OXFORD

third edition

FOUNDATIONS OF MODERN MACROECONOMICS

BEN J. HEIJDRA

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OXFORD
UNIVERSITY PRESS

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Great Clarendon Street, Oxford, OX2 6DP,
United Kingdom

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries

First edition published in 2002
Second edition published in 2009
Third edition published in 2017

Impression: 1

Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data

Data available

Library of Congress Control Number: 2017932967

ISBN 978–0–19–878413–5

Printed and bound by
CPI Group (UK) Ltd, Croydon, CR0 4YY

In memoriam

Jenny E. Ligthart

November 6, 1967 – November 21, 2012

Walter H. Fisher

December 9, 1961 – November 15, 2012

Preface

What can the book do for you?

As was the case for the first two editions of this book, this new edition tries to present a balanced overview of modern macroeconomic theory. I follow two guiding principles. First, I adopt a rather eclectic approach by paying attention not just to the most recent insights in the field but also to developments that are currently less popular. In doing so, I hope to provide students with a better overview of current *and* past debates in macroeconomic theory. History can teach us useful lessons, provided we are willing to listen! For example, I continue to include discussions of the IS-LM model, the adaptive expectations hypothesis, and the Solow-Swan growth model (to mention a few). Though these theories are currently less fashionable (and, as some economists argue, may even be "outdated"), it is my firm conviction that they nevertheless provide important insights. For example, to fully appreciate the importance of the rational expectations hypothesis, a good understanding of the adaptive expectations hypothesis (its immediate predecessor) is indispensable. Similarly, to really understand the contributions made in recent years by New Keynesian Dynamic Stochastic General Equilibrium (DSGE) economists, it is very useful to have a firm understanding of the IS-LM model. Also, a good grasp of the Solow-Swan model helps in appreciating the Ramsey model and the endogenous growth models formulated in the 1980s and 1990s. Of course, as the saying goes, "old habits die slowly" and the IS-LM model is still used extensively even though, as Blanchard has pointed out, many people may not even know they are using it (2000, p. 1405).

The second guiding principle concerns the expositional style of the book. In addition to introducing the different theories by verbal and graphical means, I have also aimed to successively develop "the tools of the trade" of modern macroeconomics. In this aspect the book is related to Allen's (1967) marvellous macroeconomic toolbook. So instead of only providing students with a verbal/intuitive understanding of the material (valuable as it is), I also explain the basic modelling tricks of modern macroeconomics. Where needed the full details of both the models and their solutions are presented. Students who have worked through the textbook (and its accompanying manual) should have little or no problems reading the recent journal literature in macroeconomics or building their own macro models.

How can the book be used?

Depending on the background of students, the book can be used in the undergraduate and/or the graduate curriculum. Part I, consisting of Chapters 1-9, can be used in an intermediate macroeconomics course in the undergraduate curriculum. For example, I use Chapters 1-6 in my seven-week macroeconomics course in the third-year of the bachelor program at the University of Groningen. Economics students in this course have been exposed to Blanchard et al. (2013) in their first two years, whilst econometrics students have studied Gärtner (2016) in their second year of studies. In addition, these students have studied basic mathematical methods at the level discussed, for example, in Hoy et al. (2011).

Parts II and III of the book consist of Chapters 10-19. They are aimed at advanced bachelor students, first-year master students, and beginning doctoral students. In the graduate curriculum, the book can be used as the main text in a first-semester macroeconomics course or as a supplementary text for an advanced graduate macro course. At the University of Groningen, for example, I use Chapters 12-15 in my half-semester macroeconomics course in the regular masters programs. In the research master courses I also cover Chapter 10 and most of Chapters 16-19. The book is also well-suited for beginning doctoral students with no (or insufficient) previous training in macroeconomic theory. Parts of Chapters 12-16 were used in the various graduate courses I have taught over the years for the Netherlands Network of Economics (NAKE), the Tinbergen Institute, CESifo, and the Institute for Advanced Studies (Vienna).

Intermezzos

The book contains a number of so-called intermezzos. I use the term 'intermezzo' in an extended and unusual sense. Recall that in music an intermezzo is a composition that is played in between acts of a play or movements of a much larger musical piece. In this book, the intermezzos do not make any sound but, like in music, they are 'small morsels in between big chunks'. They serve a number of purposes. First of all, they ensure that upon first reading students are not distracted by complex technical intricacies. Second, they allow for in-depth coverage of a number of key results in theoretical macroeconomics. Furthermore, in combination with the chapter appendices and the mathematical appendix at the end the book, they cover all technicalities necessary for a sound understanding of modern macroeconomics. Whereas the appendices are purely aimed at mathematical results, the intermezzos focus more on the fault line between mathematics and theoretical macroeconomics. Finally, the intermezzos serve as reference tools for readers who wish to reacquaint themselves with things they used to know but have forgotten.

Starred sections

In this edition I have also included sections marked with a superscript star (★). These sections contain material that is more difficult than the rest of the chapter in which they are located. Students may choose to skip the starred material when first reading the chapter. Upon completion of the book the successful student will find that most (or even all) stars have become invisible.

Changes for the Third Edition

The book has been thoroughly rewritten. Compared to the second edition, it has grown in size by about one hundred pages. The main changes are as follows.

