

# MODERN PHYSICS

The background of the cover is a complex, abstract design. It features a dark blue and purple color palette. Overlaid on this are various geometric shapes, including circles, ellipses, and polygons, some of which are semi-transparent. A network of glowing yellow and green lines, resembling particle tracks or orbits, crisscrosses the entire cover. These lines are of varying thickness and brightness, creating a sense of dynamic movement. The overall effect is one of scientific complexity and modern aesthetics.

THIRD EDITION

**INSTRUCTOR SOLUTIONS MANUAL**

**KENNETH KRANE**

Instructor's Manual  
to accompany  
Modern Physics, 3<sup>rd</sup> Edition

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## Preface

This Instructor's Manual accompanies the 3<sup>rd</sup> edition of the textbook *Modern Physics* (John Wiley & Sons, 2012). It includes (1) explanatory material for each chapter; (2) suggested outside readings for instructor or student; (3) references to web sites or other generally available simulations of phenomena; (4) exercises that can be used in various active-engagement classroom strategies; (5) sample exam questions; and (6) complete solutions to the end-of-chapter problems in the text.

Perhaps the greatest influence on my teaching in the time since the publication of the 2<sup>nd</sup> edition of this textbook (1996) has been the growth into maturity of the field of physics education research (PER). Rather than indicating specific areas of misunderstanding, PER has demonstrated that student comprehension is enhanced by any of a number of interactive techniques that are designed to engage the students and make them active participants in the learning process. The demonstrated learning improvements are robust and replicable, and they transcend differences among instructors and institutional types. In my own trajectory in this process, I have been especially influenced by the work of Lillian McDermott and her group at the University of Washington<sup>1</sup> and Eric Mazur at Harvard University.<sup>2</sup> I am grateful to them not only for their contributions to PER but also for their friendship over the years.

With the support of a Course, Curriculum, and Laboratory Improvement grant from the National Science Foundation<sup>3</sup>, I have developed and tested a set of exercises that can be used either in class as group activities or outside of class (for example, in a Peer Instruction mode following Mazur's format or in a Just-In-Time Teaching<sup>4</sup> mode). These exercises are included in this Instructor's Manual. I am grateful for the support of the National Science Foundation in enabling this project to be carried out. Two Oregon State University graduate students assisted in the implementations of these reformed teaching methods: K. C. Walsh helped with producing several simulations and illustrative materials, with implementing an interactive web site, and with corresponding developments in the laboratory that accompanies our course, and Pornrat Wattanakasiwich undertook a PER project<sup>5</sup> for her Ph.D. that involved the observation of student reasoning about probability, which lies at the heart of most topics in modern physics.

One of the major themes that has emerged from PER in the past two decades is that students can often learn successful algorithms for solving problems while lacking a fundamental understanding of the underlying concepts. The importance of the in-class or pre-class exercises is to force students to consider these concepts and to apply them to diverse situations that often cannot be analyzed with an equation. It is absolutely essential to devote class time to these exercises and to follow through with exam questions that require similar analysis and a similar articulation of the conceptual reasoning. I strongly believe that conceptual understanding is a necessary prerequisite to successful problem solving. In my own classes at Oregon State University I have repeatedly observed that improved conceptual understanding leads directly to improved problem-solving skills.

In training students to reason conceptually, it is necessary to force them to verbalize their reasons for selecting a particular answer to a conceptual or qualitative question, and you will learn much from listening to or reading their arguments. A simple

multiple-choice conceptual question, either as a class exercise or a test problem, gives you insufficient insight into the students' reasoning patterns unless you also ask them to justify their choice. Even when I have teaching assistants grade the exams in my class, I always grade the conceptual questions myself, if only to gather insight into how students reason. To save time I generally grade such questions with either full credit (correct choice of answer and more-or-less correct reasoning) or no credit (wrong choice or correct choice with incorrect reasoning).

