

Solutions Manual
for
Heat and Mass Transfer: Fundamentals & Applications
5th Edition
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Chapter 2
HEAT CONDUCTION EQUATION

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Introduction

2-1C The term *steady* implies *no change with time* at any point within the medium while *transient* implies *variation with time* or *time dependence*. Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location although both quantities may vary from one location to another. During transient heat transfer, the temperature and heat flux may vary with time as well as location. Heat transfer is one-dimensional if it occurs primarily in one direction. It is two-dimensional if heat transfer in the third dimension is negligible.

2-2C Heat transfer is a *vector* quantity since it has direction as well as magnitude. Therefore, we must specify both direction and magnitude in order to describe heat transfer completely at a point. Temperature, on the other hand, is a scalar quantity.

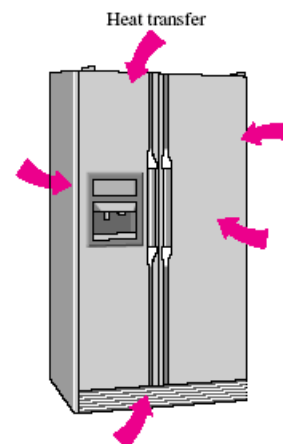
2-3C Yes, the heat flux vector at a point P on an isothermal surface of a medium has to be perpendicular to the surface at that point.

2-4C Isotropic materials have the same properties in all directions, and we do not need to be concerned about the variation of properties with direction for such materials. The properties of anisotropic materials such as the fibrous or composite materials, however, may change with direction.

2-5C In heat conduction analysis, the conversion of electrical, chemical, or nuclear energy into heat (or thermal) energy in solids is called heat generation.

2-6C The phrase “thermal energy generation” is equivalent to “heat generation,” and they are used interchangeably. They imply the conversion of some other form of energy into thermal energy. The phrase “energy generation,” however, is vague since the form of energy generated is not clear.

2-7C The heat transfer process from the kitchen air to the refrigerated space is transient in nature since the thermal conditions in the kitchen and the refrigerator, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the lowest thermostat setting for the refrigerated space, and the anticipated highest temperature in the kitchen (the so-called design conditions). If the compressor is large enough to keep the refrigerated space at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off. Heat transfer into the refrigerated space is three-dimensional in nature since heat will be entering through all six sides of the refrigerator. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer to be one-dimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfer at each surface.



2-8C Heat transfer through the walls, door, and the top and bottom sections of an oven is transient in nature since the thermal conditions in the kitchen and the oven, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the highest temperature setting for the oven, and the anticipated lowest temperature in the kitchen (the so called “design” conditions). If the heating element of the oven is large enough to keep the oven at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off.

Heat transfer from the oven is three-dimensional in nature since heat will be entering through all six sides of the oven. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer as being one-dimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfers at each surface.

2-9C Heat transfer to a potato in an oven can be modeled as one-dimensional since temperature differences (and thus heat transfer) will exist in the radial direction only because of symmetry about the center point. This would be a transient heat transfer process since the temperature at any point within the potato will change with time during cooking. Also, we would use the spherical coordinate system to solve this problem since the entire outer surface of a spherical body can be described by a constant value of the radius in spherical coordinates. We would place the origin at the center of the potato.

2-10C Assuming the egg to be round, heat transfer to an egg in boiling water can be modeled as one-dimensional since temperature differences (and thus heat transfer) will primarily exist in the radial direction only because of symmetry about the center point. This would be a transient heat transfer process since the temperature at any point within the egg will change with time during cooking. Also, we would use the spherical coordinate system to solve this problem since the entire outer surface of a spherical body can be described by a constant value of the radius in spherical coordinates. We would place the origin at the center of the egg.

2-11C Heat transfer to a hot dog can be modeled as two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction). This would be a transient heat transfer process since the temperature at any point within the hot dog will change with time during cooking. Also, we would use the cylindrical coordinate system to solve this problem since a cylinder is best described in cylindrical coordinates. Also, we would place the origin somewhere on the center line, possibly at the center of the hot dog. Heat transfer in a very long hot dog could be considered to be one-dimensional in preliminary calculations.

2-12C Heat transfer to a roast beef in an oven would be transient since the temperature at any point within the roast will change with time during cooking. Also, by approximating the roast as a spherical object, this heat transfer process can be modeled as one-dimensional since temperature differences (and thus heat transfer) will primarily exist in the radial direction because of symmetry about the center point.

2-13C Heat loss from a hot water tank in a house to the surrounding medium can be considered to be a steady heat transfer problem. Also, it can be considered to be two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction.)

2-31 For a medium in which the heat conduction equation is given by $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

(a) Heat transfer is transient, (b) it is two-dimensional, (c) there is no heat generation, and (d) the thermal conductivity is constant.