- The current book includes forty-seven intermezzos, of which sixteen are new. All of these have been extensively checked and streamlined. They are numbered and carry an informative title. A List of Intermezzos is included in the preamble of the book which facilitates cross-referencing. The numbering system is as before, with the first digit denoting the chapter in which the intermezzo is located. The new intermezzos are 1.1, 1.2, 5.1, 5.2, 8.2, 8.3, 9.1, 12.2, 13.3, 16.1, 17.1, 18.1, 18.2, 19.1, 19.2 and 19.3.
- The new Chapter 2 deals exclusively with the open economy. It follows logically from the first chapter and contains material from Sections 1 and 2 of the old Chapter 10.
- Chapter 3 is a rewritten version of the old Chapter 2.
- Chapter 4 has been renamed to better reflect its contents. It contains a rewritten version of the old Chapter 4 as well as Section 3 (on the Dornbusch model) from the old Chapter 10.
- Chapter 5 is an expanded and rewritten version of the old Chapter 3. It now includes a small open economy model and explains the Dynare software package that can be used to solve rational expectations models.
- Chapter 6 is a lightly rewritten version of the old Chapter 5.
- Chapter 7 is a thoroughly edited and shortened version of the old Chapters 6 and 7. It also contains some new material on union- and efficiency-wage models in general equilibrium.
- Chapter 8 is an expanded version of the old Chapter 8. It now contains a section on endogenous job destruction.
- Chapter 9 has been renamed to better reflect its contents. In addition it has been expanded and now includes a discussion of dynamic inconsistency of individual choices resulting from present-biased (or quasi-hyperbolic) preferences.
- Chapters 10 and 11 are lightly edited versions of the old Chapters 11 and 12.
- The old Chapter 12 (on exogenous growth) has been split into two much expanded chapters. The new Chapter 12 deals exclusively with Solow-Swan style growth models. It has been expanded somewhat and now also features a section of the two-sector Meade-Uzawa model.
- Chapter 13 contains Sections 13.5–13.7 from the old Chapter 13. In addition it has been expanded dramatically. It now includes models with endogenous labour supply (using material from the old Chapter 15), search unemployment, and money balances entering the felicity function. This is the pivotal chapter in the book as the Ramsey-Cass-Koopmans model that it covers in all its guises plays a central role in the material that follows from there on.
- Chapter 14 is a lightly edited version of the old Chapter 14. Similarly, Chapters 15 and 16 are lightly edited versions of the old Chapters 16 and 17.
- Chapter 17 is brand new. It provides a brief (and mostly intuitive) discussion of the method of dynamic programming (DP). In addition it introduces the concept of complete markets and shows how one can construct a "representative

agent" in such a setting. Whilst a deep knowledge of DP is not really essential to understand Chapters 18–19, it is indispensable if one wants to proceed to the more advanced literature in macroeconomics, e.g. the graduate textbook by Ljungqvist and Sargent (2012).

- Chapter 18 is the first chapter on the DSGE approach. It contains material from the old Section 15.5. It has been edited thoroughly and now includes discussions of the stochastic discount factor and shows how DSGE models can be simulated with the aid of the Dynare software package (introduced in Chapter 5).
- Chapter 19 is brand new. It contains a thorough discussion of the New Keynesian DSGE approach and finishes with a brief assessment of the state of the art at the time of writing. This assessment replaces the Epilogue from the second edition.

Visible means of support

It somehow seems impossible to produce a book of this size without generating (free of charge) some typos and errors. Needless to say, all such errors and typos will be published as I become aware of them. I will make the errata documents available through the website for the book:

`http://www.heijdra.org/fomm3`

So please let me know about any typos and/or errors that you may spot. This is what you can do for the book! The contact address is: `info@heijdra.org`. As a (weak substitute for a) reward, I will mention your name prominently on the website (as having contributed to the public good). Of course, your name will also feature in the Acknowledgements section in any future edition of the book.

The website also includes ready-to-use slides for all chapters in PDF format. Teachers who wish to adapt these slides to their own purpose or software platform can download the LaTeX 2_ε code and all figures (in EPS and EMF formats) and proceed from there.

I have updated and streamlined the accompanying *Exercise and Solutions Manual* which is published by Oxford University Press. This hands-on exercise book contains a large number of problems plus model answers. These problem sets allow the interested student to further develop his/her skills.

Acknowledgements

In preparing the *third edition*, I received useful comments from many people, including Pieter Ijtsma, Gerard van der Meijden, Laurie Reijnders, Girum Dagnachew Abate, Wilma Huitema, Christien de Kort, Stine Celius, Kengo Tahara, Carolien Calkhoven, Mika Kortelainen, Matthijs Katz, Yoni Schirris, Bastiaan Quast, Jelle van Essen, Marc Boom, Annemarije Santman, Bart Rutjes, Lisan Spiegelaar, Gert-Jan Romensen, Jitka Vavra, and Vesa-Matti Heikkuri. One of the great privileges of working in a university is that – surrounded by young and enthusiastic people – one never really grows old. Life-long learning is the norm rather than the exception in academia. Over the years I have greatly benefited from the interaction with

