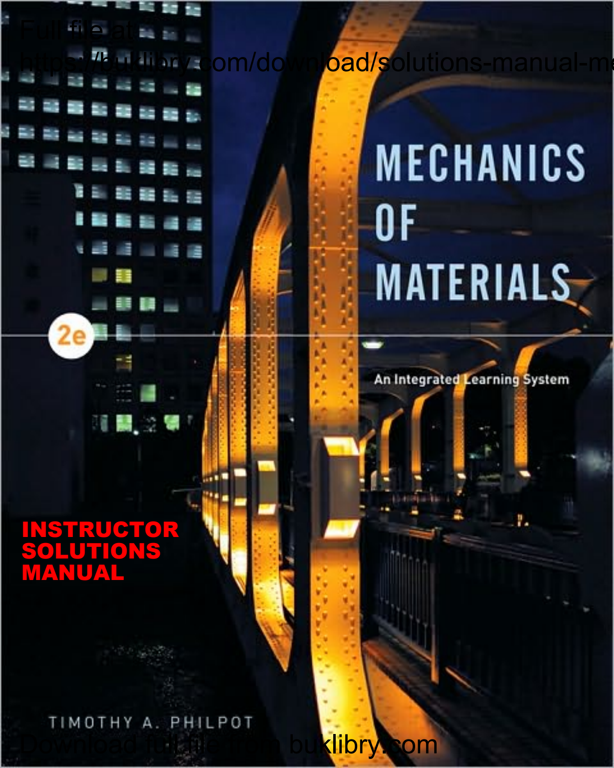
MECHANICS OF MATERIALS
2e
An Integrated Learning System
INSTRUCTOR SOLUTIONS MANUAL
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1.1 A stainless steel tube with an outside diameter of 60 mm and a wall thickness of 5 mm is used as a compression member. If the normal stress in the member must be limited to 200 MPa, determine the maximum load *P* that the member can support.

Solution

The cross-sectional area of the stainless steel tube is

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(60 \text{ mm})^2 - (50 \text{ mm})^2] = 863.938 \text{ mm}^2$$

The normal stress in the tube can be expressed as

$$\sigma = \frac{P}{A}$$

The maximum normal stress in the tube must be limited to 200 MPa. Using 200 MPa as the allowable normal stress, rearrange this expression to solve for the maximum load *P*

$$P_{\text{max}} \le \sigma_{\text{allow}} A = (200 \text{ N/mm}^2)(863.938 \text{ mm}^2) = 172{,}788 \text{ N} = \boxed{172.8 \text{ kN}}$$ **Ans.**

5.28 The concrete [E = 29 GPa] pier shown in Fig. P5.28 is reinforced using four steel [E = 200 GPa] reinforcing rods, each having a diameter of 19 mm. If the pier is subjected to an axial load of 670 kN, determine:
(a) the normal stress in the concrete and in the steel reinforcing rods.
(b) the shortening of the pier.

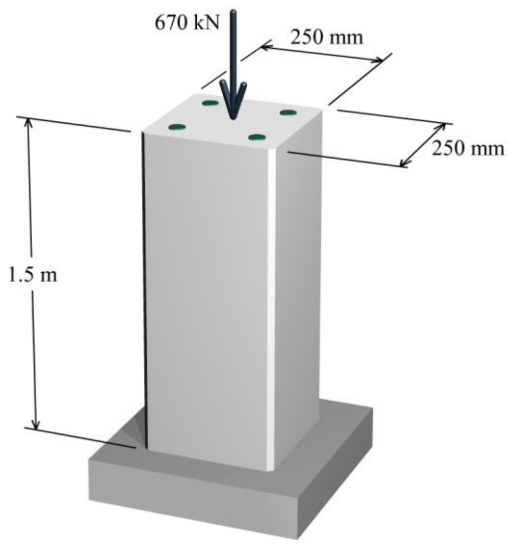
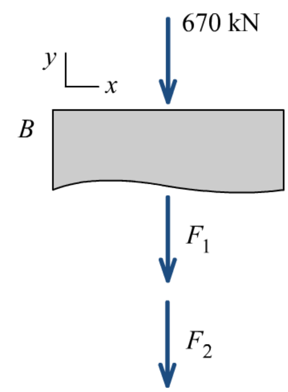


Fig. P5.28

Solution

(a) Equilibrium: Consider a FBD of the pier, cut around the upper end. The concrete will be designated member (1) and the reinforcing steel bars will be designated member (2). Sum forces in the vertical direction to obtain:



$$\Sigma F_y = -F_1 - F_2 - 670 \text{ kN} = 0 \qquad \text{(a)}$$

Geometry of Deformations:

$$\delta_1 = \delta_2 \qquad \text{(b)}$$

Force-Deformation Relationships:

