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**Complete Solutions Manual to Accompany**

# A First Course in Differential Equations with Modeling Applications

**ELEVENTH EDITION, METRIC VERSION**

**And**

# Differential Equations with Boundary-Value Problems

**NINTH EDITION, METRIC VERSION**

**Dennis G. Zill**
Loyola Marymount University,
Los Angeles, CA

Prepared by

**Roberto Martinez**
Loyola Marymount University, Los Angeles, CA

Metric Version Prepared by

**Aly El-Iraki**
Professor Emeritus, Alexandria University, Egypt

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200 First Stamford Place, 4th Floor
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# Chapter 1

# Introduction to Differential Equations

## 1.1 Definitions and Terminology

**1.** Second order; linear

**2.** Third order; nonlinear because of $(dy/dx)^4$

**3.** Fourth order; linear

**4.** Second order; nonlinear because of $\cos(r+u)$

**5.** Second order; nonlinear because of $(dy/dx)^2$ or $\sqrt{1+(dy/dx)^2}$

**6.** Second order; nonlinear because of $R^2$

**7.** Third order; linear

**8.** Second order; nonlinear because of $\dot{x}^2$

**9.** Writing the differential equation in the form $x(dy/dx)+y^2=1$, we see that it is nonlinear in $y$ because of $y^2$. However, writing it in the form $(y^2-1)(dx/dy)+x=0$, we see that it is linear in $x$.

**10.** Writing the differential equation in the form $u(dv/du)+(1+u)v=ue^u$ we see that it is linear in $v$. However, writing it in the form $(v+uv-ue^u)(du/dv)+u=0$, we see that it is nonlinear in $u$.

**11.** From $y=e^{-x/2}$ we obtain $y'=-\frac{1}{2}e^{-x/2}$. Then $2y'+y=-e^{-x/2}+e^{-x/2}=0$.

**12.** From $y=\frac{6}{5}-\frac{6}{5}e^{-20t}$ we obtain $dy/dt=24e^{-20t}$, so that

$$\frac{dy}{dt}+20y=24e^{-20t}+20\left(\frac{6}{5}-\frac{6}{5}e^{-20t}\right)=24.$$

**13.** From $y=e^{3x}\cos 2x$ we obtain $y'=3e^{3x}\cos 2x-2e^{3x}\sin 2x$ and $y''=5e^{3x}\cos 2x-12e^{3x}\sin 2x$, so that $y''-6y'+13y=0$.



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**58.** Writing $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ and applying $D(D^2+4)$ to the differential equation we obtain

$$D(D^2+4)(D^2+4) = D(D^2+4)^2 = 0.$$

Then

$$y = \underbrace{c_1 \cos 2x + c_2 \sin 2x}_{y_c} + c_3 x\cos 2x + c_4 x \sin 2x + c_5$$

and $y_p = Ax\cos 2x + Bx\sin 2x + C$. Substituting $y_p$ into the differential equation yields

$$-4A\sin 2x + 4B\cos 2x + 4C = \frac{1}{2} + \frac{1}{2}\cos 2x.$$

Equating coefficients gives $A = 0$, $B = 1/8$, and $C = 1/8$. The general solution is

$$y = c_1\cos 2x + c_2 \sin 2x + \frac{1}{8}x\sin 2x + \frac{1}{8}.$$

**59.** Applying $D^3$ to the differential equation we obtain

$$D^3(D^3+8D^2) = D^5(D+8) = 0.$$

Then

$$y = \underbrace{c_1 + c_2 x + c_3 e^{-8x}}_{y_c} + c_4 x^2 + c_5 x^3 + c_6 x^4$$

and $y_p = Ax^2 + Bx^3 + Cx^4$. Substituting $y_p$ into the differential equation yields

$$16A + 6B + (48B + 24C)x + 96Cx^2 = 2 + 9x - 6x^2.$$

Equating coefficients gives

$$\begin{aligned} 16A + 6B &= 2 \\ 48B + 24C &= 9 \\ 96C &= -6. \end{aligned}$$

Then $A = 11/256$, $B = 7/32$, and $C = -1/16$, and the general solution is

$$y = c_1 + c_2 x + c_3 e^{-8x} + \frac{11}{256}x^2 + \frac{7}{32}x^3 - \frac{1}{16}x^4.$$