some outstanding colleagues. The collaboration with Fabian Kindermann (University of Bonn) has resulted in a significant upgrade of my computing skills – something that was long overdue. Since he is a great teacher of computational economics, I highly recommend his forthcoming textbook on Fortran computing (Fehr and Kindermann, 2017). My friend and colleague, Pim Heijnen (University of Groningen) not only accompanied me to the pub quite regularly (for work-related meetings) but also had a very significant effect on my computing skills. In addition, he suggested the cake-eating example of dynamic inconsistency (and hyperbolic discounting) that is discussed in Chapter 9. Another colleague, Allard van der Made, has been a great sounding board on mathematical issues. I also thank my Groningen colleagues Lammertjan Dam for discussions on the consumption-based asset pricing model employed in Chapters 18–19, Christiaan van der Kwaak for comments on Chapters 17 and 19, and Gerard Kuper for plowing through Chapter 18. Just as for the second edition, Jochen Mierau has read the entire manuscript and has provided useful advice on many aspects of the book (both content and exposition). To the extent that I have not followed some (or all) of their suggestions, this is not because I disagreed with them but rather because of a binding time constraint on my part.

As with the previous two editions of the book, Siep Kroonenberg has assisted at crucial instances with the more complicated aspects of the LaTeX 2_ε codes used to produce this book. Of course, Leslie Lamport and Donald E. Knuth are thanked implicitly too for producing, respectively, LaTeX 2_ε and TeX.

Over the years the following people from Oxford University Press have been of great assistance in the production and marketing of this book: Andrew Schuller, Rebecca Bryant, Sarah Dobson, Jennifer Wilkinson, Sarah Caro, Aimee Wright, T.W. Bartel, and Adam Swallow. During the fine-tuning of the book I benefited tremendously from the efforts of Katie Bishop, Elisa Cozzi, and Joshua Hey. I thank all of them for their efforts.

Dedication

I dedicate this new edition of the book to Jenny E. Ligthart and Walter H. Fisher who passed away during a *hebdomas horribilis* in November 2012. I first met Jenny in December 1992. I moved to the University of Amsterdam (where she was a Ph.D. student) in May 1993 and together with Rick van der Ploeg I ended up supervising Jenny's doctoral work. Following her successful thesis defence in November 1997 we continued to work together on various projects until her passing. I first met Walter in May 2002 when he invited me to the Institute for Advanced Studies (IHS) in Vienna. From 2006-2013 I held a Visiting Research Professorship at the institute and our collaboration became much more intensive. Jenny and Walter were much more than nice colleagues and co-authors to me. Over time they became close personal friends. Some of their work finds its way into this book. The loyalty and friendship I received from them does not. I will cherish their memory for as long as I live.

Ben J. Heijdra
University of Groningen, The Netherlands
February 2017

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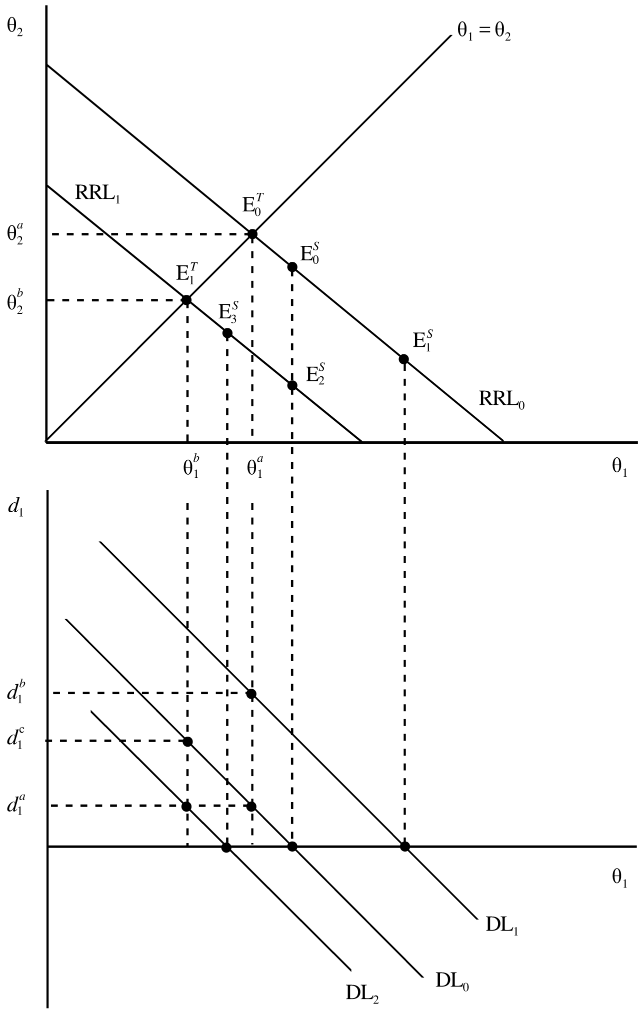

Figure 6.5: Optimal taxation, tax smoothing, and deficit financing

in infrastructural government spending is accommodated by an increase in the first-period deficit from d_1^a to d_1^b.

Rule #2 Public consumption spending and losses on public investment projects should be financed by means of taxation. Of course, by the same logic, profits on public investments must be used to reduce taxes.

