

Solutions Manual

Advanced Modern Engineering Mathematics

fourth edition

Glyn James

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Advanced Modern Engineering Mathematics

4th edition

Glyn James

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TABLE OF CONTENTS

	Page
Chapter 1. Matrix Analysis	1
Chapter 2. Numerical Solution of Ordinary Differential Equations	86
Chapter 3. Vector Calculus	126
Chapter 4. Functions of a Complex Variable	194
Chapter 5. Laplace Transforms	270
Chapter 6. The z Transform	369
Chapter 7. Fourier Series	413
Chapter 8. The Fourier Transform	489
Chapter 9. Partial Differential Equations	512
Chapter 10. Optimization	573
Chapter 11. Applied Probability and Statistics	639

6 Glyn James, Advanced Modern Engineering Mathematics, 4th Edition

so the eigenvalues are $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1$.

Eigenvectors are the corresponding solutions of $(\mathbf{A} - \lambda_i \mathbf{I})\mathbf{e}_i = 0$

When $\lambda = \lambda_1 = 3$ we have

$$\begin{bmatrix} -2 & 1 & 2 \\ 0 & -1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = 0$$

leading to the solution

$$\frac{e_{11}}{-2} = -\frac{e_{12}}{2} = \frac{e_{13}}{-1} = \beta_1$$

so the eigenvector corresponding to $\lambda_1 = 3$ is $\mathbf{e}_1 = \beta_1 [2 \ 2 \ 1]^T, \beta_1$ constant.

When $\lambda = \lambda_2 = 2$ we have

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} = 0$$

leading to the solution

$$\frac{e_{21}}{-2} = -\frac{e_{22}}{2} = \frac{e_{23}}{0} = \beta_2$$

so the eigenvector corresponding to $\lambda_2 = 2$ is $\mathbf{e}_2 = \beta_2 [1 \ 1 \ 0]^T, \beta_2$ constant.

When $\lambda = \lambda_3 = 1$ we have

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} = 0$$

leading to the solution

$$\frac{e_{31}}{0} = -\frac{e_{32}}{2} = \frac{e_{33}}{1} = \beta_3$$

so the eigenvector corresponding to $\lambda_3 = 1$ is $\mathbf{e}_3 = \beta_3 [0 \ -2 \ 1]^T, \beta_3$ constant.

6(e) Eigenvalues given by

$$\begin{vmatrix} 5 - \lambda & 0 & 6 \\ 0 & 11 - \lambda & 6 \\ 6 & 6 & -2 - \lambda \end{vmatrix} = \lambda^3 - 14\lambda^2 - 23\lambda - 686 = (\lambda - 14)(\lambda - 7)(\lambda + 7) = 0$$

so eigenvalues are $\lambda_1 = 14, \lambda_2 = 7, \lambda_3 = -7$

96 Glyn James, Advanced Modern Engineering Mathematics, 4th Edition

n	t_n	X_n	$f(t_n, X_n)$	\hat{X}_{n+1}	$f(t_{n+1}, \hat{X}_{n+1})$	X_{n+1}
0	0.0	-2.0000	-1.3072	-1.8812	0.9887	-1.8911
1	0.1	-1.8911	-1.2343	-1.7912	0.8488	-1.7987
2	0.2	-1.7987	-1.1802	-1.7130	0.7400	-1.7189
3	0.3	-1.7189	-1.1416	-1.6443	0.6538	-1.6489
4	0.4	-1.6489	-1.1162	-1.5830	0.5845	-1.5867
5	0.5	-1.5867	-1.1028	-1.5279	0.5281	-1.5309
6	0.6	-1.5309	-1.1005	-1.4778	0.4818	-1.4803
7	0.7	-1.4803	-1.1092	-1.4318	0.4434	-1.4339
8	0.8	-1.4339	-1.1295	-1.3893	0.4114	-1.3910
9	0.9	-1.3910	-1.1624	1.3497	0.3846	1.3511
10	1.0	-1.3511				

Hence $X(1.0) = -1.3511$.

■ **11** Taylor's theorem states that

$$f(t+h) = f(t) + h \frac{df}{dt}(t) + \frac{h^2}{2!} \frac{d^2f}{dt^2}(t) + \frac{h^3}{3!} \frac{d^3f}{dt^3}(t) + \frac{h^4}{4!} \frac{d^4f}{dt^4}(t) + K$$

Applying this to $\frac{dx}{dt}(t-h)$ and $\frac{dx}{dt}(t-2h)$ yields

$$\begin{aligned} \frac{dx}{dt}(t-h) &= \frac{dx}{dt}(t) - h \frac{d^2x}{dt^2}(t) + \frac{h^2}{2!} \frac{d^3x}{dt^3}(t) + O(h^3) \\ \frac{dx}{dt}(t-2h) &= \frac{dx}{dt}(t) - 2h \frac{d^2x}{dt^2}(t) + \frac{4h^2}{2!} \frac{d^3x}{dt^3}(t) + O(h^3) \end{aligned}$$