**60.** Applying $D(D-1)^2(D+1)$ to the differential equation we obtain

$$D(D-1)^2(D+1)(D^3 - D^2 + D - 1) = D(D-1)^3(D+1)(D^2+1) = 0.$$

Then

$$y = \underbrace{c_1 e^x + c_2\cos x + c_3 \sin x}_{y_c} + c_4 + c_5 e^{-x} + c_6 x e^x + c_7 x^2 e^x$$

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**57.** Solving $\frac{1}{2}q'' + 10q' + 100q = 150$ we obtain $q(t) = e^{-10t}(c_1 \cos 10t + c_2 \sin 10t) + 3/2$. The initial conditions $q(0) = 1$ and $q'(0) = 0$ imply $c_1 = c_2 = -1/2$. Thus

$$q(t) = -\frac{1}{2}e^{-10t}(\cos 10t + \sin 10t) + \frac{3}{2}.$$

As $t \to \infty$, $q(t) \to 3/2$.

**58.** In Problem 54 it is shown that the amplitude of the steady-state current is $E_0/Z$, where $Z = \sqrt{X^2 + R^2}$ and $X = L\gamma - 1/C\gamma$. Since $E_0$ is constant the amplitude will be a maximum when $Z$ is a minimum. Since $R$ is constant, $Z$ will be a minimum when $X = 0$. Solving $L\gamma - 1/C\gamma = 0$ for $\gamma$ we obtain $\gamma = 1/\sqrt{LC}$. The maximum amplitude will be $E_0/R$.

**59.** By Problem 54 the amplitude of the steady-state current is $E_0/Z$, where $Z = \sqrt{X^2 + R^2}$ and $X = L\gamma - 1/C\gamma$. Since $E_0$ is constant the amplitude will be a maximum when $Z$ is a minimum. Since $R$ is constant, $Z$ will be a minimum when $X = 0$. Solving $L\gamma - 1/C\gamma = 0$ for $C$ we obtain $C = 1/L\gamma^2$.

**60.** Solving $0.1q'' + 10q = 100 \sin \gamma t$ we obtain

$$q(t) = c_1 \cos 10t + c_2 \sin 10t + q_p(t)$$

where $q_p(t) = A \sin \gamma t + B \cos \gamma t$. Substituting $q_p(t)$ into the differential equation we find

$$(100 - \gamma^2)A \sin \gamma t + (100 - \gamma^2)B \cos \gamma t = 100 \sin \gamma t.$$

Equating coefficients we obtain $A = 100/(100-\gamma^2)$ and $B = 0$. Thus, $q_p(t) = \dfrac{100}{100 - \gamma^2} \sin \gamma t$. The initial conditions $q(0) = q'(0) = 0$ imply $c_1 = 0$ and $c_2 = -10\gamma/(100 - \gamma^2)$. The charge is

$$q(t) = \frac{10}{100 - \gamma^2}(10 \sin \gamma t - \gamma \sin 10t)$$

and the current is

$$i(t) = \frac{100\gamma}{100 - \gamma^2}(\cos \gamma t - \cos 10t).$$

**61.** In an $LC$-series circuit there is no resistor, so the differential equation is

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = E(t).$$

Then $q(t) = c_1 \cos\left(t/\sqrt{LC}\right) + c_2 \sin\left(t/\sqrt{LC}\right) + q_p(t)$ where $q_p(t) = A \sin \gamma t + B \cos \gamma t$. Substituting $q_p(t)$ into the differential equation we find

$$\left(\frac{1}{C} - L\gamma^2\right) A \sin \gamma t + \left(\frac{1}{C} - L\gamma^2\right) B \cos \gamma t = E_0 \cos \gamma t.$$

Equating coefficients we obtain $A = 0$ and $B = E_0C/(1 - LC\gamma^2)$. Thus, the charge is

$$q(t) = c_1 \cos \frac{1}{\sqrt{LC}}t + c_2 \sin \frac{1}{\sqrt{LC}}t + \frac{E_0C}{1 - LC\gamma^2} \cos \gamma t.$$

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From the last result and using $\nu = 3/2$ we obtain

$$\begin{aligned} 3J_{3/2}(x) &= xJ_{5/2}(x) + xJ_{1/2}(x) \\ J_{5/2}(x) &= \frac{3}{x}J_{3/2}(x) - J_{1/2}(x) \\ &= \frac{3}{x}\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right) - \sqrt{\frac{2}{\pi x}}\sin x \\ &= \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3}{x^2} - 1\right)\sin x - \frac{3\cos x}{x}\right] \end{aligned}$$

