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CALCULUS

ANTON BIVENS DAVIS

SOLUTION MANUAL

EARLY TRANSCENDENTALS 10TH EDITION



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Before Calculus

Exercise Set 0.1

- (a) $-2.9, -2.0, 2.35, 2.9$ (b) None (c) $y = 0$ (d) $-1.75 \leq x \leq 2.15, x = -3, x = 3$

(e) $y_{\max} = 2.8$ at $x = -2.6$; $y_{\min} = -2.2$ at $x = 1.2$
- (a) $x = -1, 4$ (b) None (c) $y = -1$ (d) $x = 0, 3, 5$

(e) $y_{\max} = 9$ at $x = 6$; $y_{\min} = -2$ at $x = 0$
- (a) Yes (b) Yes (c) No (vertical line test fails) (d) No (vertical line test fails)
- (a) The natural domain of f is $x \neq -1$, and for g it is the set of all x . $f(x) = g(x)$ on the intersection of their domains.

(b) The domain of f is the set of all $x \geq 0$; the domain of g is the same, and $f(x) = g(x)$.
- (a) 1999, \$47,700 (b) 1993, \$41,600

(c) The slope between 2000 and 2001 is steeper than the slope between 2001 and 2002, so the median income was declining more rapidly during the first year of the 2-year period.
- (a) In thousands, approximately $\frac{47.7 - 41.6}{6} = \frac{6.1}{6}$ per yr, or \$1017/yr.

(b) From 1993 to 1996 the median income increased from \$41.6K to \$44K (K for 'kilodollars'; all figures approximate); the average rate of increase during this time was $(44 - 41.6)/3$ K/yr = $2.4/3$ K/yr = \$800/year. From 1996 to 1999 the average rate of increase was $(47.7 - 44)/3$ K/yr = $3.7/3$ K/yr \approx \$1233/year. The increase was larger during the last 3 years of the period.

(c) 1994 and 2005.
- (a) $f(0) = 3(0)^2 - 2 = -2$; $f(2) = 3(2)^2 - 2 = 10$; $f(-2) = 3(-2)^2 - 2 = 10$; $f(3) = 3(3)^2 - 2 = 25$; $f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$; $f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$.

(b) $f(0) = 2(0) = 0$; $f(2) = 2(2) = 4$; $f(-2) = 2(-2) = -4$; $f(3) = 2(3) = 6$; $f(\sqrt{2}) = 2\sqrt{2}$; $f(3t) = 1/(3t)$ for $t > 1$ and $f(3t) = 6t$ for $t \leq 1$.
- (a) $g(3) = \frac{3+1}{3-1} = 2$; $g(-1) = \frac{-1+1}{-1-1} = 0$; $g(\pi) = \frac{\pi+1}{\pi-1}$; $g(-1.1) = \frac{-1.1+1}{-1.1-1} = \frac{-0.1}{-2.1} = \frac{1}{21}$; $g(t^2 - 1) = \frac{t^2 - 1 + 1}{t^2 - 1 - 1} = \frac{t^2}{t^2 - 2}$.

(b) $g(3) = \sqrt{3+1} = 2$; $g(-1) = 3$; $g(\pi) = \sqrt{\pi+1}$; $g(-1.1) = 3$; $g(t^2 - 1) = 3$ if $t^2 < 2$ and $g(t^2 - 1) = \sqrt{t^2 - 1 + 1} = |t|$ if $t^2 \geq 2$.

46. $\frac{dy}{dx} = 3 \left(x - \frac{1}{x} \right)^2 \left(1 + \frac{1}{x^2} \right)$; if $x = 2$ then $y = \frac{27}{8}$, $\frac{dy}{dx} = 3 \frac{9}{4} \frac{5}{4} = \frac{135}{16}$, so the equation of the tangent line is $y - 27/8 = (135/16)(x - 2)$, or $y = \frac{135}{16}x - \frac{27}{2}$.

