Contents

Before Calculus

Exercise Set 0.1

1. **(a)** $-2.9, -2.0, 2.35, 2.9$ **(b)** None **(c)** $y = 0$ **(d)** $-1.75 \le x \le 2.15$, $x = -3$, $x = 3$

(e) $y_{\text{max}} = 2.8$ at $x = -2.6$; $y_{\text{min}} = -2.2$ at $x = 1.2$

2. **(a)** $x = -1, 4$ **(b)** None **(c)** $y = -1$ **(d)** $x = 0, 3, 5$

(e) $y_{\text{max}} = 9$ at $x = 6$; $y_{\text{min}} = -2$ at $x = 0$

3. **(a)** Yes **(b)** Yes **(c)** No (vertical line test fails) **(d)** No (vertical line test fails)

4. **(a)** The natural domain of f is $x \neq -1$, and for g it is the set of all x. $f(x) = g(x)$ on the intersection of their domains.

(b) The domain of f is the set of all $x \ge 0$; the domain of g is the same, and $f(x) = g(x)$.

5. **(a)** 1999, \$47,700 **(b)** 1993, \$41,600

(c) The slope between 2000 and 2001 is steeper than the slope between 2001 and 2002, so the median income was declining more rapidly during the first year of the 2-year period.

6. **(a)** In thousands, approximately $\dfrac{47.7 - 41.6}{6} = \dfrac{6.1}{6}$ per yr, or \$1017/yr.

(b) From 1993 to 1996 the median income increased from \$41.6K to \$44K (K for 'kilodollars'; all figures approximate); the average rate of increase during this time was $(44 - 41.6)/3$ K/yr $= 2.4/3$ K/yr $=$ \$800/year. From 1996 to 1999 the average rate of increase was $(47.7 - 44)/3$ K/yr $= 3.7/3$ K/yr $\approx$ \$1233/year. The increase was larger during the last 3 years of the period.

(c) 1994 and 2005.

7. **(a)** $f(0) = 3(0)^2 - 2 = -2$; $f(2) = 3(2)^2 - 2 = 10$; $f(-2) = 3(-2)^2 - 2 = 10$; $f(3) = 3(3)^2 - 2 = 25$; $f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$; $f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$.

(b) $f(0) = 2(0) = 0$; $f(2) = 2(2) = 4$; $f(-2) = 2(-2) = -4$; $f(3) = 2(3) = 6$; $f(\sqrt{2}) = 2\sqrt{2}$; $f(3t) = 1/(3t)$ for $t > 1$ and $f(3t) = 6t$ for $t \le 1$.

8. **(a)** $g(3) = \dfrac{3+1}{3-1} = 2$; $g(-1) = \dfrac{-1+1}{-1-1} = 0$; $g(\pi) = \dfrac{\pi+1}{\pi-1}$; $g(-1.1) = \dfrac{-1.1+1}{-1.1-1} = \dfrac{-0.1}{-2.1} = \dfrac{1}{21}$; $g(t^2-1) = \dfrac{t^2-1+1}{t^2-1-1} = \dfrac{t^2}{t^2-2}$.

(b) $g(3) = \sqrt{3+1} = 2$; $g(-1) = 3$; $g(\pi) = \sqrt{\pi+1}$; $g(-1.1) = 3$; $g(t^2-1) = 3$ if $t^2 < 2$ and $g(t^2-1) = \sqrt{t^2-1+1} = |t|$ if $t^2 \ge 2$.

9. **(a)** Natural domain: $x \neq 3$. Range: $y \neq 0$. **(b)** Natural domain: $x \neq 0$. Range: $\{1, -1\}$.

(c) Natural domain: $x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$. Range: $y \geq 0$.

(d) $x^2 - 2x + 5 = (x-1)^2 + 4 \geq 4$. So $G(x)$ is defined for all x, and is $\geq \sqrt{4} = 2$. Natural domain: all x. Range: $y \geq 2$.

(e) Natural domain: $\sin x \neq 1$, so $x \neq (2n+\frac{1}{2})\pi$, $n = 0, \pm 1, \pm 2, \ldots$. For such x, $-1 \leq \sin x < 1$, so $0 < 1 - \sin x \leq 2$, and $\frac{1}{1-\sin x} \geq \frac{1}{2}$. Range: $y \geq \frac{1}{2}$.