Here's an example of why it is necessary to require students to provide conceptual arguments. After a unit on the Schrödinger equation, I gave the following conceptual test question: Consider a particle in the first excited state of a one-dimensional infinite potential energy well that extends from  $x = 0$  to  $x = L$ . At what locations is the particle most likely to be found? The students were required to state an answer and to give their reasoning. One student drew a nice sketch of the probability density in the first excited state, correctly showing maxima at  $x = L/4$  and  $x = 3L/4$ , and stated that those locations were the most likely ones at which to find the particle. Had I not required the reasoning, the student would have received full credit, and I would have been satisfied with the student's understanding of the material. However, in stating the reasoning, the student demonstrated what turned out to be a surprisingly common incorrect mode of reasoning. The student apparently confused the graph of probability density with a similar sort of roller-coaster potential energy diagram from introductory physics and reasoned as follows: The particle is moving more slowly at the peaks of the distribution, so it spends more time at those locations and is thus more likely to be found there. PER follow-up work indicated that the confusion was caused in part by combining probability distributions with energy level diagrams – students were unsure of what the ordinate represented. As a result, I adopted a policy in class (and in this edition of the textbook) of never showing the wave functions or probability distributions on the same plot as the energy levels.

The overwhelming majority of PER work has concerned the introductory course, but the effective pedagogic techniques revealed by that research carry over directly into the modern physics course. The collection of research directly linked to topics in modern physics is much smaller but no less revealing. The University of Washington group has produced several papers impacting modern physics, including the understanding of interference and diffraction of particles<sup>5</sup>, time and simultaneity in special relativity, and the photoelectric effect (see the papers listed on their web site, ref. 1). The PER group of Edward F. Redish at the University of Maryland has also been involved in studying the learning of quantum concepts, including the student's prejudices from classical physics, probability, and conductivity.<sup>6</sup> (Further work on the learning of quantum concepts has been carried out by the research groups of two of Redish's Ph.D. students, Lei Bao at Ohio State University<sup>7</sup> and Michael Wittmann at the University of Maine.<sup>8</sup>) Dean Zollman's group at Kansas State University has developed tutorials and visualizations to enhance the teaching of quantum concepts at many levels (from pre-college through advanced undergraduate).<sup>9</sup> The physics education group at the University of Colorado, led by Noah Finkelstein and Carl Wieman, is actively pursuing several research areas involving modern physics and has produced numerous research papers as well as simulations on topics in modern physics.<sup>10</sup> Others who have conducted research on the teaching of quantum mechanics and developed interactive or evaluative materials include

## **Classroom Materials for Active Engagement**

### **1. Reading Quizzes**

I started developing the interactive classroom materials for modern physics after successfully introducing Eric Mazur's Peer Instruction techniques into my calculus-based introductory course. Daily reading quizzes were a part of Mazur's original classroom strategy, but recently he has adopted a system that is more like Just-in-Time Teaching. Nevertheless, I have found the reading quizzes to work effectively in both my introductory and modern physics classes, and I have continued using them. We use electronic classroom communication devices ("clickers") to collect the responses, but in a small class paper quizzes work just as well. Originally the quizzes were intended to get students to read the textbook before coming to class, and I have over the years collected evidence that the quizzes in fact accomplish that goal. The quizzes are given just at the start of class, and I have found that they have two other salutary effects: (1) In the few minutes before the bell rings at the start of class, the students are not reading the campus newspaper or discussing last week's football game – they are reading their physics books. (2) It takes no time at the start of class for me to focus the students' attention or put them "in the mood" for physics; the quiz gets them settled into class and thinking about physics. The multiple-choice quizzes must be very straightforward – no complex thinking or reasoning should be required, and if a student has done the assigned reading the quiz should be automatic and should take no more than a minute or so to read and answer. Nearly all students get at least 80% of the quizzes correct, so ultimately they have little impact on the grade distribution. The quizzes count only a few percent toward the student's total grade, so even if they miss a few their grade is not affected.

### **2. Conceptual Questions**

I spend relatively little class time "lecturing" in the traditional sense. I prefer an approach in which I prod and coach the students into learning and understanding the material. The students' reading of the textbook is an important component of this process – I do not see the need to repeat orally everything that is already written in the textbook. (Of course, there are some topics in any course that can be elucidated only by a well constructed and delivered lecture. Separating those topics from those that the students can mostly grasp from reading the text and associated in-class follow-ups comes only from experience. Feedback obtained from the results of the conceptual exercises and from student surveys is invaluable in this process.) I usually take about 10 minutes at the beginning of class to summarize the important elements from that day's reading. In the process I list on the board new or unfamiliar words and important formulas. These remain visible during the entire class so I can refer back to them as often as necessary. I explain any special or restrictive circumstances that accompany the use of any equation. I do not do formal mathematical derivations in class – they cause a rapid drop-off in student attention. However, I do discuss or explain mathematical processes or techniques

that might be unfamiliar to students. I encourage students to e-mail me with questions about the reading before class, and at this point I answer those questions and any new questions that may puzzle the students.