47. $\frac{dy}{dx} = \sec^2(4x^2) \frac{d}{dx}(4x^2) = 8x \sec^2(4x^2)$, $\frac{dy}{dx} \Big|_{x=\sqrt{\pi}} = 8\sqrt{\pi} \sec^2(4\pi) = 8\sqrt{\pi}$. When $x = \sqrt{\pi}$, $y = \tan(4\pi) = 0$, so the equation of the tangent line is $y = 8\sqrt{\pi}(x - \sqrt{\pi}) = 8\sqrt{\pi}x - 8\pi$.

48. $\frac{dy}{dx} = 12 \cot^3 x \frac{d}{dx} \cot x = -12 \cot^3 x \csc^2 x$, $\frac{dy}{dx} \Big|_{x=\pi/4} = -24$. When $x = \pi/4$, $y = 3$, so the equation of the tangent line is $y - 3 = -24(x - \pi/4)$, or $y = -24x + 3 + 6\pi$.

49. $\frac{dy}{dx} = 2x\sqrt{5-x^2} + \frac{x^2}{2\sqrt{5-x^2}}(-2x)$, $\frac{dy}{dx} \Big|_{x=1} = 4 - 1/2 = 7/2$. When $x = 1$, $y = 2$, so the equation of the tangent line is $y - 2 = (7/2)(x - 1)$, or $y = \frac{7}{2}x - \frac{3}{2}$.

50. $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{x}{2}(1-x^2)^{3/2}(-2x)$, $\frac{dy}{dx} \Big|_{x=0} = 1$. When $x = 0$, $y = 0$, so the equation of the tangent line is $y = x$.

51. $\frac{dy}{dx} = x(-\sin(5x)) \frac{d}{dx}(5x) + \cos(5x) - 2 \sin x \frac{d}{dx}(\sin x) = -5x \sin(5x) + \cos(5x) - 2 \sin x \cos x =$
 $= -5x \sin(5x) + \cos(5x) - \sin(2x)$,
 $\frac{d^2y}{dx^2} = -5x \cos(5x) \frac{d}{dx}(5x) - 5 \sin(5x) - \sin(5x) \frac{d}{dx}(5x) - \cos(2x) \frac{d}{dx}(2x) = -25x \cos(5x) - 10 \sin(5x) - 2 \cos(2x)$.

52. $\frac{dy}{dx} = \cos(3x^2) \frac{d}{dx}(3x^2) = 6x \cos(3x^2)$, $\frac{d^2y}{dx^2} = 6x(-\sin(3x^2)) \frac{d}{dx}(3x^2) + 6 \cos(3x^2) = -36x^2 \sin(3x^2) + 6 \cos(3x^2)$.

53. $\frac{dy}{dx} = \frac{(1-x) + (1+x)}{(1-x)^2} = \frac{2}{(1-x)^2} = 2(1-x)^{-2}$ and $\frac{d^2y}{dx^2} = -2(2)(-1)(1-x)^{-3} = 4(1-x)^{-3}$.

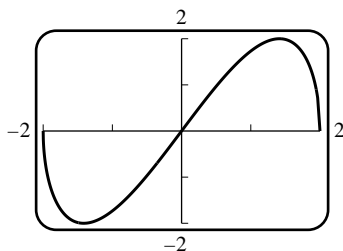
54. $\frac{dy}{dx} = x \sec^2 \left(\frac{1}{x} \right) \frac{d}{dx} \left(\frac{1}{x} \right) + \tan \left(\frac{1}{x} \right) = -\frac{1}{x} \sec^2 \left(\frac{1}{x} \right) + \tan \left(\frac{1}{x} \right)$,
 $\frac{d^2y}{dx^2} = -\frac{2}{x} \sec \left(\frac{1}{x} \right) \frac{d}{dx} \sec \left(\frac{1}{x} \right) + \frac{1}{x^2} \sec^2 \left(\frac{1}{x} \right) + \sec^2 \left(\frac{1}{x} \right) \frac{d}{dx} \left(\frac{1}{x} \right) = \frac{2}{x^3} \sec^2 \left(\frac{1}{x} \right) \tan \left(\frac{1}{x} \right)$.

