C H A P T E R 1
Preparation for Calculus

CHAPTER 1
Preparation for Calculus

Section 1.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

2. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

3. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

4. $y = x^3 - x$

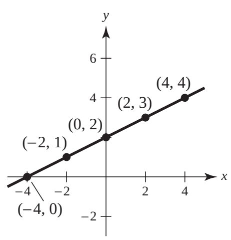
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

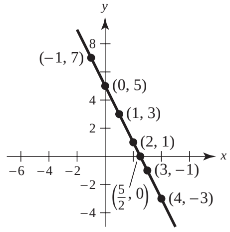
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



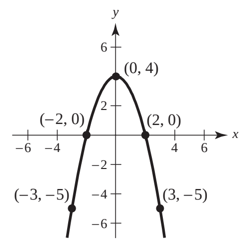
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



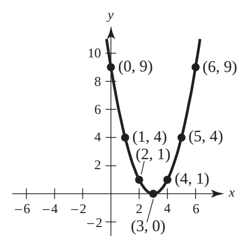
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



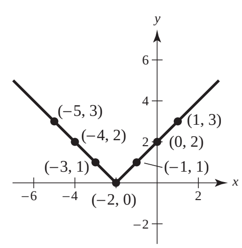
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



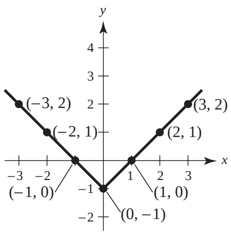
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



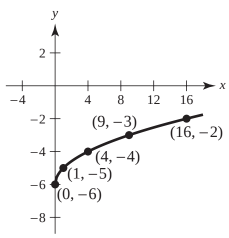
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



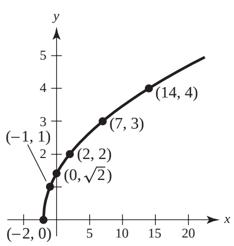
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



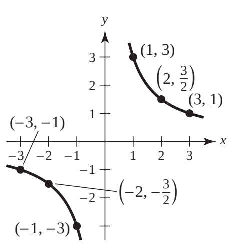
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



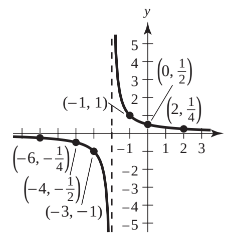
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

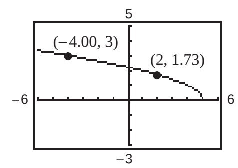


14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



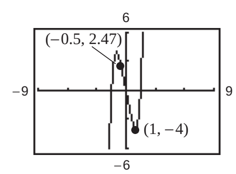
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2}$; $\left(\frac{5}{2}, 0\right)$

18. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

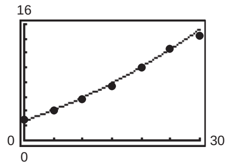
x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

67. (a) Using a graphing utility, you obtain
$y = 0.005t^2 + 0.27t + 2.7.$

(b)



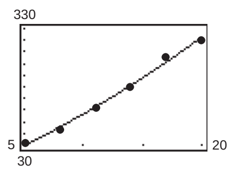
(c) For 2020, $t = 40.$

$$y = 0.005(40)^2 + 0.27(40) + 2.7 = 21.5$$

The GDP in 2020 will be $21.5 trillion.

68. (a) Using a graphing utility, you obtain
$y = 0.24t^2 + 12.6t - 40.$

(b)



The model is a good fit for the data.

(c) For 2020, $t = 30.$

$$y = 0.24(30)^2 + 12.6(30) - 40 = 554$$

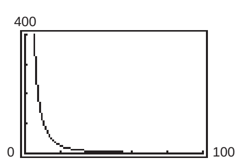
The number of cellular phone subscribers in 2020 will be 554 million.

69.

$$\begin{aligned} C &= R \\ 2.04x + 5600 &= 3.29x \\ 5600 &= 3.29x - 2.04x \\ 5600 &= 1.25x \\ x &= \frac{5600}{1.25} = 4480 \end{aligned}$$

To break even, 4480 units must be sold.

70. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

71. $y = kx^3$

(a) $(1, 4)$: $\quad 4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $\quad 1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $\quad 0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $\quad -1 = k(-1)^3 = -k \Rightarrow k = 1$

72. $y^2 = 4kx$

(a) $(1, 1)$:
$$\begin{aligned} 1^2 &= 4k(1) \\ 1 &= 4k \\ k &= \tfrac{1}{4} \end{aligned}$$

(b) $(2, 4)$:
$$\begin{aligned} (4)^2 &= 4k(2) \\ 16 &= 8k \\ k &= 2 \end{aligned}$$

(c) $(0, 0)$: $\quad 0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$:
$$\begin{aligned} (3)^2 &= 4k(3) \\ 9 &= 12k \\ k &= \tfrac{9}{12} = \tfrac{3}{4} \end{aligned}$$

73. Answers may vary. *Sample answer*:
$y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

74. Answers may vary. *Sample answer*:
$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

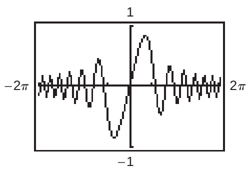
75. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

87. $f(x) = \dfrac{\sin x^2}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	–0.099998	–0.01	–0.001	?	0.001	0.01	0.099998

It appears that the limit is 0.



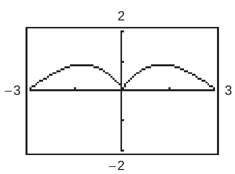
The graph has a hole at $x = 0$.

Analytically, $\lim\limits_{x\to 0}\dfrac{\sin x^2}{x} = \lim\limits_{x\to 0} x\left(\dfrac{\sin x^2}{x}\right) = 0(1) = 0.$

88. $f(x) = \dfrac{\sin x}{\sqrt[3]{x}}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.215	0.0464	0.01	?	0.01	0.0464	0.215

It appears that the limit is 0.

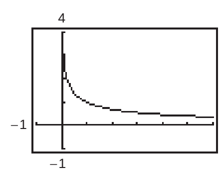


The graph has a hole at $x = 0$.

Analytically, $\lim\limits_{x\to 0}\dfrac{\sin x}{\sqrt[3]{x}} = \lim\limits_{x\to 0} \sqrt[3]{x^2}\left(\dfrac{\sin x}{x}\right) = (0)(1) = 0.$

89. $f(x) = \dfrac{\ln x}{x - 1}$

x	0.5	0.9	0.99	1.01	1.1	1.5
$f(x)$	1.3863	1.0536	1.0050	0.9950	0.9531	0.8109

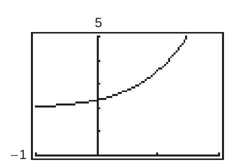


It appears that the limit is 1.

Analytically, $\lim\limits_{x\to 1}\dfrac{\ln x}{x - 1} = 1.$

90. $f(x) = \dfrac{e^{3x} - 8}{e^{2x} - 4}$

x	0.5	0.6	0.69	0.70	0.8	0.9
$f(x)$	2.7450	2.8687	2.9953	3.0103	3.1722	3.3565



It appears that the limit is 3.

Analytically, $\lim\limits_{x\to \ln 2}\dfrac{e^{3x} - 8}{e^{2x} - 4} = \lim\limits_{x\to \ln 2}\dfrac{(e^x - 2)(e^{2x} + 2e^x + 4)}{(e^x - 2)(e^x + 2)} = \lim\limits_{x\to \ln 2}\dfrac{e^{2x} + 2e^x + 4}{e^x + 2} = \dfrac{4 + 4 + 4}{2 + 2} = 3.$

7.
$$\begin{aligned} x^3y^3 - y - x &= 0 \\ 3x^3y^2y' + 3x^2y^3 - y' - 1 &= 0 \\ \left(3x^3y^2 - 1\right)y' &= 1 - 3x^2y^3 \\ y' &= \frac{1 - 3x^2y^3}{3x^3y^2 - 1} \end{aligned}$$

8.
$$\begin{aligned} \sqrt{xy} &= x^2y + 1 \\ \frac{1}{2}(xy)^{-1/2}(xy' + y) &= 2xy + x^2y' \\ \frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} &= 2xy + x^2y' \\ \left(\frac{x}{2\sqrt{xy}} - x^2\right)y' &= 2xy - \frac{y}{2\sqrt{xy}} \\ y' &= \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2} \\ y' &= \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}} \end{aligned}$$

