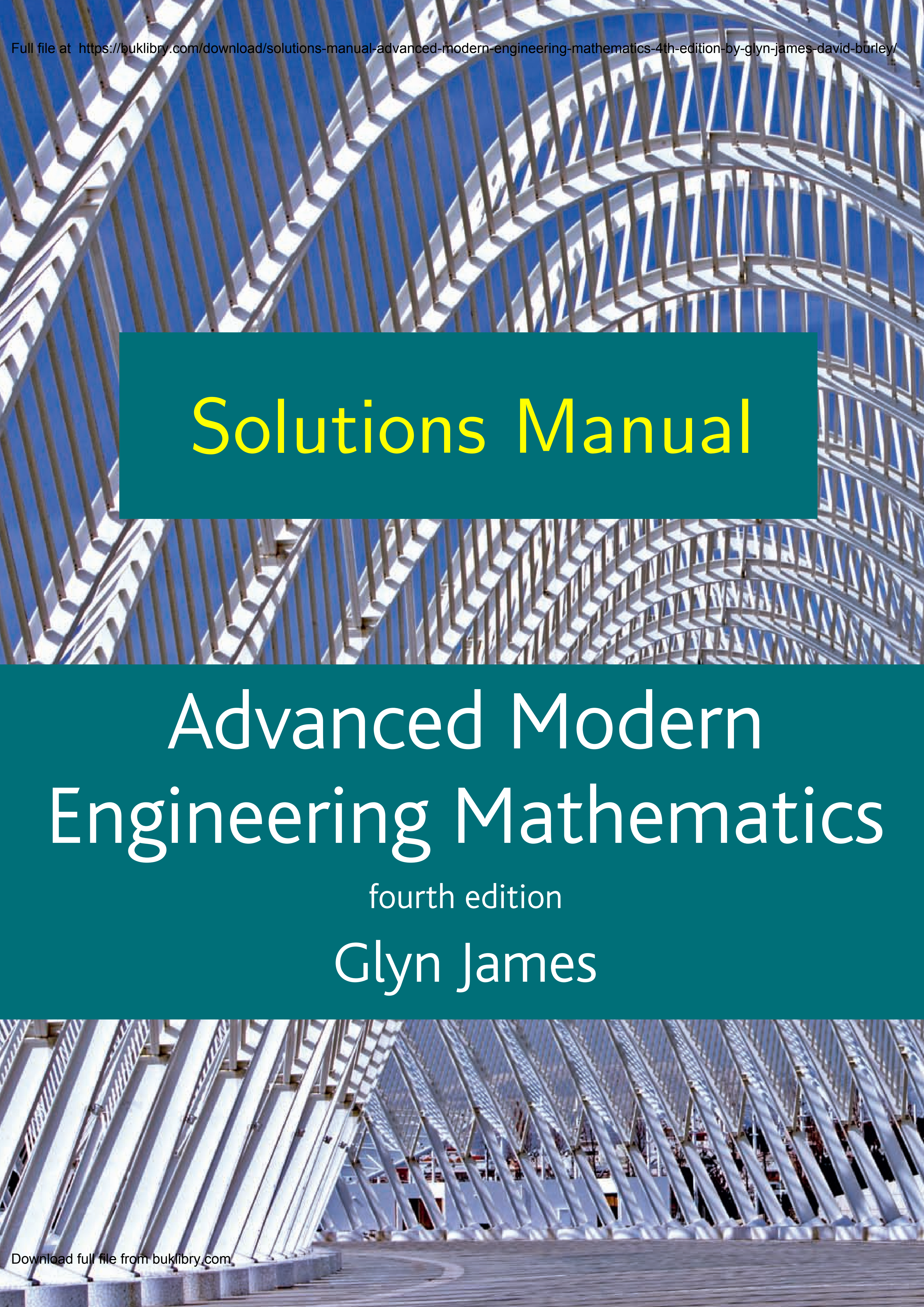

Solutions Manual

Advanced Modern Engineering Mathematics

fourth edition

Glyn James

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4^{th} edition

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so the eigenvalues are $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1$.
Eigenvectors are the corresponding solutions of $(\mathbf{A} - \lambda_i \mathbf{I})\mathbf{e}_i = 0$
When $\lambda = \lambda_1 = 3$ we have

$$\begin{bmatrix} -2 & 1 & 2 \\ 0 & -1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = 0$$

leading to the solution

$$\frac{e_{11}}{-2} = -\frac{e_{12}}{2} = \frac{e_{13}}{-1} = \beta_1$$

so the eigenvector corresponding to $\lambda_2 = 3$ is $\mathbf{e}_1 = \beta_1[2\ 2\ 1]^T, \beta_1$ constant.
When $\lambda = \lambda_2 = 2$ we have