55. $y = \cot^3(\pi - \theta) = -\cot^3 \theta$ so $dy/dx = 3 \cot^2 \theta \csc^2 \theta$.

56. $6 \left(\frac{au+b}{cu+d} \right)^5 \frac{ad-bc}{(cu+d)^2}$.

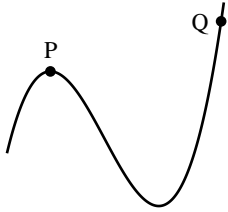
57. $\frac{d}{d\omega} [a \cos^2 \pi\omega + b \sin^2 \pi\omega] = -2\pi a \cos \pi\omega \sin \pi\omega + 2\pi b \sin \pi\omega \cos \pi\omega = \pi(b-a)(2 \sin \pi\omega \cos \pi\omega) = \pi(b-a) \sin 2\pi\omega$.

58. $2 \csc^2(\pi/3 - y) \cot(\pi/3 - y)$.

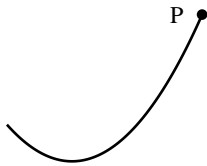


59. (a)

This function has a relative maximum at P which is not an absolute maximum, since the value of the function at Q is larger than at P:

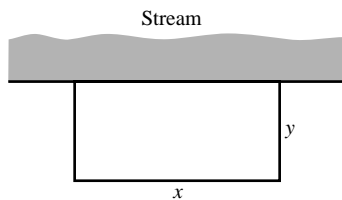


This function has an absolute maximum at P. It is not a relative maximum, since it occurs at an endpoint of the interval where the function is defined.



Exercise Set 4.5

- If $y = x + 1/x$ for $1/2 \leq x \leq 3/2$, then $dy/dx = 1 - 1/x^2 = (x^2 - 1)/x^2$, $dy/dx = 0$ when $x = 1$. If $x = 1/2, 1, 3/2$, then $y = 5/2, 2, 13/6$ so
 - y is as small as possible when $x = 1$.
 - y is as large as possible when $x = 1/2$.
- Let x and y be nonnegative numbers and z the sum of their squares, then $z = x^2 + y^2$. But $x + y = 1$, $y = 1 - x$ so $z = x^2 + (1 - x)^2 = 2x^2 - 2x + 1$ for $0 \leq x \leq 1$. $dz/dx = 4x - 2$, $dz/dx = 0$ when $x = 1/2$. If $x = 0, 1/2, 1$ then $z = 1, 1/2, 1$ so
 - z is as large as possible when one number is 0 and the other is 1.
 - z is as small as possible when both numbers are $1/2$.
- $A = xy$ where $x + 2y = 1000$ so $y = 500 - x/2$ and $A = 500x - x^2/2$ for x in $[0, 1000]$; $dA/dx = 500 - x$, $dA/dx = 0$ when $x = 500$. If $x = 0$ or 1000 then $A = 0$, if $x = 500$ then $A = 125,000$ so the area is maximum when $x = 500$ ft and $y = 500 - 500/2 = 250$ ft.



- Let the length of one fenced side be x feet. Then the other fenced side has length $1000 - x$ feet, and the area of the triangle is $A(x) = \frac{1}{2}x(1000 - x) = 500x - \frac{1}{2}x^2$ square feet. We wish to maximize this for x in the interval $[0, 1000]$. The derivative $A'(x) = 500 - x$ equals 0 when $x = 500$, so the maximum area occurs for either $x = 0$, $x = 500$, or $x = 1000$. Since $A(0) = A(1000) = 0$ and $A(500) = 125,000$, the maximum area occurs when both fenced sides are 500 feet long.