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1} \qquad \delta_2 = \frac{F_2 L_2}{A_2 E_2} \qquad \text{(c)}$$

Compatibility Equation: Substitute Eqs. (c) into Eq. (b) to derive the compatibility equation:

$$\frac{F_1 L_1}{A_1 E_1} = \frac{F_2 L_2}{A_2 E_2} \qquad \text{(d)}$$

Solve the Equations: Solve Eq. (d) for F_2:

$$F_2 = F_1 \frac{L_1}{A_1 E_1} \frac{A_2 E_2}{L_2} = F_1 \frac{L_1}{L_2} \frac{A_2}{A_1} \frac{E_2}{E_1} \qquad \text{(e)}$$

and substitute this expression into Eq. (a) to determine F_1:

$$\Sigma F_y = -F_1 - F_2 = -F_1 - \left[F_1 \frac{L_1}{L_2} \frac{A_2}{A_1} \frac{E_2}{E_1} \right] = -F_1 \left[1 + \frac{L_1}{L_2} \frac{A_2}{A_1} \frac{E_2}{E_1} \right] = 670 \text{ kN} \qquad \text{(f)}$$

Since $L_1 = L_2$, the term $L_1/L_2 = 1$ can be eliminated and Eq. (f) simplified to

$$F_1 \left[1 + \frac{A_2}{A_1} \frac{E_2}{E_1} \right] = -670 \text{ kN} \qquad \text{(g)}$$

The gross cross-sectional area of the pier is (250 mm)^2 = 62,500 mm^2; however, the reinforcing bars take up a portion of this area. The cross-sectional area of four 19-mm-diameter reinforcing bars is

6.11 A solid circular steel shaft having an outside diameter of 35 mm is subjected to a pure torque of T = 640 N-m. The shear modulus of the steel is G = 80 GPa. Determine:
(a) the maximum shear stress in the shaft.
(b) the magnitude of the angle of twist in a 1.5-m length of shaft.

Solution

The polar moment of inertia for the shaft is

$$J = \frac{\pi}{32}(35 \text{ mm})^4 = 147{,}323.515 \text{ mm}^4$$

(a) The maximum shear stress in the steel shaft is found from the elastic torsion formula:

$$\tau_{\max} = \frac{Tc}{J} = \frac{(640 \text{ N-m})(35 \text{ mm} / 2)(1{,}000 \text{ mm/m})}{147{,}323.515 \text{ mm}^4} = 76.023 \text{ MPa} = \boxed{76.0 \text{ MPa}}$$ **Ans.**

(b) The magnitude of the angle of twist in a 1.5-m length of shaft is

$$\phi = \frac{TL}{JG} = \frac{(640 \text{ N-m})(1.5 \text{ m})(1{,}000 \text{ mm/m})^2}{(147{,}323.515 \text{ mm}^4)(80{,}000 \text{ N/mm}^2)} = 0.081453 \text{ rad} = \boxed{0.0815 \text{ rad}} = \boxed{4.67°}$$ **Ans.**

$$-T_1 + T_2 = -(-0.291784T_2) + T_2 = 1.291784T_2 = -42 \text{ kip-in.}$$

$$\therefore T_2 = -32.5132 \text{ kip-in.}$$

The torque in member (1) is therefore:

$$T_1 = T_2 + 42 \text{ kip-in.} = -32.5132 \text{ kip-in.} + 42 \text{ kip-in.} = 9.4868 \text{ kip-in.}$$

(a) Maximum Shear Stress: The maximum shear stress magnitude in member (1) is:

$$\tau_1 = \frac{T_1 c_1}{J_1} = \frac{(9.4868 \text{ kip-in.})(2.250 \text{ in.}/2)}{1.595340 \text{ in.}^4} = \boxed{6.69 \text{ ksi}}$$ **Ans.**

The maximum shear stress magnitude in member (2) is:

$$\tau_2 = \frac{T_2 c_2}{J_2} = \frac{(32.5132 \text{ kip-in.})(3.500 \text{ in.}/2)}{6.102156 \text{ in.}^4} = \boxed{9.32 \text{ ksi}}$$ **Ans.**

(b) Rotation Angle of Flange *B* Relative to Support *A*: The angle of twist in member (1) can be defined by the difference in rotation angles at the two ends; hence,

$$\phi_1 = \phi_B - \phi_A$$

Since joint *A* is restrained from rotating, $\phi_A = 0$ and thus

$$\phi_1 = \phi_B$$

The rotation angle at *B* can be determined by computing the angle of twist in member (1):

$$\phi_1 = \frac{T_1 L_1}{J_1 G_1}$$

$$= \frac{(9.4868 \text{ kip-in.})(40 \text{ in.})}{(1.595340 \text{ in.}^4)(12{,}500 \text{ ksi})}$$

$$= 0.019029 \text{ rad} = \boxed{0.01903 \text{ rad}}$$ **Ans.**

7.20 Use the graphical method to construct the shear-force and bending-moment diagrams for the beam shown. Label all significant points on each diagram and identify the maximum moments (both positive and negative) along with their respective locations. Clearly differentiate straight-line and curved portions of the diagrams.