2-32 We consider a small rectangular element of length Δx , width Δy , and height $\Delta z = 1$ (similar to the one in Fig. 2-20). The density of the body is ρ and the specific heat is c . Noting that heat conduction is two-dimensional and assuming no heat generation, an *energy balance* on this element during a small time interval Δt can be expressed as

$$\left(\begin{array}{c} \text{Rate of heat} \\ \text{conduction at the} \\ \text{surfaces at } x \text{ and } y \end{array} \right) - \left(\begin{array}{c} \text{Rate of heat conduction} \\ \text{at the surfaces at} \\ x + \Delta x \text{ and } y + \Delta y \end{array} \right) = \left(\begin{array}{c} \text{Rate of change of} \\ \text{the energy content} \\ \text{of the element} \end{array} \right)$$

or $\dot{Q}_x + \dot{Q}_y - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} = \frac{\Delta E_{\text{element}}}{\Delta t}$

Noting that the volume of the element is $V_{\text{element}} = \Delta x \Delta y \Delta z = \Delta x \Delta y \times 1$, the change in the energy content of the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c \Delta x \Delta y (T_{t+\Delta t} - T_t)$$

Substituting, $\dot{Q}_x + \dot{Q}_y - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} = \rho c \Delta x \Delta y \frac{T_{t+\Delta t} - T_t}{\Delta t}$

Dividing by $\Delta x \Delta y$ gives

$$-\frac{1}{\Delta y} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x} \frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the thermal conductivity k to be constant and noting that the heat transfer surface areas of the element for heat conduction in the x and y directions are $A_x = \Delta y \times 1$ and $A_y = \Delta x \times 1$, respectively, and taking the limit as Δx , Δy , and $\Delta t \rightarrow 0$ yields

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta y \Delta z} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} &= \frac{1}{\Delta y \Delta z} \frac{\partial \dot{Q}_x}{\partial x} = \frac{1}{\Delta y \Delta z} \frac{\partial}{\partial x} \left(-k \Delta y \Delta z \frac{\partial T}{\partial x} \right) = -\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) = -k \frac{\partial^2 T}{\partial x^2} \\ \lim_{\Delta y \rightarrow 0} \frac{1}{\Delta x \Delta z} \frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} &= \frac{1}{\Delta x \Delta z} \frac{\partial \dot{Q}_y}{\partial y} = \frac{1}{\Delta x \Delta z} \frac{\partial}{\partial y} \left(-k \Delta x \Delta z \frac{\partial T}{\partial y} \right) = -\frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = -k \frac{\partial^2 T}{\partial y^2} \end{aligned}$$

Here the property $\alpha = k / \rho c$ is the thermal diffusivity of the material.

2-168 Heat is generated in a long 0.3-cm-diameter cylindrical electric heater at a rate of 180 W/cm^3 . The heat flux at the surface of the heater in steady operation is

- (a) 12.7 W/cm^2 (b) 13.5 W/cm^2 (c) 64.7 W/cm^2 (d) 180 W/cm^2 (e) 191 W/cm^2

Answer (b) 13.5 W/cm^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

"Consider a 1-cm long heater:"

$L=1 \text{ [cm]}$

$e=180 \text{ [W/cm}^3\text{]}$

$D=0.3 \text{ [cm]}$

$V=\pi*(D^2/4)*L$

$A=\pi*D*L \text{ "[cm}^2\text{]"}$

$E_{\text{gen}}=e*V \text{ "[W]"}$

$Q_{\text{flux}}=E_{\text{gen}}/A \text{ "[W/cm}^2\text{]"}$

"Some Wrong Solutions with Common Mistakes:"

$W1=E_{\text{gen}}$ "Ignoring area effect and using the total"

$W2=e/A$ "Threating g as total generation rate"

$W3=e$ "ignoring volume and area effects"

2-169 Heat is generated uniformly in a 4-cm-diameter, 12-cm-long solid bar ($k = 2.4 \text{ W/m}\cdot^\circ\text{C}$). The temperatures at the center and at the surface of the bar are measured to be 210°C and 45°C , respectively. The rate of heat generation within the bar is

- (a) 597 W (b) 760 W (b) 826 W (c) 928 W (d) 1020 W

Answer (a) 597 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

$D=0.04 \text{ [m]}$

$L=0.12 \text{ [m]}$

$k=2.4 \text{ [W/m}\cdot^\circ\text{C]}$

$T_0=210 \text{ [}^\circ\text{C]}$

$T_s=45 \text{ [}^\circ\text{C]}$

$T_0-T_s=(e*(D/2)^2)/(4*k)$

$V=\pi*D^2/4*L$

$E_{\text{dot_gen}}=e*V$

"Some Wrong Solutions with Common Mistakes"

$W1_V=\pi*D*L$ "Using surface area equation for volume"

$W1_E_{\text{dot_gen}}=e*W1_V$

$T_0=(W2_e*(D/2)^2)/(4*k)$ "Using center temperature instead of temperature difference"

$W2_Q_{\text{dot_gen}}=W2_e*V$

$W3_Q_{\text{dot_gen}}=e$ "Using heat generation per unit volume instead of total heat generation as the result"