$$34. \left(3x^{5/3} + \frac{4}{x}\right) \Big|_1^8 = 179/2.$$

$$35. \left(\frac{1}{2}x^2 - \sec x\right) \Big|_0^1 = 3/2 - \sec(1).$$

$$36. \left(6\sqrt{t} - \frac{10}{3}t^{3/2} + \frac{2}{\sqrt{t}}\right) \Big|_1^4 = -55/3.$$

$$37. \int_0^{3/2} (3-2x)dx + \int_{3/2}^2 (2x-3)dx = (3x-x^2) \Big|_0^{3/2} + (x^2-3x) \Big|_{3/2}^2 = 9/4 + 1/4 = 5/2.$$

$$38. \int_0^{\pi/6} (1/2 - \sin x) dx + \int_{\pi/6}^{\pi/2} (\sin x - 1/2) dx = (x/2 + \cos x) \Big|_0^{\pi/6} - (\cos x + x/2) \Big|_{\pi/6}^{\pi/2} = (\pi/12 + \sqrt{3}/2) - 1 - \pi/4 + (\sqrt{3}/2 + \pi/12) = \sqrt{3} - \pi/12 - 1.$$

$$39. \int_1^9 \sqrt{x} dx = \frac{2}{3}x^{3/2} \Big|_1^9 = \frac{2}{3}(27-1) = 52/3.$$

$$40. \int_1^4 x^{-3/5} dx = \frac{5}{2}x^{2/5} \Big|_1^4 = \frac{5}{2}(4^{2/5} - 1).$$

$$41. \int_1^3 e^x dx = e^x \Big|_1^3 = e^3 - e.$$

$$42. \int_1^{e^3} \frac{1}{x} dx = \ln x \Big|_1^{e^3} = 3 - \ln 1 = 3.$$

$$43. A = \int_1^2 (-x^2 + 3x - 2) dx = \left(-\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x\right) \Big|_1^2 = 1/6.$$

$$44. \text{The only positive zero of } f \text{ is } b = \frac{1+\sqrt{5}}{2}, \text{ and the area is given by } A = \int_0^b f(x) dx = \frac{13+5\sqrt{5}}{24}.$$

$$45. A = A_1 + A_2 = \int_0^1 (1-x^2) dx + \int_1^3 (x^2-1) dx = 2/3 + 20/3 = 22/3.$$

$$46. A = A_1 + A_2 = \int_{-1}^0 [1 - \sqrt{x+1}] dx + \int_0^1 [\sqrt{x+1} - 1] dx = \left(x - \frac{2}{3}(x+1)^{3/2}\right) \Big|_{-1}^0 + \left(\frac{2}{3}(x+1)^{3/2} - x\right) \Big|_0^1 = -\frac{2}{3} + 1 + \frac{4\sqrt{2}}{3} - 1 - \frac{2}{3} = 4\frac{\sqrt{2}-1}{3}.$$

$$47. \text{(a) } x^3 + 1 \quad \text{(b) } F(x) = \left(\frac{1}{4}t^4 + t\right) \Big|_1^x = \frac{1}{4}x^4 + x - \frac{5}{4}; F'(x) = x^3 + 1.$$

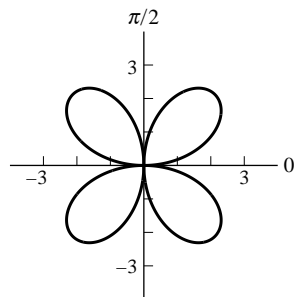
$$48. \text{(a) } F'(x) = \frac{1}{\sqrt{x}}. \quad \text{(b) } F(x) = 2\sqrt{t} \Big|_4^x = 2\sqrt{x} - 2; F'(x) = \frac{1}{\sqrt{x}}.$$