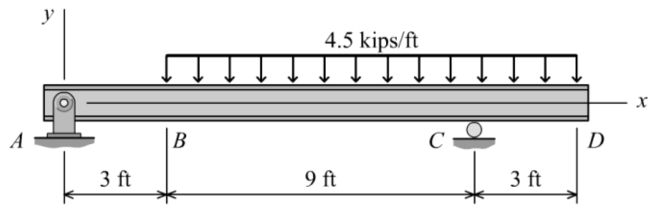


Fig. P7.20

Solution

Beam equilibrium:

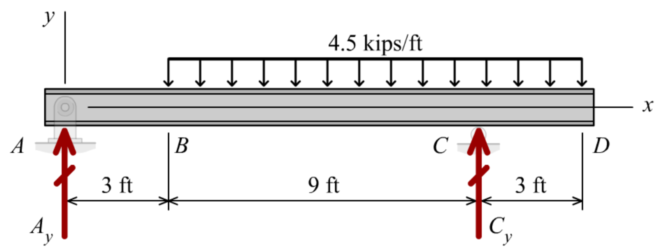
$$\Sigma M_A = -\ 4.5\text{ kips/ft}\quad 12\text{ ft}\quad 9\text{ ft}\ + C_y\ \ 12\text{ ft}\ = 0$$

$$\therefore C_y = 40.50\text{ kips}$$

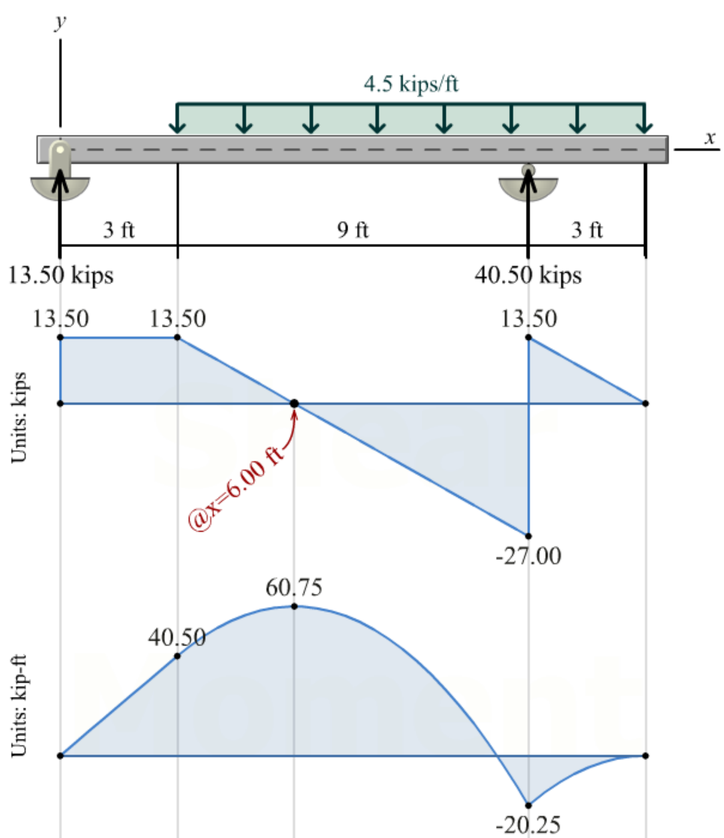
$$\Sigma F_y = A_y + C_y -\ \ 4.5\text{ kips/ft}\quad 12\text{ ft}$$

$$= A_y + 40.50\text{ kips} -\ \ 4.5\text{ kips/ft}\quad 12\text{ ft}\ = 0$$

$$\therefore A_y = 13.50\text{ kips}$$



Shear-force and bending-moment diagrams:



9.11 A 5-m long simply supported timber beam carries a uniformly distributed load of 12 kN/m, as shown in Fig. P9.11*a*. The cross-sectional dimensions of the beam are shown in Fig. P9.11*b*.
(a) At section *a*–*a*, determine the magnitude of the shear stress in the beam at point *H*.
(b) At section *a*–*a*, determine the magnitude of the shear stress in the beam at point *K*.
(c) Determine the maximum horizontal shear stress that occurs in the beam at any location within the 5-m span length.
(d) Determine the maximum compression bending stress that occurs in the beam at any location within the 5-m span length.

