SOLUTIONS MANUAL

Electricity and Magnetism

Third Edition

Edward M. Purcell and David J. Morin

TO THE INSTRUCTOR: I have tried to pay as much attention to detail in these exercise solutions as I did in the problem solutions in the text. But despite working through each solution numerous times during the various stages of completion, there are bound to be errors. So please let me know if anything looks amiss.

Also, to keep this pdf file from escaping to the web, PLEASE don't distribute it to anyone, with the exception of your teaching assistants. And please make sure they also agree to this. Once this file gets free, there's no going back.

In addition to any comments you have on these solutions, I welcome any comments on the book in general. I hope you're enjoying using it!

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(Version 1, January 2013)

Chapter 1

Electrostatics

Solutions manual for *Electricity and Magnetism, 3rd edition,* E. Purcell, D. Morin.

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1.34. **Aircraft carriers and specks of gold**

The volume of a cube 1 mm on a side is $10^{-3}\,\text{cm}^3$. So the mass of this 1 mm cube is $1.93 \cdot 10^{-2}$ g. The number of atoms in the cube is therefore

$$6.02 \cdot 10^{23} \cdot \frac{1.93 \cdot 10^{-2}\,\text{g}}{197\,\text{g}} = 5.9 \cdot 10^{19}. \tag{1}$$

Each atom has a positive charge of $1\,e = 1.6 \cdot 10^{-19}$ C, so the total charge in the cube is $(5.9 \cdot 10^{19})(1.6 \cdot 10^{-19}\,\text{C}) = 9.4\,\text{C}$. The repulsive force between two such cubes 1 m apart is therefore

$$F = k\frac{q^2}{r^2} = \left(9 \cdot 10^9\,\frac{\text{kg}\,\text{m}^3}{\text{s}^2\,\text{C}^2}\right)\frac{(9.4\,\text{C})^2}{(1\,\text{m})^2} = 8 \cdot 10^{11}\ \text{N}. \tag{2}$$

The weight of an aircraft carrier is $mg = (10^8\,\text{kg})(9.8\,\text{m/s}^2) \approx 10^9\,\text{N}$. The above F is therefore equal to the weight of 800 aircraft carriers. This is just another example of the fact that the electrostatic force is enormously larger than the gravitational force.

1.35. **Balancing the weight**

Let the desired distance be d. We want the upward electric force $e^2/4\pi\epsilon_0 d^2$ to equal the downward gravitational force mg. Hence,

$$d^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mg} = \left(9 \cdot 10^9\,\frac{\text{kg}\,\text{m}^3}{\text{s}^2\,\text{C}^2}\right)\frac{(1.6 \cdot 10^{-19}\,\text{C})^2}{(9 \cdot 10^{-31}\,\text{kg})(9.8\,\text{m/s}^2)} = 26\,\text{m}^2, \tag{3}$$

which gives $d = 5.1\,\text{m}$. The non-infinitesimal size of this answer is indicative of the feebleness of the gravitational force compared with the electric force. It takes about $3.6 \cdot 10^{51}$ nucleons (that's roughly how many are in the earth) to produce a gravitational force at an effective distance of $6.4 \cdot 10^6\,\text{m}$ (the radius of the earth) that cancels the electrical force from *one* proton at a distance of 5 m. The difference in these distances accounts for a factor of only $1.6 \cdot 10^{12}$ between the forces (the square of the ratio of the distances). So even if all the earth's mass were somehow located the same distance away from the electron as the single proton is, we would still need about $2 \cdot 10^{39}$ nucleons to produce the necessary gravitational force.

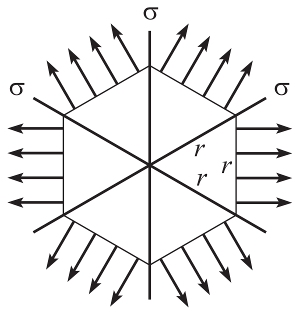

Figure 26

(b) Fig. 26 shows the field at points on a symmetrically-located hexagon. Let the "radius" of the hexagon be r, and consider a hexagonal tube with length ℓ perpendicular to the page. The surface area of this tube is $6r\ell$, and the charge enclosed is $6r\ell\sigma$. Since the electric field is everywhere perpendicular to the surface, Gauss's law gives

$$\int \mathbf{E}\cdot d\mathbf{a} = \frac{Q}{\epsilon_0} \implies E\cdot 6r\ell = \frac{6r\ell\sigma}{\epsilon_0} \implies E = \frac{\sigma}{\epsilon_0}\,, \tag{71}$$

in agreement with the result in part (a). Again, note that E is independent of r. While Gauss's law is always valid, it was actually useful in the present setup because we were able to find a simple surface that is everywhere perpendicular to the electric field (because the electric field is uniform in each pie piece).