Multiplying the first equation by 2 and subtracting the second yields

$$\begin{aligned} 2 \frac{dx}{dt}(t-h) - \frac{dx}{dt}(t-2h) &= \frac{dx}{dt}(t) - h^2 \frac{d^3x}{dt^3}(t) + O(h^3) \\ \text{that is, } h^2 \frac{d^3x}{dt^3}(t) &= -2 \frac{dx}{dt}(t-h) + \frac{dx}{dt}(t-2h) + \frac{dx}{dt}(t) + O(h^3) \end{aligned}$$

Multiplying the first equation by 4 and subtracting the second yields

$$\begin{aligned} 4 \frac{dx}{dt}(t-h) - \frac{dx}{dt}(t-2h) &= 3 \frac{dx}{dt}(t) - 2h \frac{d^2x}{dt^2}(t) + O(h^3) \\ \text{that is, } 2h \frac{d^2x}{dt^2}(t) &= -4 \frac{dx}{dt}(t-h) + \frac{dx}{dt}(t-2h) + 3 \frac{dx}{dt}(t) + O(h^3) \end{aligned}$$

Now Taylor's theorem yields

$$x(t+h) = x(t) + h \frac{dx}{dt}(t) + \frac{h^2}{2!} \frac{d^2x}{dt^2}(t) + \frac{h^3}{3!} \frac{d^3x}{dt^3}(t) + O(h^3)$$

- 4 Splitting the mapping $w = (1 - j)z$ into real and imaginary parts gives

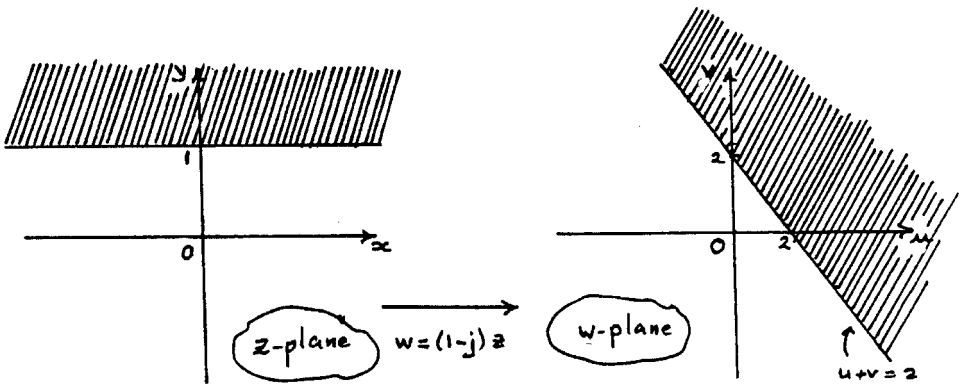
$$\begin{aligned}u + jv &= (1 - j)(x + jy) \\ &= x + y + j(y - x)\end{aligned}$$

that is, $u = x + y$

$$v = y - x$$

so that, $u + v = 2y$

Therefore $y > 1$ corresponds to $u + v > 2$.



-
- 5 Since $w = jz + j$
 $x = v - 1, y = -u$
so that $x > 0$ corresponds to $v > 1$.
-

- 6 Since $w = jz + 1$
 $v = x$
 $u = -y + 1$

so that $x > 0 \Rightarrow v > 0$
and $0 < y < 2 \Rightarrow -1 < u < 1$ or $|u| < 1$.
This is illustrated below

396 Glyn James, Advanced Modern Engineering Mathematics, 4th Edition

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.368 & -0.1185 \\ 0.632 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.1185 & 0 \\ 0.069 & -1 \end{bmatrix} \begin{bmatrix} k_c \\ 1.1x_1(0) \end{bmatrix}$$

(c) Adopting the feedback control policy

$$u_1(t) = k_c - x_2(t)$$

the given continuous-time state model becomes

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & k_1 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} k_1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} k_c \\ u_2 \end{bmatrix}$$