Rule #3 The composition of a given level of ξ_1 does not matter. Consider, for example, a *temporary* rise in government consumption, i.e. an increase now that is exactly offset by a decrease in the future, or $(1+r_1)\Delta g_1^C = -(1+\gamma)\,\Delta g_2^C$. Since ξ_1 is unchanged, optimal tax rates are unchanged and debt financing is called for. In terms of Figure 6.5 the change only shifts the spending point (say from E_0^S to E_1^S) but leaves the optimal taxation point unaffected. The temporary increase in government spending is thus accommodated by an increase in the first-period deficit (and hence debt). This is a neoclassical policy prescription that looks a lot like old-fashioned Keynesian countercyclical policy. During (temporary) recessions government consumption may be higher and there is no harm in letting the debt increase a little bit provided future government consumption is curbed appropriately. (Of course, the tax smoothing model employed here does not include a description of the macro-economy so the similarity between the neoclassical and Keynesian prescriptions is only suggestive.)

Rule #4 If there is a change in the government's net liabilities, ξ_1, then it is optimal to adjust both tax rates immediately. For example, assume that the government credibly announces that it will lower its consumption spending in the future ($\Delta g_2^C < 0$). Then *both* tax rates should be lowered immediately. In terms of Figure 6.5, the revenue requirement line shifts from RRL_0 to RRL_1, the optimal taxation point shifts to E_1^T, and the spending point moves from E_0^S to E_2^S directly below it. The first-period deficit increases from d_1^a to d_1^c as a result. Mathematically, we obtain $\Delta d_1/\Delta g_2^C = -(1+\gamma)/(2+r_1+\gamma)$.

Rule #5 If the government decides to implement a so-called "balanced decline" of the public sector, for which $\Delta g_1^c = \Delta g_2^c = \Delta g < 0$, then both tax rates should be reduced and there is no effect on the first-period deficit, i.e. $\Delta\theta_1 = \Delta\theta_2 = \Delta g$ and $\Delta d_1 = 0$. In terms of Figure 6.5 the spending point shifts from E_0^S to E_3^S, the optimal taxation point moves from E_0^T to E_1^T, and the deficit line shifts from DL_0 to DL_2.

6.3 Punchlines

In this chapter two concepts, both relating to the government budget constraint, are introduced and analysed, namely the so-called Ricardian equivalence theorem (RET) and the theory of tax smoothing.

Starting with the first of these, the RET can be defined as follows. *For a given path of government spending*, the particular financing method used by the government (bonds or taxes) does not matter. More precisely, when the RET is valid, the financing method of the government does not affect real consumption, investment, output, and welfare, and government debt is seen as a form of delayed taxation. It must be stressed that the RET is not a statement about the effects of government consumption but rather deals with the way these expenditures are paid for by the government.

The intuition behind the RET is quite simple. If the government cuts taxes today and finances the resulting deficit by means of debt, then households will realize that, since total resources claimed by the government have not changed in present value terms, eventually the tax will have to be raised again sometime in the future. To ensure that it will be able to meet its future tax bills, the household reacts to the tax cut by saving it. The tax cut does not affect the lifetime resources available to the households and thus does not affect their consumption plans either.

Although the RET was not taken seriously by David Ricardo himself, it was (and still is) taken seriously by most new classical economists. A lot of objections have, however, been raised against the strict validity of the RET. First, if the Ricardian experiment involves changing one or more taxes which distort economic decisions (for example, because labour supply is endogenous and reacts to the timing of taxes) then the RET will fail. Intuitively, the lifetime resources available to the households will in that case depend on the particular time path of taxes and not just on the present value of taxes.

Second, if the household is unable to borrow freely, for example because future labour income cannot be used as collateral, then the RET fails. Again, the reason for this failure is that the household choice set (and the severity of the household's borrowing constraints) is affected by the time path of taxes chosen by the government.

Third, if households have finite lives whilst the government (and the economy as a whole) is infinitely lived, the RET may or may not be valid. It turns out that it matters whether the overlapping generations which populate the economy are altruistically linked with each other or not. Generations are altruistically linked if they care about each other's welfare (like children caring for their parents or vice versa). In the absence of intergenerational altruism, the the RET fails. Intuitively, a tax cut now matched (in present value terms) by a tax hike later on will make present generations wealthier and future generations poorer. With intergenerational altruism it is possible that the RET holds because transfers between generations will take place. Intuitively, a tax cut today will be passed on to future generations in the form of an (additional) inheritance.

Other objections to the RET relate to net population growth, informational problems (irrationality, myopia, and lack of information), and the so-called "bird in the hand" fallacy. The upshot of the discussion is that there are ample theoretical reasons to suspect that the RET is not strictly valid. Unfortunately, as is often the case, the empirical evidence regarding the approximate validity of the RET is inconclusive.

Even if one is willing to assume that the RET is valid, this does not mean that public debt has no role to play in the economy. Indeed, according to the theory of tax smoothing the government can use public debt to smooth its tax rates over time. To the extent that these tax rates are distorting the behaviour of private agents, tax smoothing is socially beneficial because it minimizes the distortions of the tax system as a whole. A number of intuitive "rules of thumb" follow from the theory.