The remainder of the class period consists of conceptual questions and worked examples. I follow the Peer Instruction model for the conceptual questions: an individual answer with no discussion, then small group discussions, and finally a second individual answer. On my computer I can see the histograms of the responses using the clickers, and if there are fewer than 30% or more than 70% correct answers on the first response, the group discussions normally don't provide much benefit so I abandon the question and move on to another. During the group discussion time, I wander throughout the class listening to the comments and occasionally asking questions or giving a small nudge if I feel a particular group is moving in the wrong direction. After the second response I ask a member of the class to give the answer and an explanation, and I will supplement the student's explanation as necessary. I generally do not show the histograms of the clicker responses to the class, neither upon the first response nor the second. The daily quiz, summary, two conceptual questions or small group projects, and one or two worked examples will normally fill a 50-minute class period, with a few minutes at the end for recapitulation or additional questions. I try to end each class period with a brief teaser regarding the next class.

Some conceptual questions listed for class discussion may appear similar to those given on exams. I never use the same question for both class discussion and examination during any single term. However, conceptual questions used during one term for examinations may find use for in-class discussions during a subsequent term.

### **3. JITT Warm-up Exercises**

Just-in-Time Teaching uses web-based "warm-up" exercises to assess the student's prior knowledge and misconceptions. The instructor can use the responses to the warm-up exercises to plan the content of the next class. The reading quizzes and conceptual questions intended for in-class activities can in many cases be used equally well for JiTT warm-up exercises.

### **Lecture Demonstrations**

Demonstrations are an important part of teaching introductory physics, and physics education research has shown that learning from the demos is enhanced if they are made interactive. (For example, you can ask students to predict the response of the apparatus, discuss the predictions with a neighbor, and then to reconcile an incorrect prediction with the observation.) Unfortunately, there are few demos that can be done in the modern physics classroom. Instead, we must rely on simulations and animations. There are many effective and interesting instructional software packages on the web that can be downloaded for your class, and you can make them available for the students to use outside of class. I have listed in this Manual some of the modern physics software that I have used in my classes. Of particular interest is the open-source collection of Physlets (physics applets) covering relativity and quantum physics produced by Mario Belloni, Wolfgang Christian, and Anne J. Cox.<sup>13</sup>

## Sample Test Questions

This Instructor's Manual includes a selection of sample test questions. A typical midterm exam in my Modern Physics class might include 4 multiple-choice questions (no reasoning arguments required) worth 20 points, 2 conceptual questions (another 20 points) requiring the student to select an answer from among 2 or 3 possibilities and to give the reasons for that choice, and 3 numerical problems worth a total of 60 points. Students have 1 hour and 15 minutes to complete the exam. The final exam is about 1.5 times the length of a midterm exam.

One point worth considering is the use of formula sheets during exams. Over the years I have gone back and forth among many different exam systems: open book, closed book and notes, and closed book with a student-generated formula sheet. I have found that in the open book format students seem to spend a lot of time leafing through the book looking for an essential formula or constant. On the other hand, I have been amazed at how many equations a student can pack onto a single sheet of paper, and I often find myself wondering how much better such students would perform on exams if they spent as much study time working on practice problems as they do miniaturizing equations. (Students often have difficulty distinguishing important formulas, which represent a fundamental concept or relationship, from mere equations which might be intermediate steps in solving a problem or deriving a formula.) I have finally settled on a closed book format in which I supply the formula sheet with each exam. I feel this has a number of advantages: (1) It equalizes the playing field. (2) Students don't need to waste time copying equations. (3) The formula sheet, a copy of which I give to students at the beginning of the term, itself serves as a kind of study guide. (4) Students use the formula sheet when working homework problems and studying for the exams, so they know what formulas are on the sheet and where they are located. (5) I can be sure that the formulas that students need to work the exams are included on the formula sheet. A sample copy of my formula sheet is included in this Instructor's Manual.

This Instructor's Manual is always a work in progress. I would be grateful to receive corrections or suggestions from users.

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## References

1. <http://www.phys.washington.edu/groups/peg/>
2. <http://mazur.harvard.edu/education/educationmenu.php>. Also see E. Mazur, *Peer Instruction: A User's Manual* (Prentice Hall, 1997).