(c) For general N, the electric field is everywhere perpendicular to a regular $2N$-gon. The surface area of this $2N$-gon is $(2N)\big(2\sin(\pi/2N)\big)r\ell$, and the charge enclosed is $(2N)r\ell\sigma$. So Gauss's law gives

$$E\cdot(2N)\big(2\sin(\pi/2N)\big)r\ell = \frac{(2N)r\ell\sigma}{\epsilon_0} \implies E = \frac{\sigma}{2\epsilon_0\sin(\pi/2N)}\,. \tag{72}$$

As expected, this is independent of r. And it agrees with the above result when $N=3$. For large N, we have $\sin(\pi/2N)\approx\pi/2N$, so $E\approx N\sigma/\pi\epsilon_0$. In the case of large N, the sheets are very close to each other, so we effectively have a continuous volume charge distribution that depends on r. The separation between adjacent sheets grows linearly with r, so we have $\rho(r)\propto 1/r$. More precisely, you can show that $\rho(r)=N\sigma/\pi r$. This is consistent with the result from Exercise 1.68, where we found that a cylinder with a density of the form $\rho(r)\propto 1/r$ produces a field whose magnitude is independent of r (inside the cylinder).

1.72. A plane and a slab

The total effective charge per unit area (looking perpendicular to the sheet/slab) is $\sigma+\rho d$, because $\rho(Ad)$ is the charge contained within an area A of the slab. Let $x=0$ be defined to be the location of the plane. Then for $x<0$ the field is $E=-(\sigma+\rho d)/2\epsilon_0$, and for $x>d$ it is $E=(\sigma+\rho d)/2\epsilon_0$. At a general point inside the slab (that is, for $0<x\le d$), there is a charge density $\sigma+\rho x$ to the left of the point and $(d-x)\rho$ to the right. So for $0<x\le d$ the field is

$$\frac{\sigma+\rho x}{2\epsilon_0}-\frac{(d-x)\rho}{2\epsilon_0}=\frac{\sigma-\rho d+2\rho x}{2\epsilon_0}\,. \tag{73}$$

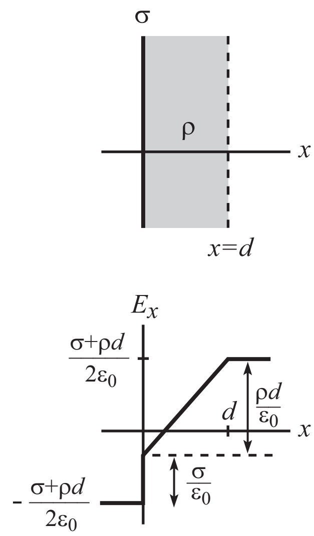

Figure 27

The plot of E as a function of x is shown in Fig. 27. E is continuous at $x=d$ but not at $x=0$. If the plane had a nonzero thickness, then the field would be continuous at $x=0$. The case shown in the plot has $\rho d>\sigma$. If we instead had $\sigma>\rho d$, then at the discontinuity at $x=0$, E would jump to a positive value.

1.73. Sphere in a cylinder

From the reasoning in the solution to Problem 1.27, the electric field inside a uniform cylinder is $\mathbf{E}=\rho\mathbf{r}/2\epsilon_0$, where $\mathbf{r}$ points away from the axis. And the electric field inside a uniform sphere is $\mathbf{E}=\rho\mathbf{r}/3\epsilon_0$, where $\mathbf{r}$ points away from the center.

The given setup may be considered to be the superposition of a uniform cylinder with density ρ and a uniform sphere with density $-3\rho/2$. This produces the desired net density of $-\rho/2$ within the sphere.

so the total field at P due to the strip is

$$E(x) = \int_x^{x+b} \frac{\sigma\, dr}{2\pi\epsilon_0 r} = \frac{\sigma}{2\pi\epsilon_0} \ln\left(\frac{x+b}{x}\right). \tag{227}$$

If $x \to 0$ this result diverges (slowly, like $\ln x$). This divergence is what causes the electric field at the corner of the square tube to diverge, for the following reason.

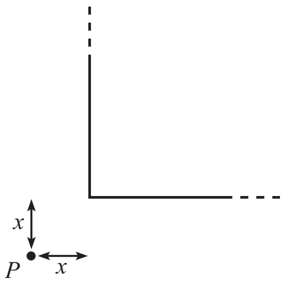

Figure 68

If we treat the corner like an exact point, then a cross section is shown in Fig. 68. The given point P is near the edges of two different strips (the two adjacent faces of the tube). P doesn't lie exactly in the plane of each strip, but this doesn't matter. The field from each strip differs from the field in part (a) by a finite additive amount, so it still diverges as $x \to 0$. This is true because if we ignore the "rods" in the strip that are within a distance of, say, $5x$ from P, then P can be treated as essentially lying in the plane of the remaining part of the strip. The effective value of x is now $6x$, but the factor of 6 doesn't matter; the field still diverges as $x \to 0$. (This reasoning holds for any location near the corner; P need not lie on the line of the angle bisector.) We are concerned only with the component of the field that lies along the angle bisector, so this brings in a factor of $\cos 45° = 1/\sqrt{2}$ in the field from each strip. But this doesn't change the fact that the total field diverges.

