Instructor's Manual

for

Numerical Analysis

Eighth Edition

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Preface

This Instructor's Manual for the Eighth of Numerical Analysis by Burden and Faires contains solutions to all the exercises in the book. Although the answers to the odd exercises are also in the back of the text, we have found that users of the book appreciate having all the solutions in one source. In addition, the results listed in this Instructor's Manual often go beyond those given in the back of the book. For example, we do not place the long solutions to theoretical and applied exercises in the book. You will find them here.

It has been our practice to include structured algorithms of all the techniques discussed in our Numerical Analysis book. The algorithms are given in a form that can be coded in any appropriate programming language, by those with even a minimal amount of programming expertise.

In earlier editions of the book, we included in the Instructor's Manual a complete FORTRAN listing for all the algorithms, and distributed to instructors using the book, upon demand, a tape (actually punched cards in the First Edition) containing all these programs.

In the Fourth Edition we supplemented this with a disk containing Pascal programs for the algorithms. In the Fifth Edition we added FORTRAN programs to the package. In the Sixth Edition we placed the disk in the text itself, and added C programs, as well as worksheets in Maple and Mathematica, for all the algorithms. We continued this practice for the Seventh Edition, updating the Maple programs to both versions 5.0 and 6.0 and adding MATLAB programs as well.

For the Eighth Edition, we have added new Maple programs to reflect the linear algebra package change from the original `linalg` package to the more modern `LinearAlgebra` package. In addition, we now also have the programs coded in Java.

You will not find a disk with this edition of the book. Instead, our reviewers suggested, and we agree, that it is more convenient to have the programs available for downloading from the web. At the website for the book,

http://www.as.ysu.edu/~faires/Numerical-Analysis/

you will find all the programs that used to be on the disk that came with the book. This site also contains additional information about the book and will be updated regularly to reflect any modifications that need to be made. For example, we will list a copy of the adoption list for the book so that potential users can ask colleagues for suggestions, and any changes made when a new printing is produced.

Placing the programs on the web site also permits us to more easily updated programs as the software changes, and to give responses to comments made by users of the book. We can also add new material that might be included in a subsequent edition in the form of PDF files that users can download. Our hope is that this will extend the life of the Eighth Edition while keeping the material up to date.

In addition to this Manual, we have rewritten the Student Study Guide for the Eighth Edition. The exercises that are solved in the Guide are generally those requiring insight into the methods in the text, rather than those involving computation. The Guide should be especially helpful for those doing self study of numerical techniques. Please ask your students to contact us if they are interested in this Guide.

We hope our supplement package provides flexibility for instructors teaching Numerical Analysis. If you have any suggestions for improvements that can be incorporated into future editions of the book or the supplements, we would be most grateful to receive your comments. We can be most easily contacted by electronic mail at the addresses listed below.

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January 21, 2005

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Mathematical Preliminaries

Exercise Set 1.1, page 14

1. For each part, $f \in C[a,b]$ on the given interval. Since $f(a)$ and $f(b)$ are of opposite sign, the Intermediate Value Theorem implies that a number c exists with $f(c) = 0$.

2. (a) $[0,1]$ (b) $[0,1]$, $[4,5]$, $[-1,0]$
   (c) $[-2,-1]$, $[0,1]$, $[2.5,3.5]$ (d) $[-3,-2]$, $[-1,-0.5]$, and $[-0.5,0]$

3. For each part, $f \in C[a,b]$, f' exists on (a,b) and $f(a) = f(b) = 0$. Rolle's Theorem implies that a number c exists in (a,b) with $f'(c) = 0$. For part (d), we can use $[a,b] = [-1,0]$ or $[a,b] = [0,2]$.

4. The maximum value for $|f(x)|$ is given below.
   (a) 0.4620981 (b) 0.8 (c) 5.164000 (d) 1.582572

5. For $x < 0$, $f(x) < 2x + k < 0$, provided that $x < -\frac{1}{2}k$. Similarly, for $x > 0$, $f(x) > 2x + k > 0$, provided that $x > -\frac{1}{2}k$. By Theorem 1.13, there exists a number c with $f(c) = 0$. If $f(c) = 0$ and $f(c') = 0$ for some $c' \neq c$, then by Theorem 1.7, there exists a number p between c and c' with $f'(p) = 0$. However, $f'(x) = 3x^2 + 2 > 0$ for all x.

6. Suppose p and q are in $[a,b]$ with $p \neq q$ and $f(p) = f(q) = 0$. By the Mean Value Theorem, there exists $\xi \in (a,b)$ with
$$f(p) - f(q) = f'(\xi)(p - q).$$
But, $f(p) - f(q) = 0$ and $p \neq q$. So $f'(\xi) = 0$, contradicting the hypothesis.