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} = 0$$

leading to the solution

$$\frac{e_{21}}{-2} = -\frac{e_{22}}{2} = \frac{e_{23}}{0} = \beta_3$$

so the eigenvector corresponding to $\lambda_2 = 2$ is $\mathbf{e}_2 = \beta_2[1\ 1\ 0]^T, \beta_2$ constant.
When $\lambda = \lambda_3 = 1$ we have

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} = 0$$

leading to the solution

$$\frac{e_{31}}{0} = -\frac{e_{32}}{2} = \frac{e_{33}}{1} = \beta_1$$

so the eigenvector corresponding to $\lambda_3 = 1$ is $\mathbf{e}_3 = \beta_3[0\ \ -2\ 1]^T, \beta_3$ constant.

6(e) Eigenvalues given by

$$\begin{vmatrix} 5-\lambda & 0 & 6 \\ 0 & 11-\lambda & 6 \\ 6 & 6 & -2-\lambda \end{vmatrix} = \lambda^3 - 14\lambda^2 - 23\lambda - 686 = (\lambda - 14)(\lambda - 7)(\lambda + 7) = 0$$

so eigenvalues are $\lambda_1 = 14, \lambda_2 = 7, \lambda_3 = -7$

n	t_n	X_n	$f(t_n, X_n)$	$\hat{X}_{n+1}$	$f(t_{n+1}, \hat{X}_{n+1})$	X_{n+1}
0	0.0	−2.0000	−1.3072	−1.8812	0.9887	−1.8911
1	0.1	−1.8911	−1.2343	−1.7912	0.8488	−1.7987
2	0.2	−1.7987	−1.1802	−1.7130	0.7400	−1.7189
3	0.3	−1.7189	−1.1416	−1.6443	0.6538	−1.6489
4	0.4	−1.6489	−1.1162	−1.5830	0.5845	−1.5867
5	0.5	−1.5867	−1.1028	−1.5279	0.5281	−1.5309
6	0.6	−1.5309	−1.1005	−1.4778	0.4818	−1.4803
7	0.7	−1.4803	−1.1092	−1.4318	0.4434	−1.4339
8	0.8	−1.4339	−1.1295	−1.3893	0.4114	−1.3910
9	0.9	−1.3910	−1.1624	1.3497	0.3846	1.3511
10	1.0	−1.3511				

Hence $X(1.0) = -1.3511$.

---

11 Taylor's theorem states that

$$f(t+h) = f(t) + h\frac{df}{dt}(t) + \frac{h^2}{2!}\frac{d^2f}{dt^2}(t) + \frac{h^3}{3!}\frac{d^3f}{dt^3}(t) + \frac{h^4}{4!}\frac{d^4f}{dt^4}(t) + K$$

Applying this to $\dfrac{dx}{dt}(t-h)$ and $\dfrac{dx}{dt}(t-2h)$ yields

$$\frac{dx}{dt}(t-h) = \frac{dx}{dt}(t) - h\frac{d^2x}{dt^2}(t) + \frac{h^2}{2!}\frac{d^3x}{dt^3}(t) + O(h^3)$$
$$\frac{dx}{dt}(t-2h) = \frac{dx}{dt}(t) - 2h\frac{d^2x}{dt^2}(t) + \frac{4h^2}{2!}\frac{d^3x}{dt^3}(t) + O(h^3)$$

Multiplying the first equation by 2 and subtracting the second yields

$$2\frac{dx}{dt}(t-h) - \frac{dx}{dt}(t-2h) = \frac{dx}{dt}(t) - h^2\frac{d^3x}{dt^3}(t) + O(h^3)$$
$$\text{that is, } h^2\frac{d^3x}{dt^3}(t) = -2\frac{dx}{dt}(t-h) + \frac{dx}{dt}(t-2h) + \frac{dx}{dt}(t) + O(h^3)$$