If we treat the corner more realistically as curved (like a quarter circle), then the above reasoning still applies. Ignoring the nearby part of the charge distribution still leaves us with two strips that each produce an infinite field, in the limit where the radius of curvature of the quarter circle goes to zero (assuming that P is close to the quarter circle, on the order of the radius). If the radius does *not* go to zero, then the field certainly doesn't diverge. So the "corner" of the tube needs to be sharp in order for the field to diverge.

We have been treating the charge density σ as constant. But in a conducting tube, the density increases near the corners, because of the self-repulsion of the charges. This has the effect of making the field even larger than the above reasoning would imply, so the above conclusion of a diverging field is still valid. Since the conclusion is true for both conducting and nonconducting tubes, the word "conducting" in the statement of the problem could have been omitted.

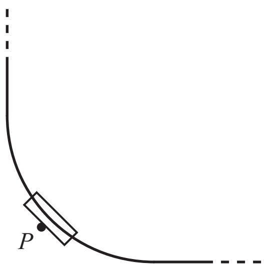

Figure 69

In the case of a curved corner, if P is very close to the quarter circle (or whatever curve), then we can draw a tiny Gaussian pillbox Fig. 69 to say that the field at P equals σ/ϵ_0. Since we just showed that the field diverges, this implies that the density also diverges at the corner. Intuitively, if it didn't diverge, then there wouldn't exist a sufficient force to keep the charges in the straight parts of Fig. 69 from flowing onto the curved part. So this would eventually lead to a very large density at the corner anyway.

All of the above reasoning still holds if the cross section of the tube is something other than a square. At any point where the direction of the surface changes abruptly, the field diverges. Even for a polygon with 100 sides, in which the surface bends by only a few degrees at each "corner," the field still diverges, because when taking the component along the angle bisector, the nonzero trig factor doesn't change the fact that the total field diverges.

If we kick things down a dimension and look at a kink in a wire, the field still diverges (even more quickly). This is true because the field near the end of a uniform stick diverges; Eq. (227) is replaced by

$$E(x) = \int_x^{x+b} \frac{\lambda\, dr}{4\pi\epsilon_0 r^2} = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{x} - \frac{1}{x+b}\right). \tag{228}$$

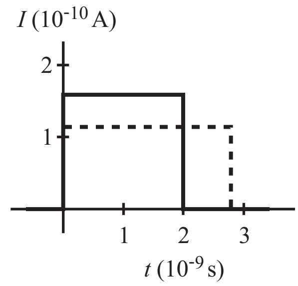

Figure 86

This current lasts for a time $t = \ell/v = (0.002\,\text{m})/(10^6\,\text{m/s}) = 2\cdot 10^{-9}\,\text{s}$, which is 2 nanoseconds. The current is constant during this time, so we have the bold line shown in Fig. 86. The total charge that flows during this time is It, which equals $2e$ as expected.

If the path slopes upward at 45°, then $dx/dt = v\cos 45°$. From above, the current pulse is therefore reduced in amplitude by a factor $1/\sqrt{2}$ and stretched out in time by a factor $\sqrt{2}$; see the dotted line in Fig. 86. Again the total charge transferred is $It = 2e$.

(b) Following the strategy of the solution to Exercise 3.37, we know that if Q_1 and Q_2 are the charges on the inner and outer electrodes (with radii a and b, respectively), then $Q_1 + Q_2 = -2e$. How is the charge of $-2e$ distributed between Q_1 and Q_2 when the alpha particle is at radius r? As in Exercise 3.37, the key points are that (1) we can smear out the alpha particle into a cylinder of charge, and (2) the potentials of the two electrodes are the same, which means that the line integrals of the electric field from radius r to the two electrodes must be equal. The field inside radius r is proportional to Q_1/r (this points inward since Q_1 is negative), and the field outside radius r is proportional to $(2e+Q_1)/r = -Q_2/r$ (this points outward since Q_2 is negative). Equating the two line integrals gives (note that both sides of the following equation are positive since dr is negative in the left integral)

$$\int_r^a \frac{Q_1}{r}\,dr = \int_r^b \frac{-Q_2}{r}\,dr \implies Q_1\ln(a/r) = -Q_2\ln(b/r)$$
$$\implies Q_1\ln(r/a) = Q_2\ln(b/r). \tag{334}$$