7. (a) $P_2(x) = 0$
   (b) $R_2(0.5) = 0.125$; actual error $= 0.125$
   (c) $P_2(x) = 1 + 3(x-1) + 3(x-1)^2$
   (d) $R_2(0.5) = -0.125$; actual error $= -0.125$

(c) We have $\sin 0.34 \approx H_7(0.34) = 0.33350$. Although the error bound is now 5.4×10^{-20}, the accuracy of the given data dominates the calculations. This result is actually less accurate than the approximation in part (b), since $\sin 0.34 = 0.333487$.

6. (a) $H(1.03) = 0.80932485$. The actual error is 1.24×10^{-6}, and error bound is 1.31×10^{-6}.

   (b) $H(1.03) = 0.809323619263$. The actual error is 3.63×10^{-10}, and an error bound is 3.86×10^{-10}.

7. For 3(a), we have an error bound of 5.9×10^{-8}. The error bound for 3(c) is 0 since $f^{(n)}(x) \equiv 0$, for $n > 3$.

8. For 4(a), we have an error bound of 1.6×10^{-3}. The error bound for 4(c) is 1.5×10^{-7}.

9. $H_3(1.25) = 1.169080403$ with an error bound of 4.81×10^{-5}, and $H_5(1.25) = 1.169016064$ with an error bound of 4.43×10^{-4}.

10. The Hermite polynomial generated from these data is

$$\begin{aligned} H_9(x) =& 75x + 0.222222x^2(x-3) - 0.0311111x^2(x-3)^2 \\ &- 0.00644444x^2(x-3)^2(x-5) + 0.00226389x^2(x-3)^2(x-5)^2 \\ &- 0.000913194x^2(x-3)^2(x-5)^2(x-8) + 0.000130527x^2(x-3)^2(x-5)^2(x-8)^2 \\ &- 0.0000202236x^2(x-3)^2(x-5)^2(x-8)^2(x-13). \end{aligned}$$

   (a) The Hermite polynomial predicts a position of $H_9(10) = 743$ ft and a speed of $H_9'(10) = 48$ ft/sec. Although the position approximation is reasonable, the low speed prediction is suspect.

   (b) To find the first time the speed exceeds 55 mi/hr, which is equivalent to $80.\overline{6}$ ft/sec, we solve for the smallest value of t in the equation $80.\overline{6} = H_9'(x)$. This gives $x \approx 5.6488092$.

   (c) The estimated maximum speed is $H_9'(12.37187) = 119.423$ ft/sec ≈ 81.425 mi/hr.

11. (a) Suppose $P(x)$ is another polynomial with $P(x_k) = f(x_k)$ and $P'(x_k) = f'(x_k)$, for $k = 0, \ldots, n$, and the degree of $P(x)$ is at most $2n+1$. Let

$$D(x) = H_{2n+1}(x) - P(x).$$

   Then $D(x)$ is a polynomial of degree at most $2n+1$ with $D(x_k) = 0$, and $D'(x_k) = 0$, for each $k = 0, 1, \ldots, n$. Thus, D has zeros of multiplicity 2 at each x_k and

$$D(x) = (x-x_0)^2 \ldots (x-x_n)^2 Q(x).$$

   Hence, $D(x)$ must be of degree $2n$ or more, which would be a contradiction, or $Q(x) \equiv 0$ which implies that $D(x) \equiv 0$. Thus, $P(x) \equiv H_{2n+1}(x)$.

   (b) First note that the error formula holds if $x = x_k$ for any choice of ξ.
   Let $x \neq x_k$, for $k = 0, \ldots, n$, and define

$$g(t) = f(t) - H_{2n+1}(t) - \frac{(t-x_0)^2 \ldots (t-x_n)^2}{(x-x_0)^2 \ldots (x-x_n)^2}\left[f(x) - H_{2n+1}(x)\right].$$

   Note that $g(x_k) = 0$, for $k = 0, \ldots, n$, and $g(x) = 0$. Thus, g has $n+2$ distinct zeros in $[a, b]$. By Rolle's Theorem, g' has $n+1$ distinct zeros $\xi_0, \ldots, \xi_n$, which are between the

7. The Trapezoidal Algorithm gives the results in the following tables.

(a)

t_i	w_i	k	y_i
0.200	0.39109643	2	0.44932896
0.500	0.02134361	2	0.03019738
0.700	0.00307084	2	0.00499159
1.000	0.00016759	2	0.00033546

(b)

t_i	w_i	k	y_i
0.200	0.04000000	2	0.04610521
0.500	0.25000000	2	0.25001513
0.700	0.49000000	2	0.49000028
1.000	1.00000000	2	1.00000000