## Problem Solutions

1. (a) Conservation of momentum gives  $p_{x,\text{initial}} = p_{x,\text{final}}$ , or

$$m_{\text{H}} v_{\text{H,initial}} + m_{\text{He}} v_{\text{He,initial}} = m_{\text{H}} v_{\text{H,final}} + m_{\text{He}} v_{\text{He,final}}$$

Solving for  $v_{\text{He,final}}$  with  $v_{\text{He,initial}} = 0$ , we obtain

$$\begin{aligned} v_{\text{He,final}} &= \frac{m_{\text{H}}(v_{\text{H,initial}} - v_{\text{H,final}})}{m_{\text{He}}} \\ &= \frac{(1.674 \times 10^{-27} \text{ kg})[1.1250 \times 10^7 \text{ m/s} - (-6.724 \times 10^6 \text{ m/s})]}{6.646 \times 10^{-27} \text{ kg}} = 4.527 \times 10^6 \text{ m/s} \end{aligned}$$

(b) Kinetic energy is the only form of energy we need to consider in this elastic collision. Conservation of energy then gives  $K_{\text{initial}} = K_{\text{final}}$ , or

$$\frac{1}{2} m_{\text{H}} v_{\text{H,initial}}^2 + \frac{1}{2} m_{\text{He}} v_{\text{He,initial}}^2 = \frac{1}{2} m_{\text{H}} v_{\text{H,final}}^2 + \frac{1}{2} m_{\text{He}} v_{\text{He,final}}^2$$

Solving for  $v_{\text{He,final}}$  with  $v_{\text{He,initial}} = 0$ , we obtain

$$\begin{aligned} v_{\text{He,final}} &= \sqrt{\frac{m_{\text{H}}(v_{\text{H,initial}}^2 - v_{\text{H,final}}^2)}{m_{\text{He}}}} \\ &= \sqrt{\frac{(1.674 \times 10^{-27} \text{ kg})[(1.1250 \times 10^7 \text{ m/s})^2 - (-6.724 \times 10^6 \text{ m/s})^2]}{6.646 \times 10^{-27} \text{ kg}}} = 4.527 \times 10^6 \text{ m/s} \end{aligned}$$

2. (a) Let the helium initially move in the  $x$  direction. Then conservation of momentum gives:

$$\begin{aligned} p_{x,\text{initial}} = p_{x,\text{final}} : \quad m_{\text{He}} v_{\text{He,initial}} &= m_{\text{He}} v_{\text{He,final}} \cos \theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \cos \theta_{\text{O}} \\ p_{y,\text{initial}} = p_{y,\text{final}} : \quad 0 &= m_{\text{He}} v_{\text{He,final}} \sin \theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \sin \theta_{\text{O}} \end{aligned}$$

From the second equation,

$$v_{\text{O,final}} = -\frac{m_{\text{He}} v_{\text{He,final}} \sin \theta_{\text{He}}}{m_{\text{O}} \sin \theta_{\text{O}}} = -\frac{(6.6465 \times 10^{-27} \text{ kg})(6.636 \times 10^6 \text{ m/s})(\sin 84.7^\circ)}{(2.6560 \times 10^{-26} \text{ kg})[\sin(-40.4^\circ)]} = 2.551 \times 10^6 \text{ m/s}$$

(b) From the first momentum equation,

30. 
$$K = E - mc^2 = \sqrt{p^2 c^2 + m^2 c^4} - mc^2$$

$$\frac{dK}{dp} = \frac{1}{2}(p^2 c^2 + m^2 c^4)^{-1/2}(2pc^2) = \frac{pc^2}{\sqrt{p^2 c^2 + m^2 c^4}} = \frac{pc^2}{E} = c^2 \frac{mv / \sqrt{1 - v^2 / c^2}}{mc^2 / \sqrt{1 - v^2 / c^2}} = v$$

31. (a) With a node at each end (say, at  $x = 0$  and  $x = L$ ) and no other nodes, we must have one half-wave between the two nodes. Thus  $L = \lambda_1 / 2$  or  $\lambda_1 = 2L$ . If there is an additional node at the midpoint ( $x = L/2$ ), then there is a full wave between the two ends, and  $L = \lambda_2$  or  $\lambda_2 = 2L / 2$ . The next shorter wavelength has (in addition to the nodes at either end) nodes at  $x = L/3$  and  $x = 2L/3$ , so there are three half-waves between the ends:  $L = 3\lambda_3 / 2$  or  $\lambda_3 = 2L / 3$ . Continuing in this way, we see that in the  $n^{\text{th}}$  case there are  $n$  half-waves in the length  $L$ , so  $L = n(\lambda_n / 2)$  or  $\lambda_n = 2L / n$ .
- (b) With  $p_n = h / \lambda_n = nh / 2L$ , we see that  $cp_n$  is of order keV, so nonrelativistic equations can safely be used:

$$K_n = \frac{p_n^2}{2m} = \frac{c^2 p_n^2}{2mc^2} = \frac{n^2 h^2 c^2}{8mc^2 L^2} = n^2 \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.50 \text{ nm})^2} = n^2 (1.50 \text{ eV})$$

Thus  $K_1 = 1.50 \text{ eV}$ ,  $K_2 = 6.00 \text{ eV}$ ,  $K_3 = 13.5 \text{ eV}$ .