Further reading

Although he did not use the term as such, the notion of Ricardian equivalence was introduced to modern macroeconomists by Barro (1974). Buchanan (1976) coined the term "Ricardian equivalence theorem," and O'Driscoll (1977) documents Ricardo's own misgivings about the result that is now known under his name. For good survey articles on Ricardian equivalence, see Bernheim (1987) and Seater (1993). Bernheim and Bagwell (1988) are very critical of the dynastic approach used by Barro and argue that it should not be used to study the effects of public policies. They take the altruistic approach as given, and demonstrate that there will be strong inter-family linkages in such a setting (due to marriages, etcetera). This in turn will produce neutrality results that are unrealistically strong (such as the equivalence of distorting taxes and lump-sum taxes, and the inability of governments to engage in redistribution). Arguing backwards, they conclude that there must be something wrong with the dynastic approach itself.

The earliest contributions to the macroeconomic theory of tax smoothing are by Prescott (1977) and Barro (1979). Subsequent contributions to the literature include Lucas and Stokey (1983), Kingston (1984, 1991), Roubini (1988), Huang and Lin (1993), Ghosh (1995), and Fisher and Kingston (2004, 2005). As was pointed out by Sargent (2001), in a stochastic framework the optimal time path of taxes depends critically on whether or not the government is able to issue state-contingent debt. Whereas the tax smoothing literature typically assumes government spending to be exogenous, Judd (1999) presents an analysis of the joint determination of optimal taxation and spending in a deterministic setting.

Readers interested in the various issues surrounding the government budget constraint and the deficit are referred to Buiter (1985, 1990). The intertemporal consumption model used in this chapter is due to Fisher (1930). Further results on the two-period consumption model are presented by Obstfeld and Rogoff (1996, ch. 1). See Deaton (1992) and Attanasio (1999) for advanced surveys of intertemporal consumption theory.

substituting the first-order condition (g) into the definition of C (given in (11.2)), we obtain the following expression for C_j:

$$C_j = \frac{N^{-\eta} C P_j^{-\theta}}{\left[\sum_{k=1}^{N} N^{-1} P_k^{1-\theta}\right]^{-\theta/(1-\theta)}}. \tag{h}$$

By substituting (h) into the constraint given in (f) the expression for the price index P is obtained:

$$\sum_{j=1}^{N} P_j C_j = \frac{N^{\theta/(\theta-1)-\eta} C \left[\sum_{j=1}^{N} P_j^{1-\theta}\right]}{\left[\sum_{j=1}^{N} P_j^{1-\theta}\right]^{-\theta/(1-\theta)}} = PC \;\Rightarrow$$

$$P \equiv N^{-\eta} \left[N^{-\theta} \sum_{j=1}^{N} P_j^{1-\theta} \right]^{1/(1-\theta)}. \tag{i}$$

By using this price index we can re-express the demand for variety j of the consumption good (given in (h)) in a more compact form as:

$$\frac{C_j}{C} = N^{-(\theta+\eta)+\eta\theta} \left(\frac{P_j}{P}\right)^{-\theta}, \quad j = 1, \ldots, N, \tag{j}$$

which is the expression used in the text (namely equation (11.7)).

It must be pointed out that we could have solved the choice problem facing the consumer in one single (and rather large) maximization problem, instead of by means of two-stage budgeting, and we would, of course, have obtained the same solutions. The advantages of two-stage budgeting are twofold: (i) it makes the computations more straightforward and mistakes easier to avoid, and (ii) it automatically yields useful definitions for true price indexes as by-products.

Finally, although we did not explicitly use the terminology, the observant reader will have noted that we have already used the method of two-stage budgeting before in Chapter 2. There we discussed the Armington approach to modelling international trade flows and assumed that a domestic composite good consists of a domestically produced good and a good produced abroad.

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The firm sector is characterized by monopolistic competition, i.e. there are very many small firms each producing a variety of the differentiated good and each enjoying market power in its own output market. The individual firm j uses labour to produce variety j and faces the following production function:

$$Y_j = \begin{cases} 0 & \text{if } L_j \le F \\ \dfrac{L_j - F}{k} & \text{if } L_j \ge F, \end{cases} \tag{11.10}$$

where Y_j is the marketable output of firm j, L_j is labour used by the firm, F is fixed where Z_X is an exogenous productivity index and L_{ij} is the amount of labour used in the production of X_{ij}. The crucial thing to note is that production costs do not depend on the quality of the input that is being produced, i.e. high and low quality producers face the same cost structure. Labour is perfectly mobile across sectors of the economy and fetches a wage rate $W(t)$, i.e. total cost of firm ij is equal to $W(t)X_{ij}(t)/Z_X$ and profit is defined as follows:

$$\Pi_{ij}(t) \equiv \left[P_{ij}(t) - \frac{W(t)}{Z_X}\right] X_{ij}(t). \tag{14.134}$$