$$49. e^{x^2}$$

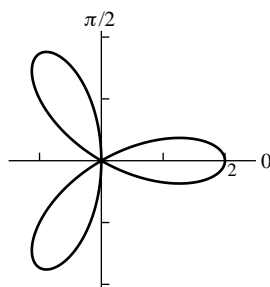
Exercise Set 7.8

1. (a) Improper; infinite discontinuity at $x = 3$. (b) Continuous integrand, not improper.
 (c) Improper; infinite discontinuity at $x = 0$. (d) Improper; infinite interval of integration.
 (e) Improper; infinite interval of integration and infinite discontinuity at $x = 1$.
 (f) Continuous integrand, not improper.
2. (a) Improper if $p > 0$. (b) Improper if $1 \leq p \leq 2$.
 (c) Integrand is continuous for all p , not improper.
3. $\lim_{\ell \rightarrow +\infty} \left(-\frac{1}{2}e^{-2x} \right) \Big|_0^\ell = \frac{1}{2} \lim_{\ell \rightarrow +\infty} (-e^{-2\ell} + 1) = \frac{1}{2}.$
4. $\lim_{\ell \rightarrow +\infty} \frac{1}{2} \ln(1 + x^2) \Big|_{-1}^\ell = \lim_{\ell \rightarrow +\infty} \frac{1}{2} [\ln(1 + \ell^2) - \ln 2] = +\infty$, divergent.
5. $\lim_{\ell \rightarrow +\infty} -2 \coth^{-1} x \Big|_3^\ell = \lim_{\ell \rightarrow +\infty} (2 \coth^{-1} 3 - 2 \coth^{-1} \ell) = 2 \coth^{-1} 3.$
6. $\lim_{\ell \rightarrow +\infty} -\frac{1}{2}e^{-x^2} \Big|_0^\ell = \lim_{\ell \rightarrow +\infty} \frac{1}{2} (-e^{-\ell^2} + 1) = 1/2.$
7. $\lim_{\ell \rightarrow +\infty} -\frac{1}{2 \ln^2 x} \Big|_e^\ell = \lim_{\ell \rightarrow +\infty} \left[-\frac{1}{2 \ln^2 \ell} + \frac{1}{2} \right] = \frac{1}{2}.$
8. $\lim_{\ell \rightarrow +\infty} 2\sqrt{\ln x} \Big|_2^\ell = \lim_{\ell \rightarrow +\infty} (2\sqrt{\ln \ell} - 2\sqrt{\ln 2}) = +\infty$, divergent.
9. $\lim_{\ell \rightarrow -\infty} -\frac{1}{4(2x-1)^2} \Big|_\ell^0 = \lim_{\ell \rightarrow -\infty} \frac{1}{4} [-1 + 1/(2\ell-1)^2] = -1/4.$
10. $\lim_{\ell \rightarrow -\infty} \frac{1}{3} \tan^{-1} \frac{x}{3} \Big|_\ell^3 = \lim_{\ell \rightarrow -\infty} \frac{1}{3} \left[\frac{\pi}{4} - \tan^{-1} \frac{\ell}{3} \right] = \frac{1}{3} [\pi/4 - (-\pi/2)] = \pi/4.$
11. $\lim_{\ell \rightarrow -\infty} \frac{1}{3} e^{3x} \Big|_\ell^0 = \lim_{\ell \rightarrow -\infty} \left[\frac{1}{3} - \frac{1}{3} e^{3\ell} \right] = \frac{1}{3}.$
12. $\lim_{\ell \rightarrow -\infty} -\frac{1}{2} \ln(3 - 2e^x) \Big|_\ell^0 = \lim_{\ell \rightarrow -\infty} \frac{1}{2} \ln(3 - 2e^\ell) = \frac{1}{2} \ln 3.$
13. $\int_{-\infty}^{+\infty} x \, dx$ converges if $\int_{-\infty}^0 x \, dx$ and $\int_0^{+\infty} x \, dx$ both converge; it diverges if either (or both) diverges. $\int_0^{+\infty} x \, dx = \lim_{\ell \rightarrow +\infty} \frac{1}{2} x^2 \Big|_0^\ell = \lim_{\ell \rightarrow +\infty} \frac{1}{2} \ell^2 = +\infty$, so $\int_{-\infty}^{+\infty} x \, dx$ is divergent.
14. $\int_0^{+\infty} \frac{x}{\sqrt{x^2+2}} dx = \lim_{\ell \rightarrow +\infty} \sqrt{x^2+2} \Big|_0^\ell = \lim_{\ell \rightarrow +\infty} (\sqrt{\ell^2+2} - \sqrt{2}) = +\infty$, so $\int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}} dx$ is divergent.