9.
$$\begin{aligned} xe^y - 10x + 3y &= 0 \\ xe^y\frac{dy}{dx} + e^y - 10 + 3\frac{dy}{dx} &= 0 \\ \frac{dy}{dx}\left(xe^y + 3\right) &= 10 - e^y \\ \frac{dy}{dx} &= \frac{10 - e^y}{xe^y + 3} \end{aligned}$$

10.
$$\begin{aligned} e^{xy} + x^2 - y^2 &= 10 \\ \left(x\frac{dy}{dx} + y\right)e^{xy} + 2x - 2y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx}\left(xe^{xy} - 2y\right) &= -ye^{xy} - 2x \\ \frac{dy}{dx} &= -\frac{ye^{xy} + 2x}{xe^{xy} - 2y} \end{aligned}$$

11.
$$\begin{aligned} \sin x + 2\cos 2y &= 1 \\ \cos x - 4(\sin 2y)y' &= 0 \\ y' &= \frac{\cos x}{4\sin 2y} \end{aligned}$$

12.
$$\begin{aligned} (\sin \pi x + \cos \pi y)^2 &= 2 \\ 2(\sin \pi x + \cos \pi y)\left[\pi \cos \pi x - \pi(\sin \pi y)y'\right] &= 0 \\ \pi \cos \pi x - \pi(\sin \pi y)y' &= 0 \\ y' &= \frac{\cos \pi x}{\sin \pi y} \end{aligned}$$

13.
$$\begin{aligned} \sin x &= x(1 + \tan y) \\ \cos x &= x\left(\sec^2 y\right)y' + (1 + \tan y)(1) \\ y' &= \frac{\cos x - \tan y - 1}{x\sec^2 y} \end{aligned}$$

14.
$$\begin{aligned} \cot y &= x - y \\ \left(-\csc^2 y\right)y' &= 1 - y' \\ y' &= \frac{1}{1 - \csc^2 y} = \frac{1}{-\cot^2 y} = -\tan^2 y \end{aligned}$$

15.
$$\begin{aligned} y &= \sin xy \\ y' &= \left[xy' + y\right]\cos(xy) \\ y' - x\cos(xy)y' &= y\cos(xy) \\ y' &= \frac{y\cos(xy)}{1 - x\cos(xy)} \end{aligned}$$

16.
$$\begin{aligned} x &= \sec\frac{1}{y} \\ 1 &= -\frac{y'}{y^2}\sec\frac{1}{y}\tan\frac{1}{y} \\ y' &= \frac{-y^2}{\sec(1/y)\tan(1/y)} = -y^2\cos\left(\frac{1}{y}\right)\cot\left(\frac{1}{y}\right) \end{aligned}$$

17.
$$\begin{aligned} x^2 - 3\ln y + y^2 &= 10 \\ 2x - \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} &= 0 \\ 2x &= \frac{dy}{dx}\left(\frac{3}{y} - 2y\right) \\ \frac{dy}{dx} &= \frac{2x}{(3/y) - 2y} = \frac{2xy}{3 - 2y^2} \end{aligned}$$

18.
$$\begin{aligned} \ln(xy) + 5x &= 30 \\ \ln x + \ln y + 5x &= 30 \\ \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + 5 &= 0 \\ \frac{1}{y}\frac{dy}{dx} &= -\frac{1}{x} - 5 \\ \frac{dy}{dx} &= -\frac{y}{x} - 5y = -\left(\frac{y + 5xy}{x}\right) \end{aligned}$$

13. $y = x\tan x$

$dy = (x\sec^2 x + \tan x)dx$

14. $y = \csc 2x$

$dy = (-2\csc 2x \cot 2x)dx$

15. $y = \dfrac{x+1}{2x-1}$

$dy = -\dfrac{3}{(2x-1)^2}\,dx$

16. $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

$dy = \left(\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2x\sqrt{x}}\right)dx = \dfrac{x-1}{2x\sqrt{x}}\,dx$

17. $y = \sqrt{9 - x^2}$

$dy = \dfrac{1}{2}(9 - x^2)^{-1/2}(-2x)\,dx = \dfrac{-x}{\sqrt{9 - x^2}}\,dx$

18. $y = x\sqrt{1 - x^2}$

$dy = \left(x\dfrac{-x}{\sqrt{1-x^2}} + \sqrt{1-x^2}\right)dx = \dfrac{1 - 2x^2}{\sqrt{1-x^2}}\,dx$