(f) Division by 0 occurs for $x = 2$. For all other x, $\frac{x^2-4}{x-2} = x + 2$, which is nonnegative for $x \geq -2$. Natural domain: $[-2, 2) \cup (2, +\infty)$. The range of $\sqrt{x+2}$ is $[0, +\infty)$. But we must exclude $x = 2$, for which $\sqrt{x+2} = 2$. Range: $[0, 2) \cup (2, +\infty)$.

10. **(a)** Natural domain: $x \leq 3$. Range: $y \geq 0$. **(b)** Natural domain: $-2 \leq x \leq 2$. Range: $0 \leq y \leq 2$.

(c) Natural domain: $x \geq 0$. Range: $y \geq 3$. **(d)** Natural domain: all x. Range: all y.

(e) Natural domain: all x. Range: $-3 \leq y \leq 3$.

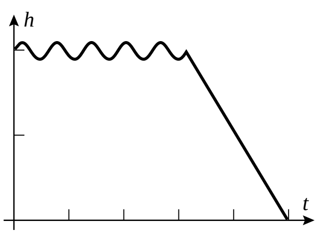
(f) For $\sqrt{x}$ to exist, we must have $x \geq 0$. For $H(x)$ to exist, we must also have $\sin \sqrt{x} \neq 0$, which is equivalent to $\sqrt{x} \neq \pi n$ for $n = 0, 1, 2, \ldots$. Natural domain: $x > 0$, $x \neq (\pi n)^2$ for $n = 1, 2, \ldots$. For such x, $0 < |\sin \sqrt{x}| \leq 1$, so $0 < (\sin \sqrt{x})^2 \leq 1$ and $H(x) \geq 1$. Range: $y \geq 1$.

11. **(a)** The curve is broken whenever someone is born or someone dies.

(b) C decreases for eight hours, increases rapidly (but continuously), and then repeats.

12. **(a)** Yes. The temperature may change quickly under some conditions, but not instantaneously.

(b) No; the number is always an integer, so the changes are in movements (jumps) of at least one unit.

13.


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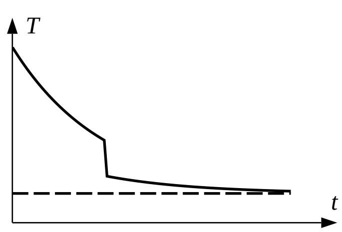
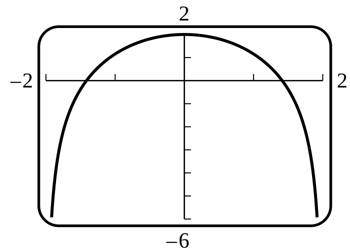


15. Yes. $y = \sqrt{25 - x^2}$.

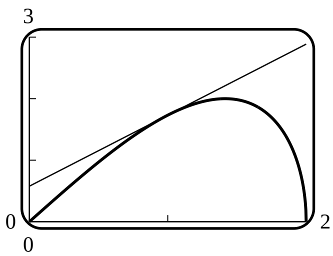
16. Yes. $y = -\sqrt{25 - x^2}$.

17. Yes. $y = \begin{cases} \sqrt{25 - x^2}, & -5 \leq x \leq 0 \\ -\sqrt{25 - x^2}, & 0 < x \leq 5 \end{cases}$

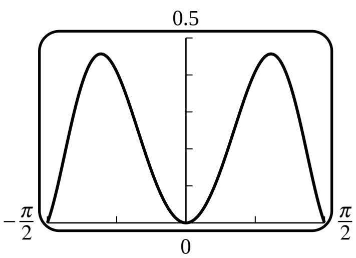


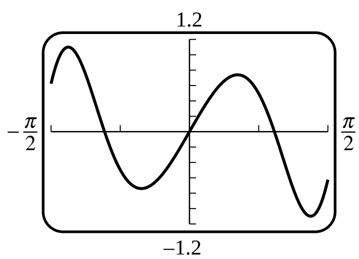
(c) $f'(x) = x\dfrac{-x}{\sqrt{4-x^2}} + \sqrt{4-x^2} = \dfrac{4-2x^2}{\sqrt{4-x^2}}$.

(d) $f(1) = \sqrt{3}$ and $f'(1) = \dfrac{2}{\sqrt{3}}$ so the tangent line has the equation $y - \sqrt{3} = \dfrac{2}{\sqrt{3}}(x-1)$.