(c)

t_i	w_i	k	y_i
0.500	0.66291133	2	0.47947094
1.000	0.87506346	2	0.84147099
1.500	1.00366141	2	0.99749499
2.000	0.91053267	2	0.90929743

(d)

t_i	w_i	k	y_i
0.200	−1.07568307	4	1.00000000
0.500	−0.97868360	4	1.00000000
0.700	−0.99046408	3	1.00000000
1.000	−1.00284456	3	1.00000000

8. The Trapezoidal Algorithm gives the results in the following tables.

(a)

t_i	w_i	k	$y(t_i)$
0.2	1.58152229	2	
0.4	1.62161373	2	
0.6	1.86902118	2	
0.8	2.24264359	2	
1.0	2.72470482	2	

(b)

t_i	w_i	k	$y(t_i)$
0.2	0.50203131	2	
0.4	0.43355903	2	
0.6	0.60372878	2	
0.8	0.80041431	2	
1.0	1.00004603	2	

(c)

t_i	w_i	k	$y(t_i)$
1.25	0.49787835	2	
1.75	0.18503499	2	
2.25	0.08754204	2	
2.75	0.04802381	2	

(d)

t_i	w_i	k	$y(t_i)$
0.25	1.39753016	2	
0.75	0.81057360	2	
1.25	0.33002435	2	
1.75	-0.17533094	2	

6. Euler's method gives the approximations in the following tables.

(a)

i	t_i	w_i
2	0.2	1.374257426
3	0.3	1.513709064
9	0.9	1.631412128
10	1.0	1.579669485

(b)

i	t_i	w_i
2	1.2	−1.253297013
3	1.3	−1.181899131
9	1.9	−0.9150285539
10	2.0	−0.8861569244

(c)

i	t_i	w_i
5	2.0	−1.248872291
6	2.2	−1.217791320
8	2.6	−1.174414016
9	2.8	−1.158657534

(d)

i	t_i	w_i
5	0.5	1.255609618
6	0.6	1.352114314
9	0.9	1.624904878
10	1.0	1.700214869

7. The actual errors for the approximations in Exercise 3 are in the following tables.

(a)

t	Actual error
1.2	0.0066879
1.5	0.0095942
1.7	0.0102229
2.0	0.0105806

(b)

t	Actual error
1.4	0.0507928
2.0	0.2240306
2.4	0.4742818
3.0	1.3598226

(c)

t	Actual error
0.4	0.0120510
1.0	0.0391546
1.4	0.0349030
2.0	0.0178206

(d)

t	Actual error
0.2	0.0542931
0.5	0.0363200
0.7	0.0273054
1.0	0.0219009

25.

	Jacobi 33 iterations	Gauss-Seidel 8 iterations	SOR ($\omega = 1.2$) 13 iterations
x_1	1.53873501	1.53873270	1.53873549
x_2	0.73142167	0.73141966	0.73142226
x_3	0.10797136	0.10796931	0.10797063
x_4	0.17328530	0.17328340	0.17328480
x_5	0.04055865	0.04055595	0.04055737
x_6	0.08525019	0.08524787	0.08524925
x_7	0.16645040	0.16644711	0.16644868
x_8	0.12198156	0.12197878	0.12198026
x_9	0.10125265	0.10124911	0.10125043
x_{10}	0.09045966	0.09045662	0.09045793
x_{11}	0.07203172	0.07202785	0.07202912
x_{12}	0.07026597	0.07026266	0.07026392
x_{13}	0.06875835	0.06875421	0.06875546
x_{14}	0.06324659	0.06324307	0.06324429
x_{15}	0.05971510	0.05971083	0.05971200
x_{16}	0.05571199	0.05570834	0.05570949
x_{17}	0.05187851	0.05187416	0.05187529
x_{18}	0.04924911	0.04924537	0.04924648
x_{19}	0.04678213	0.04677776	0.04677885
x_{20}	0.04448679	0.04448303	0.04448409
x_{21}	0.04246924	0.04246493	0.04246597
x_{22}	0.04053818	0.04053444	0.04053546
x_{23}	0.03877273	0.03876852	0.03876952
x_{24}	0.03718190	0.03717822	0.03717920
x_{25}	0.03570858	0.03570451	0.03570548
x_{26}	0.03435107	0.03434748	0.03434844
x_{27}	0.03309542	0.03309152	0.03309246
x_{28}	0.03192212	0.03191866	0.03191958
x_{29}	0.03083007	0.03082637	0.03082727
x_{30}	0.02980997	0.02980666	0.02980755
x_{31}	0.02885510	0.02885160	0.02885248
x_{32}	0.02795937	0.02795621	0.02795707
x_{33}	0.02711787	0.02711458	0.02711543
x_{34}	0.02632478	0.02632179	0.02632262