The optimal pricing decision of the quality leader in industry i can be studied with the aid of Figure 14.7. It is sufficient to look at the leading firm (for which $j = m_i(t)$) and its immediate predecessor as quality leader (the "follower", whose $j = m_i(t) - 1$). Note that we do not have to consider lower quality producers than the follower because they are even less competitive than the follower. The leader engages in *Bertrand price competition* with the follower and in the optimum will set its price such that the follower is driven out of the market altogether.[16] To see why this is the case, we note that the lowest price the follower can set without incurring losses is given by the marginal cost of production, i.e. $P_i^F = W/Z_X$, $X_{i,m_i-1} = X_{ij}^F$, and $\Pi_i^F = 0$. But the quality leader produces a better version of input X_{ij} and, as we saw above, the purchasers of this input will prefer to buy the higher quality input provided $P_i^L \equiv P_{im_i} \leq (1+\xi)\, P_{i,m_i-1} \equiv (1+\xi)\, P_i^F$. Hence, the effective demand facing the quality leader is the solid line passing through points E, B, A, and F. It makes no sense for the leader to charge a price in excess of $(1+\xi)\, W/Z_X$ because it would lose all its customers to the follower. Similarly, it also makes no sense to charge less than $(1+\xi)\, W/Z_X$ because it would leave profit opportunities unused. Using the tie-breaking rule that buyers' indifference between $j = m_i$ and $j = m_i - 1$ results in purchases only from the market leader, it follows that the price set by the quality leader equals:

$$P_i(t) = (1+\xi)\, \frac{W(t)}{Z_X}. \tag{14.135}$$

At that price, the quality leader will earn a positive profit equal to:

$$\begin{aligned}\Pi_i(t) &= \left[1 - \frac{W(t)}{P_i(t)\, Z_X}\right] P_i(t) X_i(t) \\ &= \frac{\xi}{1+\xi} \frac{P_Y(t) Y(t)}{N_0} \equiv \Pi^L(t),\end{aligned} \tag{14.136}$$

where we have used equation (14.132) and note that $\alpha = 1/N_0$ to arrive at the second expression. The leader produces a smaller amount of the input than the follower would have done, and thus drives up its price. In Figure 14.7, profit is equal to the shaded area ABCD. The crucial thing to note about (14.136) is that the profit attained by a quality leader is the same no matter which industry the leader is operating in! Similarly, equations (14.132)–(14.133), and (14.135) imply a number of symmetry results:

$$P_i(t) = \bar{P}(t) = (1+\xi)\, \frac{W(t)}{Z_X}, \tag{14.137}$$

[16] In Figure 14.6, we denote the quality leader in an industry with a solid dot (•) and the follower with a small dot in a box (⊡). In colloquial terms, the quality leader captures and incarcerates the follower (a former leader) and in the process cuts him/her to size (zero).

To check the remaining characteristic roots we note that $\Phi(s)$ can be written as:

$$\begin{aligned}\Phi(s) &= (s-\lambda_1)(s-\lambda_2) \\ &= s^2 - (\lambda_1+\lambda_2)s + \lambda_1\lambda_2 \end{aligned} \tag{19.95}$$

so it follows (from the comparison of (19.93) and (19.95)) that:

$$\lambda_1+\lambda_2 = 1+(1+\gamma)(1+\rho) > 2, \quad \lambda_1\lambda_2 = 1+\rho > 1. \tag{19.96}$$

The sum and product of the roots are both positive so we know that $\lambda_1 > 0$ and $\lambda_2 > 0$ for sure. Using (19.95) and (19.96) to compute $\Phi(1)$ results in:

$$\begin{aligned}\Phi(1) &= (1-\lambda_1)(1-\lambda_2) \\ &= 1-[1+(1+\gamma)(1+\rho)]+1+\rho \\ &= -\gamma(1+\rho) < 0. \end{aligned} \tag{19.97}$$

Hence, λ_1 and λ_2 are not only both positive but they also lie on either side of unity, say:

$$0 < \lambda_1 < 1, \qquad \lambda_2 > 1. \tag{19.98}$$

The canonical model is saddle-path stable when the money supply acts as the policy variable. The nominal interest rate is endogenous in that case and fluctuations in it will guarantee stability. Matters may not be so straightforward if the monetary policy maker wishes to set nominal interest rates.

Intermezzo 19.3

Blanchard-Kahn stability and existence conditions. To investigate stability of the rational expectations model the technique proposed by Blanchard and Kahn (1980) is particularly convenient. To illustrate their method we write the dynamic system in general as:

$$\begin{bmatrix} \boldsymbol{B}_{t+1} \\ E_t\boldsymbol{F}_{t+1} \end{bmatrix} = \Delta \begin{bmatrix} \boldsymbol{B}_t \\ \boldsymbol{F}_t \end{bmatrix} + \Psi\, \boldsymbol{X}_t, \tag{a}$$

where $\boldsymbol{B}_t$ is an $(n_b \times 1)$ vector of predetermined (backward-looking) variables, $\boldsymbol{F}_t$ is an $(n_f \times 1)$ vector of non-predetermined (forward-looking) variables, $\boldsymbol{X}_t$ is a $(k \times 1)$ vector of exogenous variables, Δ is an $(n_b + n_f) \times (n_b + n_f)$ matrix of coefficients, and Ψ is an $(n_b+n_f) \times k$ matrix of coefficients.

