Solutions Manual to Accompany

CLASSICAL GEOMETRY

Euclidean, Transformational, Inversive, and Projective

I. E. LEONARD
J. E. LEWIS
A. C. F. LIU
G. W. TOKARSKY

WILEY

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and since $2\alpha + 2\beta + z = 180 - z$, then

$$\cos(2\alpha + 2\beta + z) = \cos(180 - z) = -\cos z.$$

Also, since $AB = AC$ and $AD = AE$,

$$BE^2 = AB^2 + AE^2 + 2AB \cdot AE \cos z$$
$$CD^2 = AB^2 + AE^2 - 2AB \cdot AE \cos z.$$

Adding these two equations, we have

$$BE^2 + CD^2 = 2AB^2 + 2AE^2;$$

that is,

$$CD^2 = 2AB^2 + 2AE^2 - BE^2.$$

Now, M is the midpoint of BE in $\triangle ABE$, and from Apollonius' Theorem, we have

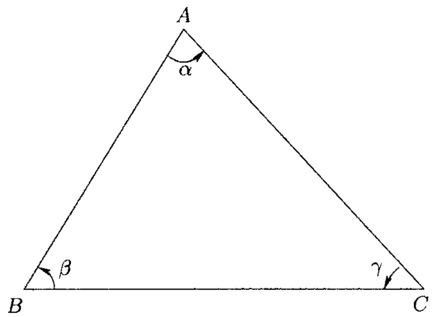
$$AB^2 + AE^2 = 2AM^2 + 2BM^2 = 2AM^2 + \frac{1}{2}BE^2,$$

and

$$CD^2 = 2AB^2 + 2AE^2 - BE^2 = 4AM^2;$$

that is, $CD = 2AM$.

Solution. In the figure,



we have

$$R_{B,2\beta}\,R_{C,2\gamma} = R_{A,2(\beta+\gamma)},$$

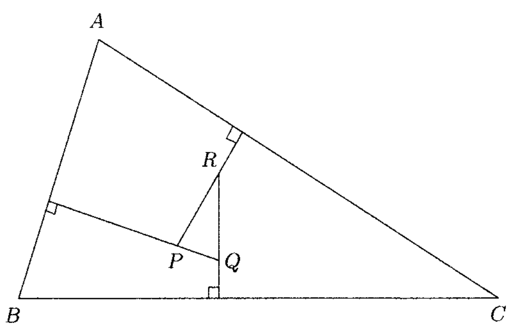
so that

$$R_{A,2\alpha}\,R_{B,2\beta}\,R_{C,2\gamma} = R_{A,2\alpha}\,R_{A,2(\beta+\gamma)} = R_{A,2(\alpha+\beta+\gamma)} = R_{A,360} = I.$$

Similarly,

$$R_{C,2\gamma}\,R_{A,2\alpha}\,R_{B,2\beta} = I \qquad \text{and} \qquad R_{B,2\beta}\,R_{C,2\gamma}\,R_{A,2\alpha} = I.$$

25. Let perpendiculars erected at arbitrary points on the sides of triangle $\triangle ABC$ meet in pairs at points P, Q, and R. Show that the triangle PQR is similar to the given triangle.



Solution. A 90° rotation of $\triangle ABC$ into $\triangle A'B'C'$ leaves the sides of $\triangle A'B'C'$ parallel to the corresponding sides of $\triangle PRQ$.

By the AAA similarity theorem,

$$\triangle A'B'C' \sim \triangle PQR,$$

d is a fixed line, F is a fixed point in the plane not on the line, dist(X, d) is the perpendicular distance from the point X to the line d, and ϵ is a fixed positve constant.

To obtain the Cartesian equations of the conic, we assume that the focus F is at the point $(0, 0)$, d is a vertical line perpendicular to the x-axis through the point $(-r, 0)$. The Cartesian equations of the conic are then

$$\left(x - \frac{r\epsilon^2}{1-\epsilon^2}\right)^2 + \frac{y^2}{1-\epsilon^2} = \left(\frac{r\epsilon}{1-\epsilon^2}\right)^2.$$

This is of the form

$$\frac{(x-f)^2}{a^2} + \frac{y^2}{b^2} = 1.$$

provided

$$a^2 = \left(\frac{r\epsilon}{1-\epsilon^2}\right)^2$$

$$b^2 = (1-\epsilon^2)\left(\frac{r\epsilon}{1-\epsilon^2}\right)^2.$$

Dividing the second of these two equations by the first, we get

$$\frac{b^2}{a^2} = \frac{(1-\epsilon^2)\left(\frac{r\epsilon}{1-\epsilon^2}\right)^2}{\left(\frac{r\epsilon}{1-\epsilon^2}\right)^2},$$

that is,

$$\frac{b^2}{a^2} = 1 - \epsilon^2,$$

and solving for ϵ we get

$$\epsilon = \sqrt{1 - \frac{b^2}{a^2}}.$$

Substituting this into the first equation, and solving for r we get

$$r = \frac{b^2}{\sqrt{a^2 - b^2}}.$$

13. Find the Cartesian equations of the following conic sections:

(a) foci: $(\pm 8, 0)$, $e = 0.2$.

(b) foci: $(\pm 4, 0)$, directrix: $x = \dfrac{16}{3}$.