32. 
$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2 K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(940 \times 10^6 \text{ eV})(0.0105 \text{ eV})}} = 0.279 \text{ nm}$$

From the Bragg scattering formula (Eq. 3.18), we have

$$\sin \theta = \frac{n\lambda}{2d} = \frac{(1)(0.279 \text{ nm})}{2(0.247 \text{ nm})} = 0.565 \quad \text{or} \quad \theta = 34.4^\circ$$

For second-order ( $n = 2$ ) scattering at that angle,  $\lambda = (2d \sin \theta) / 2 = 0.140 \text{ nm}$ . The wavelength is reduced by half, so the momentum is doubled and the kinetic energy increases by a factor of 4 to 0.0420 eV. For third-order scattering ( $n = 3$ ), the kinetic energy is 9 times as great, or 0.0945 eV. The scattered beam at that angle will consist of all energies that are  $n^2$  times the original energy ( $n = 1, 2, 3, \dots$ ).

33. (a) The mass of a nitrogen molecule is 14 u. The average molecular kinetic energy is  $\frac{3}{2}kT$ , so the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2 K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(14 \text{ u})(931.5 \times 10^6 \text{ eV/u})(1.5)(8.617 \times 10^{-5} \text{ eV/K})(293 \text{ K})}} = 0.0279 \text{ nm}$$

The experimental value for comparison is

$$\chi_{\text{expt}} = \frac{4\pi\rho}{M} \chi_{\text{molar}}^{\text{cgs}} = \frac{4\pi(3.50)}{137.3} (20.6 \times 10^{-6}) = 6.6 \times 10^{-6}$$

41. (a) For gold, the number of free electrons per unit volume is

$$\frac{N}{V} = \frac{\rho N_A}{M} = \frac{(19.3 \times 10^3 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ atoms/mole})}{0.197 \text{ kg/mole}} = 5.90 \times 10^{28} \text{ m}^{-3}$$

The Pauli paramagnetic susceptibility is

$$\chi = \frac{3\mu_0\mu_B^2}{2E_F} \frac{N}{V} = \frac{3(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(9.27 \times 10^{-24} \text{ J/T})^2 (5.90 \times 10^{28} \text{ m}^{-3})}{2(5.53 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 10.8 \times 10^{-6}$$

- (b) The experimental volume susceptibility in SI units is

$$\chi_{\text{volume}}^{\text{SI}} = 4\pi \frac{\rho}{M} \chi_{\text{molar}}^{\text{cgs}} = 4\pi \frac{19.3 \text{ g/cm}^3}{197 \text{ g/mole}} (-28.0 \times 10^{-6}) = -34.5 \times 10^{-6}$$

With  $\chi_{\text{total}} = \chi_{\text{para}} + \chi_{\text{dia}}$ , we have  $\chi_{\text{dia}} = -45.3 \times 10^{-6}$ .

42. (a) For  $\text{MnCl}_2$ ,  $\rho = 3.0 \text{ g/cm}^3$  and  $M = 126 \text{ g/mole}$ . The SI volume susceptibility is then

$$\chi_{\text{volume}}^{\text{SI}} = 4\pi \frac{\rho}{M} \chi_{\text{molar}}^{\text{cgs}} = 4\pi \frac{3.0 \text{ g/cm}^3}{126 \text{ g/mole}} (14350 \times 10^{-6}) = 4.29 \times 10^{-3}$$

$$\frac{N}{V} = \frac{\rho N_A}{M} = \frac{(3.0 \times 10^3 \text{ kg/m}^3)(6.02 \times 10^{23} \text{ atoms/mole})}{0.126 \text{ kg/mole}} = 1.43 \times 10^{28} \text{ m}^{-3}$$

$$g_J^2 J(J+1) = \frac{3kT\chi}{\mu_0(N/V)\mu_B^2} = \frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})(4.29 \times 10^{-3})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.43 \times 10^{28} \text{ m}^{-3})(9.27 \times 10^{-24} \text{ J/T})^2} = 33.7$$