Blanchard and Kahn prove the following propositions:

- **B-K Proposition 1**: If the number of eigenvalues of Δ outside the unit circle (say n_f') is equal to the number of non-predetermined variables (n_f) then there exists a unique solution.
- **B-K Proposition 2**: If n_f' exceeds n_f there no solution.
- **B-K Proposition 3**: If n_f' falls short of n_f there is an infinity of solutions ("indeterminacy").

Blanchard and Kahn also provide explicit expressions for the solution paths in the case for which B-K Proposition 1 holds (i.e. if $n'_f = n_f$) but we do not use their expressions here.

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19.3.3 Interest rate setting, instability, and Taylor rules

Up to this point we have adopted the traditional approach by assuming that monetary policy is conducted by means of changes in the nominal money supply. In reality, however, Central Banks typically set interest rates in order to achieve their policy aims. In the context of our canonical model (given in equations (19.83)–(19.86) above) this means that the interest rate $\tilde{R}_t$ becomes the policy variable whilst the nominal money supply $\tilde{M}_{t+1}$ is endogenously (and residually) determined by (19.85).

With the interest rate as the policy variable, the canonical model is fully characterized by:

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \left[\tilde{R}_t - E_t\pi_{t+1}\right], \tag{19.99}$$

$$\pi_t = \gamma\left[\tilde{Y}_t - \tilde{Z}_t\right] + \frac{1}{1+\rho}E_t\pi_{t+1}. \tag{19.100}$$

The endogenous variables are output $\tilde{Y}_t$ and inflation π_t. The exogenous variables are the nominal interest rate $\tilde{R}_t$ and the technology shock $\tilde{Z}_t$. There is no predetermined variable in this system: given the lagged price $\tilde{P}_{t-1}$ the current price follows once π_t is known (as $\pi_t \equiv \tilde{P}_t - \tilde{P}_{t-1}$). Is the model still saddle-path stable under this type of monetary policy?

Following similar steps as before we can deduce the BK format of the model. First we use (19.99)–(19.100) to obtain:

$$\Gamma\begin{bmatrix} E_t\tilde{Y}_{t+1} \\ E_t\tilde{\pi}_{t+1}\end{bmatrix} = \Delta^*\begin{bmatrix}\tilde{Y}_t \\ \tilde{\pi}_t\end{bmatrix} + \begin{bmatrix}\tilde{R}_t \\ \gamma(1+\rho)\tilde{Z}_t\end{bmatrix}, \tag{19.101}$$

where Γ, Γ^{-1}, and Δ^* are given by:

$$\Gamma \equiv \begin{bmatrix} 1 & 1 \\ 0 & 1\end{bmatrix}, \qquad \Gamma^{-1} \equiv \begin{bmatrix} 1 & -1 \\ 0 & 1\end{bmatrix}, \qquad \Delta^* \equiv \begin{bmatrix} 1 & 0 \\ -\gamma(1+\rho) & 1+\rho\end{bmatrix}. \tag{19.102}$$

It follows that equation (19.101) can be expressed in the BK form as:

$$\begin{bmatrix} E_t\tilde{Y}_{t+1} \\ E_t\tilde{\pi}_{t+1}\end{bmatrix} = \Delta\begin{bmatrix}\tilde{Y}_t \\ \tilde{\pi}_t\end{bmatrix} + \begin{bmatrix}\tilde{R}_t - \gamma(1+\rho)\tilde{Z}_t \\ \gamma(1+\rho)\tilde{Z}_t\end{bmatrix}, \tag{19.103}$$

where $\Delta \equiv \Gamma^{-1}\Delta^*$ is defined as:

$$\Delta \equiv \begin{bmatrix} 1+\gamma(1+\rho) & -(1+\rho) \\ -\gamma(1+\rho) & 1+\rho\end{bmatrix}. \tag{19.104}$$

Since output and inflation are jumping variables, stability holds if and only if the Jacobian matrix Δ features two unstable roots, i.e. both λ_1 and λ_2 must lie outside the unit circle. Our usual trick can be used to establish the signs and magnitude of the characteristic roots. The characteristic equation of Δ is:

$$\Phi(s) = s^2 - [1 + (1+\gamma)(1+\rho)]\, s + 1 + \rho,$$

and it is easy to verify that:

$$\lambda_1 + \lambda_2 = 1 + (1+\gamma)(1+\rho) > 2, \qquad \lambda_1\lambda_2 = 1+\rho > 1.$$

By computing $\Phi(1)$ we find:

$$\begin{aligned}\Phi(1) &= (1-\lambda_1)(1-\lambda_2)\\ &= 1 - [2+\rho+\gamma(1+\rho)] + 1 + \rho = -\gamma(1+\rho) < 0,\end{aligned}$$

which implies that both roots are positive and lie on either side of unity, say:

$$0 < \lambda_1 < 1, \qquad \lambda_2 > 1. \tag{19.105}$$

The root condition is clearly not satisfied as there are two non-predetermined variables and only one root outside the unit circle. Proposition 3 from Intermezzo 19.3 implies that there exist infinitely many solutions—the model suffers from *indeterminacy*. The economic lesson is that using the nominal interest rate as a policy variable induces indeterminacy in an economy that is stable if the Central Bank would just control the money supply. Loosely put, the intuition behind this result is as follows. Assume that expected future inflation rises ($E_t\tilde{\pi}_{t+1} \uparrow$). Since $\tilde{R}_t$ does not react to this change the real interest rate falls. This prompts an increase in output via the dynamic IS curve (19.99) ($\tilde{Y}_t \uparrow$). But this output change in turn boosts current inflation via the New Keynesian Phillips curve (19.100) and ($\tilde{\pi}_t \uparrow$). A self-fulfilling increase in inflation emerges: if the public thinks the inflation rate will rise then it will.