From the last result and using $\nu = 5/2$ we obtain

$$\begin{aligned} 5J_{5/2}(x) &= xJ_{7/2}(x) + xJ_{3/2}(x) \\ J_{7/2}(x) &= \frac{5}{x}J_{5/2}(x) - J_{3/2}(x) \\ &= \frac{5}{x}\sqrt{\frac{2}{\pi x}}\left(\frac{3\sin x}{x^2} - \frac{3\cos x}{x} - \sin x\right) - \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right) \\ &= \sqrt{\frac{2}{\pi x}}\left[\left(\frac{15}{x^3} - \frac{6}{x}\right)\sin x - \left(\frac{15}{x^2} - 1\right)\cos x\right] \end{aligned}$$

**33. (a)** To find the spherical Bessel functions $j_1(x)$ and $j_2(x)$ we use the first formula in (30),

$$j_n(x) = \sqrt{\frac{\pi}{2x}}\, J_{n+1/2}$$

with $n = 1$ and $n = 2$,

$$j_1(x) = \sqrt{\frac{\pi}{2x}\, J_{3/2}(x)} \qquad \text{and} \qquad j_2(x) = \sqrt{\frac{\pi}{2x}}\, J_{5/2}(x)\,.$$

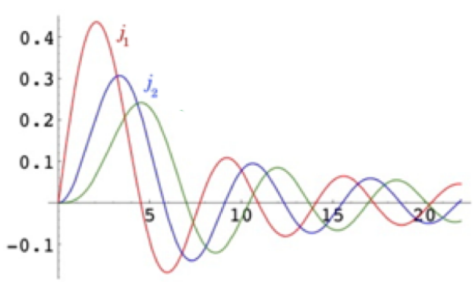
Then from Problem 32 we have

$$J_{3/2}(x) = \sqrt{2}\pi x\left(\frac{\sin x}{x} - \cos x\right) \qquad \text{so} \qquad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

and

$$J_{5/2}(x) = \sqrt{2}\pi x\left(\frac{3\sin x}{x^2} - \frac{3\cos x}{x} - \sin x\right) \qquad \text{so} \qquad j_2(x) = \left(\frac{3}{x^3} - \frac{1}{x}\right)\sin x - \frac{3\cos x}{x^2}$$

**(b)** Using a graphing utility to plot the graphs of $j_1(x)$ and $j_2(x)$, we get the red and blue graphcs in the figure to the right.

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**31.** $f(t) = 2 - 2\mathscr{U}(t-2) + [(t-2)+2]\mathscr{U}(t-2) = 2 + (t-2)\mathscr{U}(t-2)$

$$\mathscr{L}\{f(t)\} = \frac{2}{s} + \frac{1}{s^2}e^{-2s}$$

$$\mathscr{L}\left\{e^t f(t)\right\} = \frac{2}{s-1} + \frac{1}{(s-1)^2}e^{-2(s-1)}$$

**32.** $f(t) = t - t\mathscr{U}(t-1) + (2-t)\mathscr{U}(t-1) - (2-t)\mathscr{U}(t-2) = t - 2(t-1)\mathscr{U}(t-1) + (t-2)\mathscr{U}(t-2)$

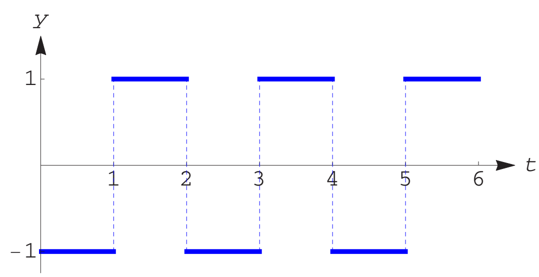
$$\mathscr{L}\{f(t)\} = \frac{1}{s^2} - \frac{2}{s^2}e^{-s} + \frac{1}{s^2}e^{-2s}$$

$$\mathscr{L}\left\{e^t f(t)\right\} = \frac{1}{(s-1)^2} - \frac{2}{(s-1)^2}e^{-(s-1)} + \frac{1}{(s-1)^2}e^{-2(s-1)}$$