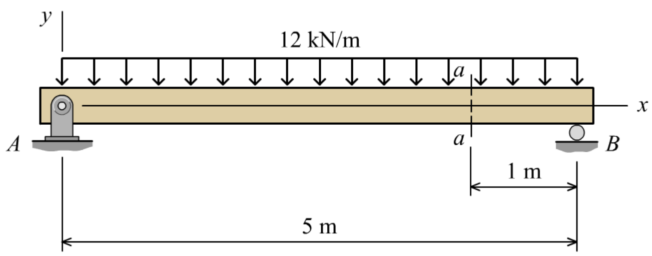


Fig. P9.11*a* Simply supported timber beam

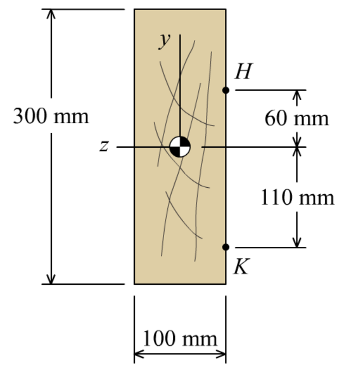
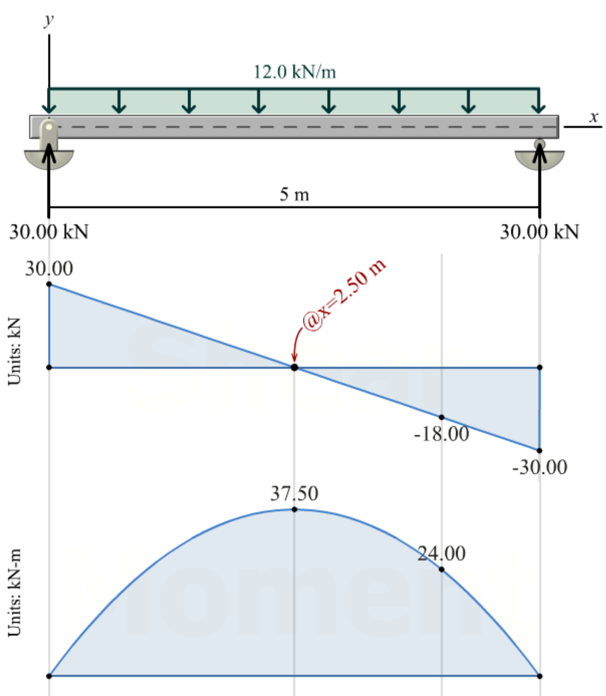


Fig. P9.11*b* Cross-sectional dimensions

Solution

Section properties:

$$I = \frac{(100\text{ mm})(300\text{ mm})^3}{12} = 225\times10^6\text{ mm}^4 \qquad t = 100\text{ mm}$$



(a) Shear stress magnitude at *H*:

$$Q = (100\text{ mm})(90\text{ mm})(105\text{ mm}) = 945{,}000\text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(18{,}000\text{ N})(945{,}000\text{ mm}^3)}{(225\times10^6\text{ mm}^4)(100\text{ mm})} = \boxed{756\text{ kPa}}$$ **Ans.**

(b) Shear stress magnitude at *K*:

$$Q = (100\text{ mm})(40\text{ mm})(130\text{ mm}) = 520{,}000\text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(18{,}000\text{ N})(520{,}000\text{ mm}^3)}{(225\times10^6\text{ mm}^4)(100\text{ mm})} = \boxed{416\text{ kPa}}$$ **Ans.**

10.18 For the beam and loading shown in Fig. P10.18, use the double-integration method to determine (a) the equation of the elastic curve for the beam, and (b) the deflection at *B*. Assume that *EI* is constant for the beam.