Multiplying the first equation by 4 and subtracting the second yields

$$4\frac{dx}{dt}(t-h) - \frac{dx}{dt}(t-2h) = 3\frac{dx}{dt}(t) - 2h\frac{d^2x}{dt^2}(t) + O(h^3)$$
$$\text{that is, } 2h\frac{d^2x}{dt^2}(t) = -4\frac{dx}{dt}(t-h) + \frac{dx}{dt}(t-2h) + 3\frac{dx}{dt}(t) + O(h^3)$$

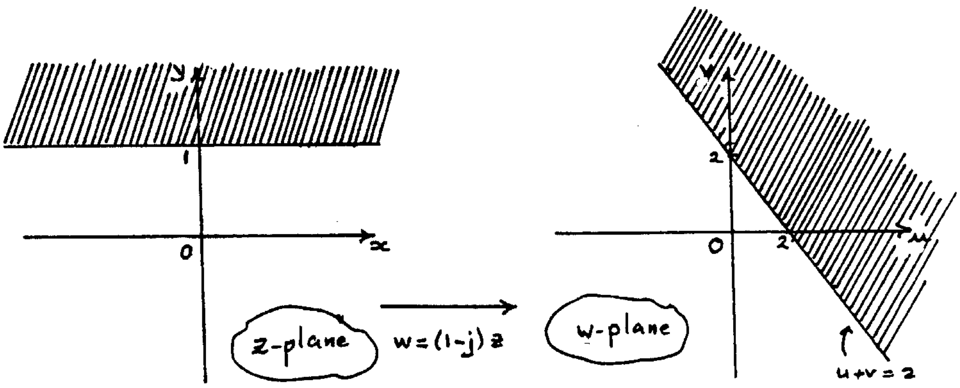
Now Taylor's theorem yields

$$x(t+h) = x(t) + h\frac{dx}{dt}(t) + \frac{h^2}{2!}\frac{d^2x}{dt^2} + \frac{h^3}{3!}\frac{d^3x}{dt^3}(t) + O(h^3)$$

■ **4** Splitting the mapping $w = (1 - j)z$ into real and imaginary parts gives

$$\begin{aligned} u + jv &= (1 - j)(x + jy) \\ &= x + y + j(y - x) \\ \text{that is,} \quad u &= x + y \\ v &= y - x \\ \text{so that,} \quad u + v &= 2y \end{aligned}$$

Therefore $y > 1$ corresponds to $u + v > 2$.



■ **5** Since $w = jz + j$

$x = v - 1, \; y = -u$

so that $x > 0$ corresponds to $v > 1$.

■ **6** Since $w = jz + 1$

$v = x$

$u = -y + 1$

so that $x > 0 \Rightarrow v > 0$

and $0 < y < 2 \Rightarrow -1 < u < 1$ or $| u |< 1$.

This is illustrated below

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.368 & -0.1185 \\ 0.632 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.1185 & 0 \\ 0.069 & -1 \end{bmatrix} \begin{bmatrix} k_c \\ 1.1x_1(0) \end{bmatrix}$$

(c) Adopting the feedback control policy

$$u_1(t) = k_c - x_2(t)$$

the given continuous-time state model becomes

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & k_1 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} k_1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} k_c \\ u_2 \end{bmatrix}$$

Taking $k_1 = \frac{3}{16}$ and $u_2 = 1.1x_1(0)$ this reduces to

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & -\frac{3}{16} \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{3}{16} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} k_c \\ 1.1x_1(0) \end{bmatrix}$$

$$(s\mathbf{I} - \mathbf{A}_c) = \begin{bmatrix} s+1 & \frac{3}{16} \\ -1 & s \end{bmatrix} \Rightarrow (s\mathbf{I} - \mathbf{A}_c)^{-1} = \frac{1}{s^2 + s + \frac{3}{16}} \begin{bmatrix} s & -\frac{3}{16} \\ 1 & s+1 \end{bmatrix}$$

$$\Rightarrow (s\mathbf{I} - \mathbf{A}_c)^{-1} = \begin{bmatrix} \dfrac{-\frac{1}{2}}{s+\frac{1}{4}} + \dfrac{\frac{3}{2}}{s+\frac{3}{4}} & \dfrac{-\frac{3}{8}}{s+\frac{1}{4}} + \dfrac{\frac{3}{8}}{s+\frac{3}{4}} \\ \dfrac{2}{s+\frac{1}{4}} - \dfrac{2}{s+\frac{3}{4}} & \dfrac{\frac{3}{2}}{s+\frac{1}{4}} - \dfrac{\frac{1}{2}}{s+\frac{3}{4}} \end{bmatrix}$$