7. (a) $(5, 0.92730)$ (b) $(10, -0.92730)$ (c) $(1.27155, 2.47582)$
8. (a) $(5, 2.21430)$ (b) $(3.44819, 2.62604)$ (c) $(2.06740, 0.25605)$
9. (a) $r^2 = x^2 + y^2 = 4$; circle.
- (b) $y = 4$; horizontal line.
- (c) $r^2 = 3r \cos \theta$, $x^2 + y^2 = 3x$, $(x - 3/2)^2 + y^2 = 9/4$; circle.
- (d) $3r \cos \theta + 2r \sin \theta = 6$, $3x + 2y = 6$; line.
10. (a) $r \cos \theta = 5$, $x = 5$; vertical line.
- (b) $r^2 = 2r \sin \theta$, $x^2 + y^2 = 2y$, $x^2 + (y - 1)^2 = 1$; circle.
- (c) $r^2 = 4r \cos \theta + 4r \sin \theta$, $x^2 + y^2 = 4x + 4y$, $(x - 2)^2 + (y - 2)^2 = 8$; circle.
- (d) $r = \frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta}$, $r \cos^2 \theta = \sin \theta$, $r^2 \cos^2 \theta = r \sin \theta$, $x^2 = y$; parabola.
11. (a) $r \cos \theta = 3$. (b) $r = \sqrt{7}$. (c) $r^2 + 6r \sin \theta = 0$, $r = -6 \sin \theta$.
- (d) $9(r \cos \theta)(r \sin \theta) = 4$, $9r^2 \sin \theta \cos \theta = 4$, $r^2 \sin 2\theta = 8/9$.
12. (a) $r \sin \theta = -3$. (b) $r = \sqrt{5}$. (c) $r^2 + 4r \cos \theta = 0$, $r = -4 \cos \theta$.
- (d) $r^4 \cos^2 \theta = r^2 \sin^2 \theta$, $r^2 = \tan^2 \theta$, $r = \pm \tan \theta$.



13. $r = 3 \sin 2\theta$.



14. $r = 2 \cos 3\theta$.

31. $3t\mathbf{i} + 2t^2\mathbf{j} + \mathbf{C}$.

32. $(t^3/3)\mathbf{i} - t^2\mathbf{j} + \ln|t|\mathbf{k} + \mathbf{C}$.

33. $\langle te^t - e^t, t \ln t - t \rangle + \mathbf{C}$.

34. $\langle -e^{-t}, e^t, t^3 \rangle + \mathbf{C}$.

35. $\left\langle \frac{1}{2} \sin 2t, -\frac{1}{2} \cos 2t \right\rangle \Big|_0^{\pi/2} = \langle 0, 1 \rangle$.

36. $\left(\frac{1}{3} t^3 \mathbf{i} + \frac{1}{4} t^4 \mathbf{j} \right) \Big|_0^1 = \frac{1}{3} \mathbf{i} + \frac{1}{4} \mathbf{j}$.

37. $\int_0^2 \sqrt{t^2 + t^4} dt = \int_0^2 t(1 + t^2)^{1/2} dt = \frac{1}{3} (1 + t^2)^{3/2} \Big|_0^2 = (5\sqrt{5} - 1)/3$.

38. $\left\langle -\frac{2}{5}(3-t)^{5/2}, \frac{2}{5}(3+t)^{5/2}, t \right\rangle \Big|_{-3}^3 = \langle 72\sqrt{6}/5, 72\sqrt{6}/5, 6 \rangle$.