**33.** The graph of

$$f(t) = -1 + 2\sum_{k=1}^{\infty}(-1)^{k+1}\,\mathscr{U}(t-k) = -1 + 2\mathscr{U}(t-1) - 2\mathscr{U}(t-2) + 2\mathscr{U}(t-3) - \cdots$$

is




One way of proceeding to find the Laplace transform is to take the transform term-by-term of the series:

$$\mathscr{L}\{f(t)\} = -\frac{1}{s} + \frac{2}{s}e^{-s} - \frac{2}{s}e^{-2s} + \frac{2}{s}e^{-3s} - \cdots \quad \longleftarrow \quad \text{geometric series}$$

For $s > 0$,

$$\mathscr{L}\{f(t)\} = -\frac{1}{s} + \frac{2}{s}\left[e^{-s} - e^{-2s} + e^{-3s} - \cdots\right] = -\frac{1}{s} + \frac{2}{s}\cdot\frac{e^{-s}}{1+e^{-s}}$$

$$= \frac{e^{-s}-1}{s\,(1+e^{-s})}$$

Alternatively, since $f$ is a periodic functions it can also be defined by

$$f(t) = \begin{cases} -1, & 0 \le t < 1 \\ 1, & 1 \le t < 2, \end{cases} \quad \text{where } f(t+2) = f(t).$$

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**I.** If $\lambda = 0$ then $X'' = 0$ and $X(x) = c_1x + c_2$. Also $Y'' - Y = 0$ and $Y(y) = c_3 \cosh y + c_4 \sinh y$ so

$$u = XY = (c_1x + c_2)(c_3 \cosh y + c_4 \sinh y).$$

**II.** If $\lambda = -\alpha^2 < 0$ then $X'' - \alpha^2 X = 0$ and $Y'' + (\alpha^2 - 1)Y = 0$. The solution of the first differential equation is $X(x) = c_5 \cosh \alpha x + c_6 \sinh \alpha x$. The solution of the second differential equation depends on the nature of $\alpha^2 - 1$. We consider three cases:

**(i)** If $\alpha^2 - 1 = 0$, or $\alpha^2 = 1$, then $Y(y) = c_7 y + c_8$ and

$$u = XY = (c_5 \cosh \alpha x + c_6 \sinh \alpha x)(c_7 y + c_8).$$

**(ii)** If $\alpha^2 - 1 < 0$, or $0 < \alpha^2 < 1$, then $Y(y) = c_9 \cosh \sqrt{1-\alpha^2}\, y + c_{10} \sinh \sqrt{1-\alpha^2}\, y$ and

$$u = XY = (c_5 \cosh \alpha x + c_6 \sinh \alpha x)\left(c_9 \cosh \sqrt{1-\alpha^2}\, y + c_{10} \sinh \sqrt{1-\alpha^2}\, y\right).$$

**(iii)** If $\alpha^2 - 1 > 0$, or $\alpha^2 > 1$, then $Y(y) = c_{11} \cos \sqrt{\alpha^2 - 1}\, y + c_{12} \sin \sqrt{\alpha^2 - 1}\, y$ and

$$u = XY = (c_5 \cosh \alpha x + c_6 \sinh \alpha x)\left(c_{11} \cos \sqrt{\alpha^2-1}\, y + c_{12} \sin \sqrt{\alpha^2-1}\, y\right).$$

**III.** If $\lambda = \alpha^2 > 0$, then $X'' + \alpha^2 X = 0$ and $X(x) = c_{13} \cos \alpha x + c_{14} \sin \alpha x$. Also,
$Y'' - (1 + \alpha^2)Y = 0$ and $Y(y) = c_{15} \cosh \sqrt{1+\alpha^2}\, y + c_{16} \sinh \sqrt{1+\alpha^2}\, y$ so

$$u = XY = (c_{13} \cos \alpha x + c_{14} \sin \alpha x)\left(c_{15} \cosh \sqrt{1+\alpha^2}\, y + c_{16} \sinh \sqrt{1+\alpha^2}\, y\right).$$

**16.** Substituting $u(x,t) = X(x)T(t)$ into the partial differential equation yields $a^2 X''T - g = XT''$, which is not separable.