Taking $k_1 = \frac{3}{16}$ and $u_2 = 1.1x_1(0)$ this reduces to

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} -1 & -\frac{3}{16} \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{3}{16} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} k_c \\ 1.1x_1(0) \end{bmatrix} \\ (s\mathbf{I} - \mathbf{A}_c) &= \begin{bmatrix} s+1 & \frac{3}{16} \\ -1 & s \end{bmatrix} \Rightarrow (s\mathbf{I} - \mathbf{A}_c)^{-1} = \frac{1}{s^2 + s + \frac{3}{16}} \begin{bmatrix} s & -\frac{3}{16} \\ 1 & s+1 \end{bmatrix} \\ \Rightarrow (s\mathbf{I} - \mathbf{A}_c)^{-1} &= \begin{bmatrix} \frac{-\frac{1}{2}}{s + \frac{1}{4}} + \frac{\frac{3}{2}}{s + \frac{3}{4}} & \frac{-\frac{3}{8}}{s + \frac{1}{4}} + \frac{\frac{3}{8}}{s + \frac{3}{4}} \\ \frac{2}{s + \frac{1}{4}} - \frac{2}{s + \frac{3}{4}} & \frac{\frac{3}{2}}{s + \frac{1}{4}} - \frac{\frac{1}{2}}{s + \frac{3}{4}} \end{bmatrix} \end{aligned}$$

giving

$$e^{\mathbf{A}_c t} = L^{-1}\{(s\mathbf{I} - \mathbf{A}_c)^{-1}\} = \begin{bmatrix} -\frac{1}{2}e^{-\frac{1}{4}t} + \frac{3}{2}e^{-\frac{3}{4}t} & -\frac{3}{8}e^{-\frac{1}{4}t} + \frac{3}{8}e^{-\frac{3}{4}t} \\ 2e^{-\frac{1}{4}t} - 2e^{-\frac{3}{4}t} & \frac{3}{2}e^{-\frac{1}{4}t} - \frac{1}{2}e^{-\frac{3}{4}t} \end{bmatrix}$$

The response of the continuous feedback system is

$$\mathbf{x}(t) = e^{\mathbf{A}_c t} \begin{bmatrix} x_1(0) \\ k_c \end{bmatrix} + \int_0^t e^{\mathbf{A}(t-\tau)} d\tau \mathbf{B} \begin{bmatrix} k_c \\ 1.1x_1(0) \end{bmatrix}$$

Carrying out the integration and simplifying gives the response

$$\begin{aligned} x_1(t) &= x_1(0)[1.1 - 2.15e^{-\frac{1}{4}t} + 2.05e^{-\frac{3}{4}t}] \\ x_2(t) &= k_c + x_1(0)[-5.867 + 8.6e^{-\frac{1}{4}t} - 2.714e^{-\frac{3}{4}t}] \end{aligned}$$

496 Glyn James, Advanced Modern Engineering Mathematics, 4th Edition

- 25 Write result 24 as

$$\int_{-\infty}^{\infty} f(\omega) \mathcal{F}\{g(t)\} d\omega = \int_{-\infty}^{\infty} \mathcal{F}\{f(t)\} g(\omega) d\omega$$

$$\text{so } \int_{-\infty}^{\infty} f(\omega) \mathcal{F}\{G(jt)\} d\omega = \int_{-\infty}^{\infty} \mathcal{F}\{f(t)\} G(j\omega) d\omega$$

$$\text{Now } \left. \begin{array}{ll} g(t) & \rightarrow G(j\omega) \\ G(jt) & \rightarrow 2\pi g(-\omega) \\ G(-jt) & \rightarrow 2\pi g(\omega) \end{array} \right\} \text{ symmetry}$$

$$\text{Thus, } \int_{-\infty}^{\infty} f(\omega) \cdot 2\pi g(\omega) d\omega = \int_{-\infty}^{\infty} F(j\omega) G(-j\omega) d\omega$$

$$\text{or } \int_{-\infty}^{\infty} f(t) g(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) G(-j\omega) d\omega$$

- 26 $\mathcal{F}\{H(t) \sin \omega_0 t\}$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \pi j [\delta(\omega - u + \omega_0) - \delta(\omega - u - \omega_0)] \left[\pi \delta(u) + \frac{1}{ju} \right] du$$

$$= \frac{j}{2} [\pi \delta(\omega + \omega_0) - \pi \delta(\omega - \omega_0)] + \frac{1}{2} \left[\frac{1}{\omega + \omega_0} - \frac{1}{\omega - \omega_0} \right]$$

$$= \frac{\pi j}{2} [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] - \frac{\omega_0}{\omega^2 - \omega_0^2}$$

- 27

$$a_n = \frac{A}{T} \int_{-d/2}^{d/2} e^{-jn\omega_0 t} dt = \frac{Ad}{T} \text{sinc} \frac{n\omega_0 d}{2}, \quad \omega_0 = 2\pi/T$$

$$f(t) = \frac{Ad}{T} \sum_{n=-\infty}^{\infty} \text{sinc} \frac{n\omega_0 d}{2} e^{jn\omega_0 t},$$

$$F(j\omega) = \frac{2\pi Ad}{T} \sum_{n=-\infty}^{\infty} \text{sinc} \frac{n\omega_0 d}{2} \delta(\omega - n\omega_0)$$

Exercises 8.6.6

- 28

$$T = 1, \quad N = 4, \quad \Delta\omega = 2\pi/(4 \times 1) = \frac{\pi}{2}$$