets must contain at least $m + 1$ distinct integers.

- (16) Note that the size of the subset is not specified, so it makes sense to consider the subsets of $\{m_1, \dots, m_n\}$ as posets under inclusion. A natural inclusion structure is ascending chains, sequences of sets where one contains the next. If we define the subsets $T_k = \{m_1, \dots, m_k\}$ for $k = 1, 2, \dots, n$ the sum corresponding to T_k is $t_k = m_1 + m_2 + \dots + m_k$ for $1 \leq k \leq n$. If t_k is a multiple of n for some k , we are done. Otherwise, let n be the pigeons and the remainders modulo n the pigeonholes. Then $|P| = n$ and $|H| = n - 1$. Therefore, by the PHP, there are at least two such sums, say t_i and t_j for $i < j$, that have the same remainder after division by n . Hence, by (CON1) of Theorem 5.5.3, n divides $t_j - t_i = m_{i+1} + \dots + m_j$ and so the set $T_j - T_i$ works.
- (17) Consider the sequence of integers

$$m \cdot 1 - 1, \quad m \cdot 2 - 1, \quad \dots, \quad m \cdot n - 1.$$

Choose any two integers in this sequence, say $m \cdot k_1 - 1$ and $m \cdot k_2 - 1 \pmod{n}$. Then $m \cdot k_1 - 1 \equiv m \cdot k_2 - 1 \pmod{n}$ iff $k_1 \equiv k_2$. Thus, no two of the n distinct members of the sequence are congruent modulo n . Therefore, the sequence represents a complete residue system modulo n . However, then one of its members is congruent to 0 modulo n , so there is a positive k such that n divides $m \cdot k - 1$.