39. $\left(\frac{2}{3} t^{3/2} \mathbf{i} + 2t^{1/2} \mathbf{j} \right) \Big|_1^9 = \frac{52}{3} \mathbf{i} + 4\mathbf{j}$.

40. $\frac{1}{2}(e^2 - 1)\mathbf{i} + (1 - e^{-1})\mathbf{j} + \frac{1}{2}\mathbf{k}$.

41. False. The limit only exists if $\mathbf{r}(t)$ is differentiable at $t = a$. As with functions of a single variable, continuity does not imply differentiability. For example, $\mathbf{r}(t) = \langle |t|, 0 \rangle$ is continuous at $t = 0$, but not differentiable there.

42. False. By Theorem 12.2.8 they are orthogonal. They are only parallel if $\mathbf{r}(t)$ is constant, in which case $\mathbf{r}'(t) = \mathbf{0}$.

43. True. Equations (11) and (12) express $\int_a^b \mathbf{r}(t) dt$ as a vector, whose components are the definite integrals of the components of $\mathbf{r}(t)$.

44. True. In 2-space, if $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ then by the Fundamental Theorem of Calculus, equation (11) implies that $\frac{d}{dt} \int_a^t \mathbf{r}(u) du = \frac{d}{dt} \left[\left(\int_a^t x(u) du \right) \mathbf{i} + \left(\int_a^t y(u) du \right) \mathbf{j} \right] = \left(\frac{d}{dt} \int_a^t x(u) du \right) \mathbf{i} + \left(\frac{d}{dt} \int_a^t y(u) du \right) \mathbf{j} = x(t)\mathbf{i} + y(t)\mathbf{j} = \mathbf{r}(t)$. The proof for vectors in 3-space is similar.

45. $\mathbf{y}(t) = \int \mathbf{y}'(t) dt = t^2\mathbf{i} + t^3\mathbf{j} + \mathbf{C}, \mathbf{y}(0) = \mathbf{C} = \mathbf{i} - \mathbf{j}, \mathbf{y}(t) = (t^2 + 1)\mathbf{i} + (t^3 - 1)\mathbf{j}$.

46. $\mathbf{y}(t) = \int \mathbf{y}'(t) dt = (\sin t)\mathbf{i} - (\cos t)\mathbf{j} + \mathbf{C}, \mathbf{y}(0) = -\mathbf{j} + \mathbf{C} = \mathbf{i} - \mathbf{j}$ so $\mathbf{C} = \mathbf{i}$ and $\mathbf{y}(t) = (1 + \sin t)\mathbf{i} - (\cos t)\mathbf{j}$.

47. $\mathbf{y}'(t) = \int \mathbf{y}''(t) dt = t\mathbf{i} + e^t\mathbf{j} + \mathbf{C}_1, \mathbf{y}'(0) = \mathbf{j} + \mathbf{C}_1 = \mathbf{j}$ so $\mathbf{C}_1 = \mathbf{0}$ and $\mathbf{y}'(t) = t\mathbf{i} + e^t\mathbf{j}$. $\mathbf{y}(t) = \int \mathbf{y}'(t) dt = \frac{1}{2}t^2\mathbf{i} + e^t\mathbf{j} + \mathbf{C}_2, \mathbf{y}(0) = \mathbf{j} + \mathbf{C}_2 = 2\mathbf{i}$ so $\mathbf{C}_2 = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{y}(t) = \left(\frac{1}{2}t^2 + 2 \right) \mathbf{i} + (e^t - 1)\mathbf{j}$.