19. $y = 3x - \sin^2 x$

$dy = (3 - 2\sin x\cos x)\,dx = (3 - \sin 2x)\,dx$

20. $y = \dfrac{\sec^2 x}{x^2 + 1}$

$dy = \left[\dfrac{(x^2+1)2\sec^2 x\tan x - \sec^2 x(2x)}{(x^2+1)^2}\right]dx$

$= \left[\dfrac{2\sec^2 x(x^2\tan x + \tan x - x)}{(x^2+1)^2}\right]dx$

21. $y = \ln\sqrt{4 - x^2} = \dfrac{1}{2}\ln(4 - x^2)$

$dy = \dfrac{1}{2}\left(\dfrac{-2x}{4 - x^2}\right)dx = \dfrac{-x}{4 - x^2}\,dx$

22. $y = e^{-0.5x}\cos 4x$

$dy = \left[e^{-0.5x}(-4\sin 4x) + (-0.5)e^{-0.5x}\cos 4x\right]dx$

$= e^{-0.5x}\left[-4\sin 4x - 0.5\cos 4x\right]dx$

23. $y = x\arcsin x$

$dy = \left(\dfrac{x}{\sqrt{1 - x^2}} + \arcsin x\right)dx$

24. $y = \arctan(x - 2)$

$dy = \dfrac{1}{1 + (x-2)^2}\,dx$

25. (a) $f(1.9) = f(2 - 0.1) \approx f(2) + f'(2)(-0.1)$

$\approx 1 + (1)(-0.1) = 0.9$

(b) $f(2.04) = f(2 + 0.04) \approx f(2) + f'(2)(0.04)$

$\approx 1 + (1)(0.04) = 1.04$

26. (a) $f(1.9) = f(2 - 0.1) \approx f(2) + f'(2)(-0.1)$

$\approx 1 + \left(-\tfrac{1}{2}\right)(-0.1) = 1.05$

(b) $f(2.04) = f(2 + 0.04) \approx f(2) + f'(2)(0.04)$

$\approx 1 + \left(-\tfrac{1}{2}\right)(0.04) = 0.98$

27. (a) $g(2.93) = g(3 - 0.07) \approx g(3) + g'(3)(-0.07)$

$\approx 8 + \left(-\tfrac{1}{2}\right)(-0.07) = 8.035$

(b) $g(3.1) = g(3 + 0.1) \approx g(3) + g'(3)(0.1)$

$\approx 8 + \left(-\tfrac{1}{2}\right)(0.1) = 7.95$

28. (a) $g(2.93) = g(3 - 0.07) \approx g(3) + g'(3)(-0.07)$

$\approx 8 + (3)(-0.07) = 7.79$

(b) $g(3.1) = g(3 + 0.1) \approx g(3) + g'(3)(0.1)$

$\approx 8 + (3)(0.1) = 8.3$

29. $x = 10$ in., $\Delta x = dx = \pm\dfrac{1}{32}$ in.

(a) $A = x^2$

$dA = 2x dx$

$\Delta A \approx dA = 2(10)\left(\pm\dfrac{1}{32}\right) = \pm\dfrac{5}{8}$ in.^{2}

(b) Percent error:

$\dfrac{dA}{A} = \dfrac{5/8}{100} = \dfrac{5}{800} = \dfrac{1}{100} = 0.00625 = 0.625\%$

30. $r = 16$ in., $\Delta r = dr = \pm\dfrac{1}{4}$ in.

(a) $A = \pi r^2$

$dA = 2\pi r\,dr$

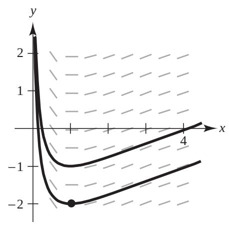
$\Delta A \approx dA = 2\pi(16)\left(\pm\dfrac{1}{4}\right) = \pm 8\pi$ in.^{2}

(b) Percent error:

$\dfrac{dA}{A} = \dfrac{8\pi}{\pi(16)^2} = \dfrac{1}{32} = 0.03125 = 3.125\%$

50. $\dfrac{dy}{dx} = \dfrac{\ln x}{x}, (1, -2)$

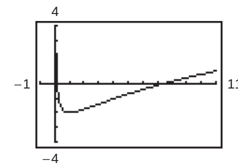
(a)



(b) $y = \displaystyle\int \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2} + C$