giving

$$e^{\mathbf{A}_c t} = L^{-1}\{(s\mathbf{I} - \mathbf{A}_c)^{-1}\} = \begin{bmatrix} -\frac{1}{2}e^{-\frac{1}{4}t} + \frac{3}{2}e^{-\frac{3}{4}t} & -\frac{3}{8}e^{-\frac{1}{4}t} + \frac{3}{8}e^{-\frac{3}{4}t} \\ 2e^{-\frac{1}{4}t} - 2e^{-\frac{3}{4}t} & \frac{3}{2}e^{-\frac{1}{4}t} - \frac{1}{2}e^{-\frac{3}{4}t} \end{bmatrix}$$

The response of the continuous feedback system is

$$\mathbf{x}(t) = e^{\mathbf{A}_c t} \begin{bmatrix} x_1(0) \\ k_c \end{bmatrix} + \int_0^t e^{\mathbf{A}(t-\tau)} d\tau \mathbf{B} \begin{bmatrix} k_c \\ 1.1x_1(0) \end{bmatrix}$$

Carrying out the integration and simplifying gives the response

$$x_1(t) = x_1(0)[1.1 - 2.15e^{-\frac{1}{4}t} + 2.05e^{-\frac{3}{4}t}]$$
$$x_2(t) = k_c + x_1(0)[-5.867 + 8.6e^{-\frac{1}{4}t} - 2.714e^{-\frac{3}{4}t}]$$

■ **25** Write result 24 as

$$\int_{-\infty}^{\infty} f(\omega)\mathcal{F}\{g(t)\}d\omega = \int_{-\infty}^{\infty} \mathcal{F}\{f(t)\}g(\omega)d\omega$$

$$\text{so} \int_{-\infty}^{\infty} f(\omega)\mathcal{F}\{G(jt)\}d\omega = \int_{-\infty}^{\infty} \mathcal{F}\{f(t)\}G(j\omega)d\omega$$

$$\text{Now} \quad \left.\begin{array}{cc} g(t) & \to G(j\omega) \\ G(jt) & \to 2\pi g(-\omega) \\ G(-jt) & \to 2\pi g(\omega) \end{array}\right\} \quad \text{symmetry}$$

$$\text{Thus,} \quad \int_{-\infty}^{\infty} f(\omega).2\pi g(\omega)d\omega = \int_{-\infty}^{\infty} F(j\omega)G(-j\omega)d\omega$$

$$\text{or} \quad \int_{-\infty}^{\infty} f(t)g(t)\ \mathrm{d}t = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)G(-j\omega)d\omega$$

■ **26** $\mathcal{F}\{H(t)\sin\omega_0 t\}$

$$\begin{aligned}
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi j\big[\delta(\omega - u + \omega_0) - \delta(\omega - u - \omega_0)\big]\left[\pi\delta(u) + \frac{1}{ju}\right]du \\
&= \frac{j}{2}\big[\pi\delta(\omega + \omega_0) - \pi\delta(\omega - \omega_0)\big] + \frac{1}{2}\left[\frac{1}{\omega + \omega_0} - \frac{1}{\omega - \omega_0}\right] \\
&= \frac{\pi j}{2}\big[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\big] - \frac{\omega_0}{\omega^2 - \omega_0^2}
\end{aligned}$$

■ **27**

$$a_n = \frac{A}{T}\int_{-d/2}^{d/2} e^{-jn\omega_0 t}\ dt = \frac{Ad}{T}\ \mathrm{sinc}\,\frac{n\omega_0 d}{2}, \quad \omega_0 = 2\pi/T$$

$$f(t) = \frac{Ad}{T}\sum_{n=-\infty}^{\infty} \mathrm{sinc}\,\frac{n\omega_0 d}{2} e^{jn\omega_0 t},$$

$$F(j\omega) = \frac{2\pi Ad}{T}\sum_{n=-\infty}^{\infty} \mathrm{sinc}\,\frac{n\omega_0 d}{2}\delta(\omega - n\omega_0)$$

Exercises 8.6.6

■ **28**

$$T = 1, \quad N = 4, \quad \Delta\omega = 2\pi/(4 \times 1) = \frac{\pi}{2}$$