(b) The neutral Mn atom has the electronic configuration  $3d^5 4s^2$ . If we remove the two outer  $4s$  electrons, the  $\text{Mn}^{++}$  ion will have the configuration  $3d^5$ . The  $3d$  subshell can accommodate a total of 10 electrons with 5 different  $m_l$  values. Without duplicating any  $m_l$  values, each of the five  $3d$  electrons in  $\text{Mn}^{++}$  can have  $m_s = +1/2$ , so we have  $S = 5/2$ . The 5 electrons with  $m_s = +1/2$  must use up all of the allowed  $m_l$  values for  $l = 2$  ( $+2, +1, 0, -1, -2$ ), so the total  $L$  is zero.

(c) With  $J = L + S = 0 + 5/2 = 5/2$ , we have

$$g_J = \sqrt{33.7 / J(J+1)} = \sqrt{33.7 / (2.5)(3.5)} = 1.96$$

$$\begin{aligned}
R &= \frac{h^2(9/32\pi^2)^{2/3}}{GN^{1/3}m_n^3} = \frac{h^2(9/32\pi^2)^{2/3}}{GN_\odot^{1/3}(M/M_\odot)^{1/3}m_n^3} \\
&= \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2(9/32\pi^2)^{2/3}}{(6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.188 \times 10^{57})^{1/3}(1.675 \times 10^{-27} \text{ kg})^3} \left(\frac{M}{M_\odot}\right)^{-1/3} \\
&= (12.3 \text{ km}) \left(\frac{M}{M_\odot}\right)^{-1/3}
\end{aligned}$$

(b) The radius of the neutron star is  $R = (12.3 \text{ km})(1.5)^{-1/3} = 11 \text{ km}$ . The angular momentum  $L = I\omega$  is conserved during the collapse, so  $I_i\omega_i = I_f\omega_f$ :

$$\omega_f = \omega_i \frac{I_i}{I_f} = \omega_i \frac{\frac{2}{5}MR_i^2}{\frac{2}{5}MR_f^2} = \omega_i \left(\frac{R_i}{R_f}\right)^2 = (1 \text{ rev/y}) \left(\frac{7 \times 10^5 \text{ km}}{11 \text{ km}}\right)^2 = 4.0 \times 10^9 \text{ rev/y} = 128 \text{ rev/s}$$

12. (a) For the matter-dominated universe,  $R = At^{2/3}$ , so  $dR/dt = \frac{2}{3}At^{-1/3}$  and  $d^2R/dt^2 = -\frac{2}{9}At^{-4/3}$ . The deceleration parameter is

$$q = -\frac{R(d^2R/dt^2)}{(dR/dt)^2} = -\frac{(At^{2/3})(-\frac{2}{9}At^{-4/3})}{(\frac{2}{3}At^{-1/3})^2} = 0.5$$

For the radiation-dominated universe,  $R = A't^{1/2}$ , so  $dR/dt = \frac{1}{2}A't^{-1/2}$  and  $d^2R/dt^2 = -\frac{1}{4}A't^{-3/2}$ . The deceleration parameter is

$$q = -\frac{R(d^2R/dt^2)}{(dR/dt)^2} = -\frac{(A't^{1/2})(-\frac{1}{4}A't^{-3/2})}{(\frac{1}{2}A't^{-1/2})^2} = 1.0$$

(b) Starting with  $(dR/dt)^2 = \frac{8\pi}{3}G\rho_m R^2$ , we take the derivative with respect to time:

$$2\frac{dR}{dt}\frac{d^2R}{dt^2} = \frac{8\pi}{3}G\frac{d\rho_m}{dt}R^2 + \frac{8\pi}{3}G\rho_m\left(2R\frac{dR}{dt}\right)$$

With  $\rho_m = CR^{-3}$  where  $C$  is a constant,  $d\rho_m/dt = -3CR^{-4}(dR/dt) = -3\rho_m R^{-1}(dR/dt)$  and

$$2\frac{dR}{dt}\frac{d^2R}{dt^2} = \frac{8\pi}{3}G\left(-3\rho_m R\frac{dR}{dt} + 2\rho_m R\frac{dR}{dt}\right) = -\frac{8\pi}{3}G\rho_m R\frac{dR}{dt}$$