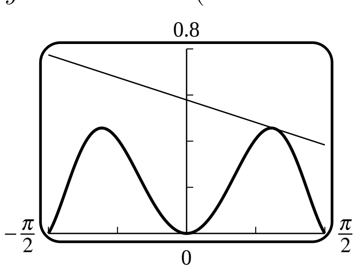
60. (a)





(c) $f'(x) = 2x\cos(x^2)\cos x - \sin x \sin(x^2)$.

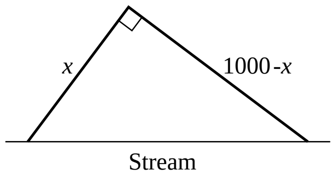
(d) $f(1) = \sin 1 \cos 1$ and $f'(1) = 2\cos^2 1 - \sin^2 1$, so the tangent line has the equation $y - \sin 1 \cos 1 = (2\cos^2 1 - \sin^2 1)(x-1)$.



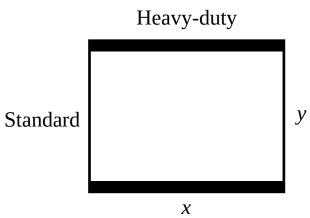
61. False. $\dfrac{d}{dx}[\sqrt{y}] = \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

62. False. $dy/dx = f'(u)g'(x) = f'(g(x))\,g'(x)$.

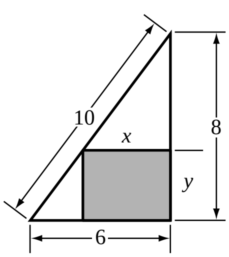
63. False. $dy/dx = -\sin[g(x)]\,g'(x)$.



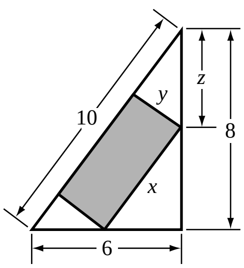
5. Let x and y be the dimensions shown in the figure and A the area, then $A = xy$ subject to the cost condition $3(2x) + 2(2y) = 6000$, or $y = 1500 - 3x/2$. Thus $A = x(1500 - 3x/2) = 1500x - 3x^2/2$ for x in $[0, 1000]$. $dA/dx = 1500 - 3x$, $dA/dx = 0$ when $x = 500$. If $x = 0$ or 1000 then $A = 0$, if $x = 500$ then $A = 375,000$ so the area is greatest when $x = 500$ ft and (from $y = 1500 - 3x/2$) when $y = 750$ ft.



6. Let x and y be the dimensions shown in the figure and A the area of the rectangle, then $A = xy$ and, by similar triangles, $x/6 = (8 - y)/8$, $y = 8 - 4x/3$ so $A = x(8 - 4x/3) = 8x - 4x^2/3$ for x in $[0, 6]$. $dA/dx = 8 - 8x/3$, $dA/dx = 0$ when $x = 3$. If $x = 0, 3, 6$ then $A = 0, 12, 0$ so the area is greatest when $x = 3$ in and (from $y = 8-4x/3$) $y = 4$ in.



7. Let x, y, and z be as shown in the figure and A the area of the rectangle, then $A = xy$ and, by similar triangles, $z/10 = y/6$, $z = 5y/3$; also $x/10 = (8 - z)/8 = (8 - 5y/3)/8$ thus $y = 24/5 - 12x/25$ so $A = x(24/5 - 12x/25) = 24x/5 - 12x^2/25$ for x in $[0, 10]$. $dA/dx = 24/5 - 24x/25$, $dA/dx = 0$ when $x = 5$. If $x = 0, 5, 10$ then $A = 0, 12, 0$ so the area is greatest when $x = 5$ in and $y = 12/5$ in.



8. $A = (2x)y = 2xy$ where $y = 16 - x^2$ so $A = 32x - 2x^3$ for $0 \le x \le 4$; $dA/dx = 32 - 6x^2$, $dA/dx = 0$ when $x = 4/\sqrt{3}$. If $x = 0, 4/\sqrt{3}, 4$ then $A = 0, 256/(3\sqrt{3}), 0$ so the area is largest when $x = 4/\sqrt{3}$ and $y = 32/3$. The dimensions of the rectangle with largest area are $8/\sqrt{3}$ by $32/3$.