The indeterminacy problem arises because the interest rate set by the Central Bank is independent of the state of the economy, i.e. we have implicitly assumed that the Central Bank uses a passive policy rule (PPR hereafter) like:

$$\tilde{R}_t = \tilde{U}_t, \qquad \text{PPR}, \tag{19.106}$$

where $\tilde{U}_t$ is an exogenously given stationary stochastic process. In reality policy makers may not be that passive. To see if it could help eliminate indeterminacy let us try a feed-back policy rule (FPR hereafter) that chokes off the inflation spiral mentioned above:

$$\tilde{R}_t = \delta_\pi \pi_t + \tilde{U}_t, \qquad \delta_\pi > 0, \qquad \text{FPR}_1. \tag{19.107}$$

There is some hope such a rule may do the job as an increase in inflation will result in an increase in the nominal interest rate. To see whether this hunch works we write the system consisting of (19.99)–(19.100) and (19.107) in the form of (19.103) and find that element $(1,2)$ of Δ^* in (19.102) changes from 0 to δ_π and Δ becomes:

$$\Delta \equiv \begin{bmatrix} 1+\gamma(1+\rho) & \delta_\pi - (1+\rho) \\ -\gamma(1+\rho) & 1+\rho \end{bmatrix}. \tag{19.108}$$

We easily find that the characteristic roots satisfy:

$$\lambda_1 + \lambda_2 = 2 + \rho + \gamma(1+\rho) > 2, \qquad \lambda_1\lambda_2 = (1+\rho)[1+\delta_\pi\gamma] > 1+\rho,$$

whilst the characteristic equation evaluated at $s = 1$ gives:

$$\Phi(1) = (1-\lambda_1)(1-\lambda_2) = (\delta_\pi - 1)\,\gamma\,(1+\rho)\,.$$

Provided the feedback coefficient exceeds unity ($\delta_\pi > 1$) we find that $\Phi(1) > 0$ so that both roots are larger than unity, i.e.:

$$\lambda_1 > 1, \qquad \lambda_2 > 1. \tag{19.109}$$

The root condition is clearly satisfied because we have two non-predetermined variables and two characteristic roots outside the unit circle. Proposition 1 from Intermezzo 19.3 implies that there exist unique solution paths for output and inflation. The economic lesson is that a feed-back policy rule such as FPR_1 eliminates indeterminacy provided $\delta_\pi > 1$. This is called the *Taylor principle* after John B. Taylor (1993) who was one of the first economists to stress that the interest rate rule should react more than one-for-one to inflation. Intuitively, under this rule an increase in inflation prompts a change in the nominal interest rate large enough to cause an increase in the *real* interest rate.

As the icing of the cake, let us try a more complicated feedback rule that responds both to inflation and to output:

$$\tilde{R}_t = \delta_\pi \pi_t + \delta_y \tilde{Y}_t + \tilde{U}_t, \qquad \delta_\pi,\, \delta_y > 0, \qquad \text{FPR}_2. \tag{19.110}$$

This is an example of a *Taylor rule*, and empirically such a rule seems to have been followed by many Central Banks in the world. To see how this policy rule works we write the system consisting of (19.99)–(19.100) and (19.110) in the form of (19.103) and find that element $(1,1)$ of Δ^* in (19.102) changes from 1 to $1+\delta_y$ whilst (just as before) element $(1,2)$ of that matrix changes from 0 to δ_π. As a result Δ becomes:

$$\Delta \equiv \begin{bmatrix} 1+\delta_y+\gamma\,(1+\rho) & \delta_\pi - (1+\rho) \\ -\gamma\,(1+\rho) & 1+\rho \end{bmatrix}. \tag{19.111}$$

We easily find that:

$$\lambda_1 + \lambda_2 = 2 + \delta_y + \rho + \gamma\,(1+\rho) > 2, \qquad \lambda_1\lambda_2 = (1+\rho)\left[1+\delta_y+\delta_\pi\gamma\right] > 1+\rho,$$

and that:

$$\Phi(1) = (1-\lambda_1)(1-\lambda_2) = \rho\delta_y + (\delta_\pi - 1)\,\gamma\,(1+\rho)\,. \tag{19.112}$$

The stability condition is that $\Phi(1) > 0$ so that (19.109) holds. In view of (19.112) we conclude that, since $\delta_y > 0$ we now have that $\delta_\pi > 1$ is no longer a necessary condition for determinacy. Even if δ_π falls short of unity, the model is still determinate provided δ_y is large enough.

19.4 Back to the general case

The canonical New Keynesian DSGE model is quite useful in that its basic mechanism are quite transparent and it allows us to analytically investigate stability issues arising from the particular form of the monetary policy rules. Of course the canonical model has severe limitations, the most important of which is the absence of capital accumulation effects in response to economic shocks. And in Chapter 18 we saw

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