Combining this equation with $Q_1 + Q_2 = -2e$ and solving for Q_1 and Q_2 gives

$$Q_1 = \frac{-(2e)\ln(b/r)}{\ln(b/a)} \qquad \text{and} \qquad Q_2 = \frac{-(2e)\ln(r/a)}{\ln(b/a)}. \tag{335}$$

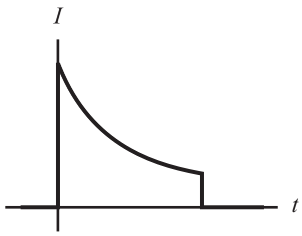

Figure 87

The current flowing out of the outer cylinder is then

$$I = -\frac{dQ_2}{dt} = \frac{2e}{\ln(b/a)}\frac{d(\ln r)}{dt} = \frac{2e}{\ln(b/a)}\frac{1}{r}\frac{dr}{dt} = \frac{2ev}{\ln(b/a)}\frac{1}{a+vt}, \tag{336}$$

where we have used $r = a + vt$. We see that $I(t)$ is not constant. A plot of the general shape of $I(t)$ is shown in Fig. 87 (with b chosen to equal $4a$). For a given value of b, if a is very small then the current starts out very large, because at $t = 0$ the smallness of a in the denominator in Eq. (336) wins out over the largeness of $\ln(b/a)$.

In the case of a 45° angle of the path, the same modifications that applied in part (a) also apply here. That is, the curve is stretched horizontally by a factor of $\sqrt{2}$, and squashed vertically by a factor of $1/\sqrt{2}$.

4.22. **Transatlantic cable**

(a) The resistance of the seven wires together is

$$R = \frac{\rho L}{A} = \frac{(3\cdot 10^{-8}\,\Omega\,\text{m})(3\cdot 10^6\,\text{m})}{7\cdot\pi(3.65\cdot 10^{-4}\,\text{m})^2} = 3.1\cdot 10^4\,\Omega. \tag{337}$$

Adding seven resistors in parallel would give the same answer.

Chapter 11

Magnetic fields in matter

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11.12. **Earth dipole**

(a) Equation (11.15) gives the field at position R along the axis of a dipole as $B_r = \mu_0 m/2\pi R^3$, so

$$m = \frac{2\pi R^3 B_r}{\mu_0} = \frac{2\pi(6.4\cdot 10^6\,\mathrm{m})^3(6.2\cdot 10^{-5}\,\mathrm{T})}{4\pi\cdot 10^{-7}\,\frac{\mathrm{kg\,m}}{\mathrm{C^2}}} = 8.1\cdot 10^{22}\ \mathrm{J/T}. \tag{697}$$

(b) Equation (6.53) gives the field at position z on the axis of a current ring as $B_z = \mu_0 I b^2/2(z^2+b^2)^{3/2}$. If R is the radius of the earth, then we have $z = R$ and $b \approx R/2$, so in terms of R the field is $B_z = \mu_0 I/(5^{3/2}R)$. Therefore,

$$I = \frac{5^{3/2} R B_z}{\mu_0} = \frac{5^{3/2}(6.4\cdot 10^6\,\mathrm{m})(6.2\cdot 10^{-5}\,\mathrm{T})}{4\pi\cdot 10^{-7}\,\frac{\mathrm{kg\,m}}{\mathrm{C^2}}} = 3.5\cdot 10^9\ \mathrm{A}. \tag{698}$$

If we instead treat the current ring as a dipole with moment $m = 8.1\cdot 10^{22}$ J/T, then we have

$$m = I(\pi b^2) \quad\Longrightarrow\quad I = \frac{m}{\pi(R/2)^2} = \frac{4(8.1\cdot 10^{22}\ \mathrm{J/T})}{\pi(6.4\cdot 10^6\,\mathrm{m})^2} = 2.5\cdot 10^9\ \mathrm{A}, \tag{699}$$

which is a so-so approximation to the correct result of $3.5\cdot 10^9$ A.

11.13. **Disk dipole**

Let's divide the disk into rings and then add up the magnetic moments of all the rings. The surface current density at radius r is σv, where $v = \omega r$. This is true because $\sigma\ell(v\,dt)$ is the amount of charge that crosses a transverse segment with length ℓ in a time dt. So the charge per time per unit transverse length (that is, the surface current density) equals $\sigma\ell(v\,dt)/(\ell\,dt) = \sigma v$.

The current in a given ring with radius r and thickness dr is therefore $I_r = (\sigma v)dr = \sigma\omega r\,dr$. The magnetic moment of this ring is then $I_r(\pi r^2) = \pi\sigma\omega r^3\,dr$. Integrating from $r = 0$ to $r = R$ gives the total magnetic moment of the disk as $\pi\sigma\omega R^4/4$.