50. $\dfrac{x}{\cos x^2}$

51. $|x-1|$

52. $\cos\sqrt{x}$

53. $\dfrac{\cos x}{1+\sin^3 x}$

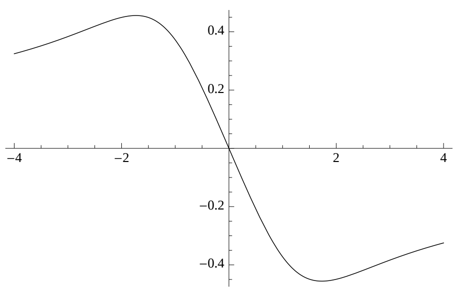
54. $\dfrac{(\ln\sqrt{x})^2}{2\sqrt{x}}$

56. **(a)** $F'(x) = \dfrac{x^2-3}{x^4+7}$; increasing on $(-\infty,-\sqrt{3}], [\sqrt{3},+\infty)$, decreasing on $[-\sqrt{3},\sqrt{3}]$.

(b) $F''(x) = \dfrac{-2x^5+12x^3+14x}{(x^4+7)^2}$; concave up on $(-\infty,-\sqrt{7})$, $(0,\sqrt{7})$ concave down on $(-\sqrt{7},0)$, $(\sqrt{7},\infty)$.

(c) Absolute maximum at $x=-\sqrt{3}$, absolute minimum at $x=\sqrt{3}$.

(d)



57. **(a)** $F'(x) = \dfrac{1}{1+x^2} + \dfrac{1}{1+(1/x)^2}(-1/x^2) = 0$ so F is constant on $(0,+\infty)$.

(b) $F(1) = \displaystyle\int_0^1 \frac{1}{1+t^2}\,dt + \int_0^1 \frac{1}{1+t^2}\,dt = 2\tan^{-1} 1 = \pi/2$, so $F(x) = \tan^{-1} x + \tan^{-1}(1/x) = \pi/2$.

58. $(-3,3)$ because f is continuous there and 1 is in $(-3,3)$.

59. **(a)** The domain is $(-\infty,+\infty)$; $F(x)$ is 0 if $x=1$, positive if $x>1$, and negative if $x<1$, because the integrand is positive, so the sign of the integral depends on the orientation (forwards or backwards).

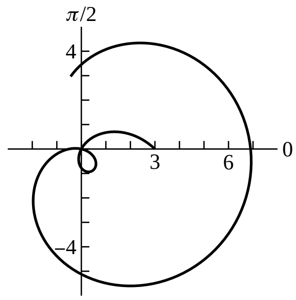
(b) The domain is $[-2,2]$; $F(x)$ is 0 if $x=-1$, positive if $-1<x\le 2$, and negative if $-2\le x<-1$; same reasons as in part (a).

60. $F(x) = \displaystyle\int_{-1}^{x} \frac{t}{\sqrt{2+t^3}}dt$, $F'(x) = \dfrac{x}{\sqrt{2+x^3}}$, so F is increasing on $[1,3]$; $F_{\max} = F(3) \approx 1.152082854$ and $F_{\min} = F(1) \approx -0.07649493141$.

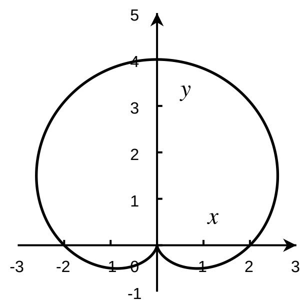
61. **(a)** $f_{\text{ave}} = \dfrac{1}{3}\displaystyle\int_0^3 x^{1/2}dx = 2\sqrt{3}/3$; $\sqrt{x^*} = 2\sqrt{3}/3$, $x^* = \dfrac{4}{3}$.

(b) $f_{\text{ave}} = \dfrac{1}{e-1}\displaystyle\int_1^e \frac{1}{x}dx = \frac{1}{e-1}\ln x\Big]_1^e = \frac{1}{e-1}$; $\dfrac{1}{x^*} = \dfrac{1}{e-1}$, $x^* = e-1$.

62. Mar 1 to Jun 7 is 14 weeks, so $w(t) = 10 + \displaystyle\int_0^t \frac{s}{7}\,ds = 10 + \frac{t^2}{14}$, so the weight on June 7 will be 24 gm.



15. $r = 3 - 4\sin\left(\frac{\pi}{4}\theta\right)$.



16. $r = 2 + 2\sin\theta$.