48. $\mathbf{y}'(t) = \int \mathbf{y}''(t) dt = 4t^3\mathbf{i} - t^2\mathbf{j} + \mathbf{C}_1, \mathbf{y}'(0) = \mathbf{C}_1 = \mathbf{0}, \mathbf{y}'(t) = 4t^3\mathbf{i} - t^2\mathbf{j}, \mathbf{y}(t) = \int \mathbf{y}'(t) dt = t^4\mathbf{i} - \frac{1}{3}t^3\mathbf{j} + \mathbf{C}_2, \mathbf{y}(0) = \mathbf{C}_2 = 2\mathbf{i} - 4\mathbf{j}, \mathbf{y}(t) = (t^4 + 2)\mathbf{i} - \left(\frac{1}{3}t^3 + 4 \right) \mathbf{j}$.

$$2. \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1 & 4v \\ 4u & -1 \end{vmatrix} = -1 - 16uv.$$

$$3. \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \cos u & -\sin v \\ \sin u & \cos v \end{vmatrix} = \cos u \cos v + \sin u \sin v = \cos(u - v).$$

$$4. \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{2(v^2 - u^2)}{(u^2 + v^2)^2} & -\frac{4uv}{(u^2 + v^2)^2} \\ \frac{4uv}{(u^2 + v^2)^2} & \frac{2(v^2 - u^2)}{(u^2 + v^2)^2} \end{vmatrix} = 4/(u^2 + v^2)^2.$$

$$5. x = \frac{2}{9}u + \frac{5}{9}v, y = -\frac{1}{9}u + \frac{2}{9}v; \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 2/9 & 5/9 \\ -1/9 & 2/9 \end{vmatrix} = \frac{1}{9}.$$

$$6. x = \ln u, y = uv; \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1/u & 0 \\ v & u \end{vmatrix} = 1.$$

$$7. x = \frac{\sqrt{u+v}}{\sqrt{2}}, y = \frac{\sqrt{v-u}}{\sqrt{2}}; \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{1}{2\sqrt{2}\sqrt{u+v}} & \frac{1}{2\sqrt{2}\sqrt{u+v}} \\ -\frac{1}{2\sqrt{2}\sqrt{v-u}} & \frac{1}{2\sqrt{2}\sqrt{v-u}} \end{vmatrix} = \frac{1}{4\sqrt{v^2 - u^2}}.$$

$$8. x = \frac{u^{3/2}}{v^{1/2}}, y = \frac{v^{1/2}}{u^{1/2}}; \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{3u^{1/2}}{2v^{1/2}} & -\frac{u^{3/2}}{2v^{3/2}} \\ -\frac{v^{1/2}}{2u^{3/2}} & \frac{1}{2u^{1/2}v^{1/2}} \end{vmatrix} = \frac{1}{2v}.$$

$$9. \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{vmatrix} = 5.$$

$$10. \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix} = u^2v.$$

$$11. y = v, x = \frac{u}{y} = \frac{u}{v}, z = w - x = w - \frac{u}{v}; \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} 1/v & -u/v^2 & 0 \\ 0 & 1 & 0 \\ -1/v & u/v^2 & 1 \end{vmatrix} = \frac{1}{v}.$$

$$12. x = \frac{v+w}{2}, y = \frac{u-w}{2}, z = \frac{u-v}{2}; \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ 1/2 & -1/2 & 0 \end{vmatrix} = -\frac{1}{4}.$$

13. False. It is the area of the parallelogram.

14. False. If the mapping is not one-to-one, then the integral may be larger than the area. For example, let $x = u$, $y = (v-3)^2$. Then R is the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 4$, with area 8, but $\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1 & 0 \\ 0 & 2(v-3) \end{vmatrix} = 2(v-3)$, so $\int_1^5 \int_0^2 \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv = \int_1^5 \int_0^2 2|v-3| du dv = \int_1^5 4|v-3| dv = \int_1^3 4(3-v) dv + \int_3^5 4(v-3) dv = (12v - 2v^2) \Big|_1^3 + (2v^2 - 12v) \Big|_3^5 = 8 + 8 = 16$.

15. False. The Jacobian is $\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r(\cos^2 \theta + \sin^2 \theta) = r$.