$y(1) = -2 \Rightarrow -2 = \dfrac{(\ln 1)^2}{2} + C \Rightarrow C = -2$

So, $y = \dfrac{(\ln x)^2}{2} - 2.$



51. $\displaystyle\int_0^4 \frac{5}{3x+1}\,dx = \left[\frac{5}{3}\ln|3x+1|\right]_0^4 = \frac{5}{3}\ln 13 \approx 4.275$

52. $\displaystyle\int_{-1}^1 \frac{1}{2x+3}\,dx = \frac{1}{2}\Big[\ln|2x+3|\Big]_{-1}^1$

$\displaystyle = \frac{1}{2}[\ln 5 - \ln 1] = \frac{1}{2}\ln 5 \approx 0.805$

53. $u = 1 + \ln x, du = \dfrac{1}{x}\,dx$

$\displaystyle\int_1^e \frac{(1+\ln x)^2}{x}\,dx = \left[\frac{1}{3}(1+\ln x)^3\right]_1^e = \frac{7}{3}$

54. $u = \ln x, du = \dfrac{1}{x}dx$

$\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx = \int_e^{e^2}\left(\frac{1}{\ln x}\right)\frac{1}{x}\,dx = \Big[\ln\big|\ln|x|\big|\Big]_e^{e^2} = \ln 2$

≈ 0.693

55. $\displaystyle\int_0^2 \frac{x^2-2}{x+1}\,dx = \int_0^2\left(x - 1 - \frac{1}{x+1}\right)dx$

$\displaystyle = \left[\frac{1}{2}x^2 - x - \ln|x+1|\right]_0^2 = -\ln 3$

≈ -1.099

56. $\displaystyle\int_0^1 \frac{x-1}{x+1}\,dx = \int_0^1 1\,dx + \int_0^1 \frac{-2}{x+1}\,dx$

$= \Big[x - 2\ln|x+1|\Big]_0^1 = 1 - 2\ln 2$

≈ -0.386

57. $\displaystyle\int_1^2 \frac{1-\cos\theta}{\theta - \sin\theta}\,d\theta = \Big[\ln|\theta - \sin\theta|\Big]_1^2$

$\displaystyle = \ln\left|\frac{2-\sin 2}{1-\sin 1}\right| \approx 1.929$

58. $u = 2\theta, du = 2\,d\theta, \theta = \dfrac{\pi}{8} \Rightarrow u = \dfrac{\pi}{4}, \theta = \dfrac{\pi}{4} \Rightarrow u = \dfrac{\pi}{2}$

$\displaystyle\int_{\pi/8}^{\pi/4}(\csc 2\theta - \cot 2\theta)\,d\theta = \frac{1}{2}\int_{\pi/4}^{\pi/2}(\csc u - \cot u)\,du$

$\displaystyle = \frac{1}{2}\Big[-\ln|\csc u + \cot u| - \ln|\sin u|\Big]_{\pi/4}^{\pi/2}$

$\displaystyle = \frac{1}{2}\left[-\ln(1+0) - \ln(1) + \ln\left(\sqrt{2}+1\right) + \ln\frac{\sqrt{2}}{2}\right]$

$\displaystyle = \frac{1}{2}\left[\ln\left(\sqrt{2}+1\right) + \ln\frac{\sqrt{2}}{2}\right]$

$\displaystyle = \frac{1}{2}\ln\left(1 + \frac{\sqrt{2}}{2}\right)$

59. $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\ln\left(1+\sqrt{x}\right) + C$

60. $\displaystyle\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx = 4\sqrt{x} - x - 4\ln\left(1+\sqrt{x}\right) + C$

61. $\displaystyle\int \frac{\sqrt{x}}{x-1}\,dx = \ln\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + 2\sqrt{x} + C$

62. $\displaystyle\int \frac{x^2}{x-1}\,dx = \ln|x-1| + \frac{x^2}{2} + x + C$

63. $\displaystyle\int_{\pi/4}^{\pi/2}(\csc x - \sin x)\,dx = \ln\left(\sqrt{2}+1\right) - \frac{\sqrt{2}}{2} \approx 0.174$

64. $\displaystyle\int_{-\pi/4}^{\pi/4}\frac{\sin^2 x - \cos^2 x}{\cos x}\,dx = \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) - 2\sqrt{2}$

≈ -1.066