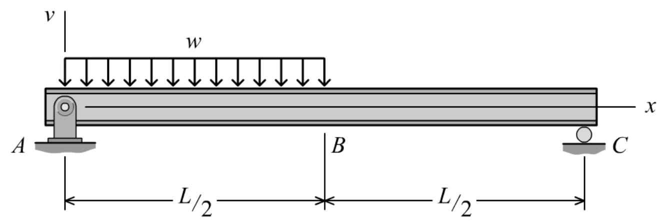
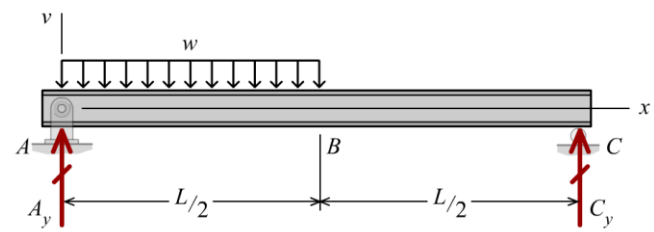


Fig. P10.18

Solution

Beam FBD:



$$\Sigma M_A = C_y(L) - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

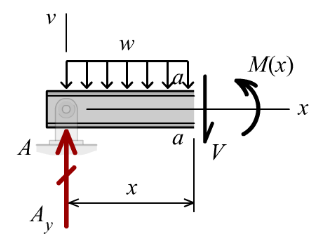
$$\therefore C_y = \frac{wL}{8}$$

$$\Sigma F_y = A_y + C_y - \frac{wL}{2} = 0$$

$$\therefore A_y = \frac{3wL}{8}$$

Consider beam segment *AB* ($0 \le x \le L/2$)

Moment equation:



$$\Sigma M_{a-a} = M(x) + \frac{wx^2}{2} - A_y x = M(x) + \frac{wx^2}{2} - \frac{3wL}{8}x = 0$$

$$\therefore M(x) = -\frac{wx^2}{2} + \frac{3wLx}{8}$$

Integration of moment equation:

$$EI\frac{d^2v}{dx^2} = M(x) = -\frac{wx^2}{2} + \frac{3wLx}{8}$$

$$EI\frac{dv}{dx} = -\frac{wx^3}{6} + \frac{3wLx^2}{16} + C_1 \qquad \text{(a)}$$

$$EI\,v = -\frac{wx^4}{24} + \frac{wLx^3}{16} + C_1 x + C_2 \qquad \text{(b)}$$

Boundary conditions:

$v = 0$ at $x = 0$

Evaluate constants:

Substitute $x = 0$ and $v = 0$ into Eq. (b) to determine $C_2 = 0$.

Slope at *B*: Let $x = L/2$ in Eq. (a).

$$EI\left.\frac{dv}{dx}\right|_B = EI\theta_B = -\frac{w(L/2)^3}{6} + \frac{3wL(L/2)^2}{16} + C_1 = -\frac{wL^3}{48} + \frac{3wL^3}{64} + C_1 = \boxed{\frac{5wL^3}{192} + C_1} \qquad \text{(c)}$$

Boundary conditions and evaluate constants:

at $x = 3\text{ m}, v = 0$

$$\therefore -\frac{420\text{ kN-m}}{2}(3\text{ m})^2 + C_1(3\text{ m}) + C_2 = 0 \tag{e}$$

at $x = 9\text{ m}, v = 0$

$$\therefore -\frac{420\text{ kN-m}}{2}(9\text{ m})^2 + \frac{B_y}{6}(6\text{ m})^3 - \frac{60\text{ kN/m}}{24}(6\text{ m})^4 + C_1(9\text{ m}) + C_2 = 0 \tag{f}$$

at $x = 15\text{ m}, v = 0$

$$\therefore -\frac{420\text{ kN-m}}{2}(15\text{ m})^2 + \frac{B_y}{6}(12\text{ m})^3 - \frac{60\text{ kN/m}}{24}(12\text{ m})^4$$
$$+\frac{C_y}{6}(6\text{ m})^3 + \frac{60\text{ kN/m}}{24}(6\text{ m})^4 + C_1(15\text{ m}) + C_2 = 0 \tag{g}$$

(a) Solve for B_y, C_y, and D_y:
Solve equations (a), (b), (e), (f), and (g) simultaneously to obtain:

$C_1 = 1{,}590.0000\text{ kN-m}^2$

$C_2 = -2{,}880.0000\text{ kN-m}^3$

$B_y = 245.0000\text{ kN} = \boxed{245\text{ kN}\uparrow}$

$C_y = 120.0000\text{ kN} = \boxed{120\text{ kN}\uparrow}$

$D_y = -5.0000\text{ kN} = \boxed{5.00\text{ kN}\downarrow}$ **Ans.**

(b) Beam deflection at A: From Eq. (d), the beam deflection at A (x = 0 m) is computed as follows:

$EI\, v_A = -2{,}880.0000\text{ kN-m}^3$

$$\therefore v_A = -\frac{2{,}880.0000\text{ kN-m}^3}{200{,}000\text{ kN-m}^2} = -0.014400\text{ m} = -14.40\text{ mm} = \boxed{14.40\text{ mm}\downarrow}$$ **Ans.**