17. (a) $r = 5$.

(b) $(x-3)^2 + y^2 = 9,\ r = 6\cos\theta$.

(c) Example 8, $r = 1 - \cos\theta$.

18. (a) From (8-9), $r = a \pm b\sin\theta$ or $r = a \pm b\cos\theta$. The curve is not symmetric about the y-axis, so Theorem 10.2.1(b) eliminates the sine function, thus $r = a \pm b\cos\theta$. The cartesian point $(-3,0)$ is either the polar point $(3,\pi)$ or $(-3,0)$, and the cartesian point $(-1,0)$ is either the polar point $(1,\pi)$ or $(-1,0)$. A solution is $a = 1, b = -2$; we may take the equation as $r = 1 - 2\cos\theta$.

(b) $x^2 + (y+3/2)^2 = 9/4, r = -3\sin\theta$.

(c) Figure 10.2.19, $a = 1, n = 3, r = \sin 3\theta$.

19. (a) Figure 10.2.19, $a = 3, n = 2, r = 3\sin 2\theta$.

(b) From (8-9), symmetry about the y-axis and Theorem 10.2.1(b), the equation is of the form $r = a \pm b\sin\theta$. The cartesian points $(3,0)$ and $(0,5)$ give $a = 3$ and $5 = a + b$, so $b = 2$ and $r = 3 + 2\sin\theta$.

(c) Example 9, $r^2 = 9\cos 2\theta$.

20. (a) Example 8 rotated through $\pi/2$ radians: $a = 3, r = 3 - 3\sin\theta$.

(b) Figure 10.2.19, $a = 1, r = \cos 5\theta$.

(c) $x^2 + (y-2)^2 = 4,\ r = 4\sin\theta$.

49. (a) $2t - t^2 - 3t = -2$, $t^2 + t - 2 = 0$, $(t+2)(t-1) = 0$ so $t = -2, 1$. The points of intersection are $(-2, 4, 6)$ and $(1, 1, -3)$.

(b) $\mathbf{r}' = \mathbf{i} + 2t\mathbf{j} - 3\mathbf{k}$; $\mathbf{r}'(-2) = \mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{r}'(1) = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, and $\mathbf{n} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ is normal to the plane. Let θ be the acute angle, then for $t = -2$: $\cos\theta = |\mathbf{n} \cdot \mathbf{r}'|/(\|\mathbf{n}\| \, \|\mathbf{r}'\|) = 3/\sqrt{156}$, $\theta \approx 76°$; for $t = 1$: $\cos\theta = |\mathbf{n} \cdot \mathbf{r}'|/(\|\mathbf{n}\| \, \|\mathbf{r}'\|) = 3/\sqrt{84}$, $\theta \approx 71°$.

50. $\mathbf{r}' = -2e^{-2t}\mathbf{i} - \sin t\mathbf{j} + 3\cos t\mathbf{k}$, $t = 0$ at the point $(1, 1, 0)$ so $\mathbf{r}'(0) = -2\mathbf{i} + 3\mathbf{k}$ and hence the tangent line is $x = 1 - 2t$, $y = 1$, $z = 3t$. But $x = 0$ in the yz-plane so $1 - 2t = 0$, $t = 1/2$. The point of intersection is $(0, 1, 3/2)$.

51. $\mathbf{r}_1(1) = \mathbf{r}_2(2) = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$ so the graphs intersect at P; $\mathbf{r}_1'(t) = 2t\mathbf{i} + \mathbf{j} + 9t^2\mathbf{k}$ and $\mathbf{r}_2'(t) = \mathbf{i} + \dfrac{1}{2}t\mathbf{j} - \mathbf{k}$ so $\mathbf{r}_1'(1) = 2\mathbf{i} + \mathbf{j} + 9\mathbf{k}$ and $\mathbf{r}_2'(2) = \mathbf{i} + \mathbf{j} - \mathbf{k}$ are tangent to the graphs at P, thus $\cos\theta = \dfrac{\mathbf{r}_1'(1) \cdot \mathbf{r}_2'(2)}{\|\mathbf{r}_1'(1)\| \, \|\mathbf{r}_2'(2)\|} = -\dfrac{6}{\sqrt{86}\sqrt{3}}$, $\theta = \cos^{-1}(6/\sqrt{258}) \approx 68°$.

52. $\mathbf{r}_1(0) = \mathbf{r}_2(-1) = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ so the graphs intersect at P; $\mathbf{r}_1'(t) = -2e^{-t}\mathbf{i} - (\sin t)\mathbf{j} + 2t\mathbf{k}$ and $\mathbf{r}_2'(t) = -\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}$ so $\mathbf{r}_1'(0) = -2\mathbf{i}$ and $\mathbf{r}_2'(-1) = -\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ are tangent to the graphs at P, thus $\cos\theta = \dfrac{\mathbf{r}_1'(0) \cdot \mathbf{r}_2'(-1)}{\|\mathbf{r}_1'(0)\| \, \|\mathbf{r}_2'(-1)\|} = \dfrac{1}{\sqrt{14}}$, $\theta \approx 74°$.

53. $\dfrac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t) + \mathbf{r}'(t) \times \mathbf{r}'(t) = \mathbf{r}(t) \times \mathbf{r}''(t) + \mathbf{0} = \mathbf{r}(t) \times \mathbf{r}''(t)$.

54. $\dfrac{d}{dt}[\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})] = \mathbf{u} \cdot \dfrac{d}{dt}[\mathbf{v} \times \mathbf{w}] + \dfrac{d\mathbf{u}}{dt} \cdot [\mathbf{v} \times \mathbf{w}] = \mathbf{u} \cdot \left(\mathbf{v} \times \dfrac{d\mathbf{w}}{dt} + \dfrac{d\mathbf{v}}{dt} \times \mathbf{w}\right) + \dfrac{d\mathbf{u}}{dt} \cdot [\mathbf{v} \times \mathbf{w}] = \mathbf{u} \cdot \left[\mathbf{v} \times \dfrac{d\mathbf{w}}{dt}\right] + \mathbf{u} \cdot \left[\dfrac{d\mathbf{v}}{dt} \times \mathbf{w}\right] + \dfrac{d\mathbf{u}}{dt} \cdot [\mathbf{v} \times \mathbf{w}]$.

55. In Exercise 54, write each scalar triple product as a determinant.

56. Let $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j}$, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $\mathbf{r}_1(t) = x_1(t)\mathbf{i} + y_1(t)\mathbf{j}$, $\mathbf{r}_2(t) = x_2(t)\mathbf{i} + y_2(t)\mathbf{j}$ and use properties of derivatives.

57. Let $\mathbf{r}_1(t) = x_1(t)\mathbf{i} + y_1(t)\mathbf{j} + z_1(t)\mathbf{k}$ and $\mathbf{r}_2(t) = x_2(t)\mathbf{i} + y_2(t)\mathbf{j} + z_2(t)\mathbf{k}$, in both (6) and (7); show that the left and right members of the equalities are the same.

58. (a) $\displaystyle\int k\mathbf{r}(t)\,dt = \int k(x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k})\,dt = k\int x(t)\,dt\,\mathbf{i} + k\int y(t)\,dt\,\mathbf{j} + k\int z(t)\,dt\,\mathbf{k} = k\int \mathbf{r}(t)\,dt$.

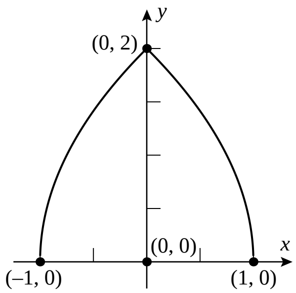
(b) Similar to part (a).

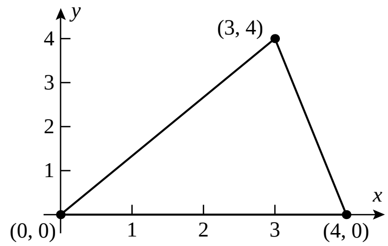
(c) Use part (a) and part (b) with $k = -1$.

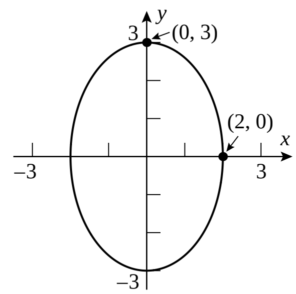
59. See discussion after Definition 12.2.3.

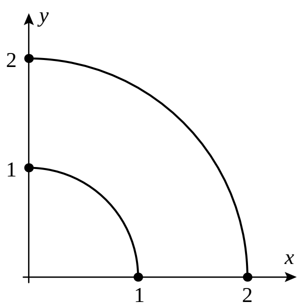
60. $\mathbf{r}'(t) = \langle 2t, 3t^2\rangle$, $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 3t^5 + 2t^3 + 3t^2$, $\|\mathbf{r}(t)\| = \sqrt{t^4 + (t^3+1)^2}$, $\|\mathbf{r}'(t)\| = \sqrt{4t^2 + 9t^4}$, so $\theta = \cos^{-1}\dfrac{3t^5 + 2t^3 + 3t^2}{\sqrt{t^4 + (t^3+1)^2}\sqrt{4t^2 + 9t^4}}$. For large negative values of t, the position and tangent vectors point in almost opposite directions, so θ is almost π. θ decreases until $t \approx -0.439$ and then increases again, approaching $\pi/2$ as $t \to 0^-$. There is a cusp in the graph at $t = 0$, so the tangent vector and θ are undefined there. As t increases from 0 to $\sqrt[3]{2}$, θ decreases; at $t = \sqrt[3]{2}$ the tangent line to the curve passes through the origin so $\theta = 0$ there. θ increases again until $t \approx 2.302$ and then decreases again, approaching 0 as $t \to +\infty$.

16. True. See the solution of Exercise 14.7.48(b).

17. 

18. 

19. 

20. 

21. $x = \dfrac{1}{5}u + \dfrac{2}{5}v,\ y = -\dfrac{2}{5}u + \dfrac{1}{5}v,\ \dfrac{\partial(x,y)}{\partial(u,v)} = \dfrac{1}{5};\ \dfrac{1}{5}\displaystyle\iint_S \frac{u}{v}\, dA_{uv} = \frac{1}{5}\int_1^3 \int_1^4 \frac{u}{v}\, du\, dv = \frac{3}{2}\ln 3.$

22. $x = \dfrac{1}{2}u + \dfrac{1}{2}v,\ y = \dfrac{1}{2}u - \dfrac{1}{2}v,\ \dfrac{\partial(x,y)}{\partial(u,v)} = -\dfrac{1}{2};\ \dfrac{1}{2}\displaystyle\iint_S ve^{uv}\, dA_{uv} = \frac{1}{2}\int_1^4 \int_0^1 ve^{uv}\, du\, dv = \frac{1}{2}(e^4 - e - 3).$

23. $x = u + v,\ y = u - v,\ \dfrac{\partial(x,y)}{\partial(u,v)} = -2$; the boundary curves of the region S in the uv-plane are $v = 0, v = u$, and $u = 1$ so $2\displaystyle\iint_S \sin u \cos v\, dA_{uv} = 2\int_0^1 \int_0^u \sin u \cos v\, dv\, du = 1 - \frac{1}{2}\sin 2.$

24. $x = \sqrt{v/u},\ y = \sqrt{uv}$ so, from Example 3, $\dfrac{\partial(x,y)}{\partial(u,v)} = -\dfrac{1}{2u}$; the boundary curves of the region S in the uv-plane are $u = 1, u = 3, v = 1$, and $v = 4$ so $\displaystyle\iint_S uv^2 \left(\frac{1}{2u}\right) dA_{uv} = \frac{1}{2}\int_1^4 \int_1^3 v^2 du\, dv = 21.$

25. $x = 3u,\ y = 4v,\ \dfrac{\partial(x,y)}{\partial(u,v)} = 12$; S is the region in the uv-plane enclosed by the circle $u^2 + v^2 = 1$. Use polar coordinates to obtain $\displaystyle\iint_S 12\sqrt{u^2+v^2}(12)\, dA_{uv} = 144\int_0^{2\pi} \int_0^1 r^2\, dr\, d\theta = 96\pi.$

26. $x = 2u, y = v,\ \dfrac{\partial(x,y)}{\partial(u,v)} = 2$; S is the region in the uv-plane enclosed by the circle $u^2+v^2 = 1$. Use polar coordinates to obtain $\displaystyle\iint_S e^{-(4u^2+4v^2)}(2)\, dA_{uv} = 2\int_0^{2\pi} \int_0^1 re^{-4r^2} dr\, d\theta = \frac{\pi}{2}(1 - e^{-4}).$

27. Let S be the region in the uv-plane bounded by $u^2 + v^2 = 1$, so $u = 2x, v = 3y, x = u/2, y = v/3,\ \dfrac{\partial(x,y)}{\partial(u,v)} =$