**17.** Identifying $A = B = C = 1$, we compute $B^2 - 4AC = -3 < 0$. The equation is elliptic.

**18.** Identifying $A = 3$, $B = 5$, and $C = 1$, we compute $B^2 - 4AC = 13 > 0$. The equation is hyperbolic.

**19.** Identifying $A = 1$, $B = 6$, and $C = 9$, we compute $B^2 - 4AC = 0$. The equation is parabolic.

**20.** Identifying $A = 1$, $B = -1$, and $C = -3$, we compute $B^2 - 4AC = 13 > 0$. The equation is hyperbolic.

**21.** Identifying $A = 1$, $B = -9$, and $C = 0$, we compute $B^2 - 4AC = 81 > 0$. The equation is hyperbolic.

**22.** Identifying $A = 0$, $B = 1$, and $C = 0$, we compute $B^2 - 4AC = 1 > 0$. The equation is hyperbolic.

**23.** Identifying $A = 1$, $B = 2$, and $C = 1$, we compute $B^2 - 4AC = 0$. The equation is parabolic.

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so

$$A_0 = 0, \quad A_1 + B_1 = 0, \quad C_1 + D_1 = 75,$$

and

$$A_n + B_n = 0, \quad C_n + D_n = 0, \qquad \text{for} \qquad n > 1.$$

When $r = 2$

$$A_0 + B_0 \ln 2 = \frac{1}{2\pi} \int_0^{2\pi} 60 \cos\theta \, d\theta = 0$$

$$A_n 2^n + B_n 2^{-n} = \frac{1}{\pi} \int_0^{2\pi} 60 \cos\theta \cos n\theta \, d\theta = \begin{cases} 0, & n > 1 \\ 60, & n = 1 \end{cases}$$

$$C_n 2^n + D_n 2^{-n} = \frac{1}{\pi} \int_0^{\infty} 60 \cos\theta \sin n\theta \, d\theta = 0, \quad n = 1, 2, \ldots,$$

so

$$B_0 = 0, \quad 2A_1 + \frac{1}{2} B_1 = 60, \quad 2C_1 + \frac{1}{2} D_1 = 0,$$

and

$$A_n 2^n + B_n 2^{-n} = 0, \quad C_n 2^n + D_n 2^{-n} = 0, \qquad \text{for} \qquad n > 1.$$

Whe have $A_0 = 0$ and $B_0 = 0$, and solving the nonhomogeneous systems for $n = 1$,

$$A_1 + B_1 = 0 \qquad\qquad C_1 + D_1 = 75$$

$$2A_1 + \frac{1}{2} B_1 = 60 \qquad\qquad 2C_1 + \frac{1}{2} D_1 = 0$$

yields $A_1 = 40$, $B_1 = -40$, $C_1 = -25$, and $D_1 = 100$. Finally, solving the homogeneous systems

$$A_n + B_n = 0 \qquad\qquad C_n + D_n = 0$$

$$A_n 2^n + B_n 2^{-n} = 0 \qquad\qquad C_n 2^n + D_n 2^{-n} = 0$$

gives $A_n = B_n = C_n = D_n = 0$ for $n > 1$. The solution is then

$$\begin{aligned} u(r,\theta) &= \left(A_1 r + B_1 r^{-1}\right) \cos\theta + \left(C_1 r + D_1 r^{-1}\right) \sin\theta \\ &= \left(4 - r - 40r^{-1}\right) \cos\theta + \left(-25r + 100r^{-1}\right) \sin\theta \\ &= 40 \left(r - \frac{1}{r}\right) \cos\theta - 25 \left(r - \frac{4}{r}\right) \sin\theta. \end{aligned}$$

**14.** We solve

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad 0 < \theta < \pi\,, \quad a < r < b,$$

$$u(a,\theta) = \theta(\pi - \theta), \quad u(b,\theta) = 0, \quad 0 < \theta < \pi\,,$$

$$u(r,0) = 0, \quad u(r,\pi) = 0, \quad a < r < b.$$

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By writing the boundary condition $x = 0$ as

$$u(0,t) = u_0 - u_0\mathscr{U}(t-1)$$

its transform is

$$\begin{aligned}
U(0,s) &= \frac{u_0}{s} - \frac{u_0}{s}\,e^{-s} \\
c_1 &= \frac{u_0}{s} - \frac{u_0}{s}\,e^{-s} \\
U(x,s) &= u_0\,\frac{e^{-\sqrt{s}\,x}}{s} - u_0\,\frac{e^{-\sqrt{s}\,x}}{s}\,e^{-s} \\
u(x,t) &= u_0\mathscr{L}^{-1}\left\{\frac{e^{-\sqrt{s}\,x}}{s}\right\} - u_0\mathscr{L}^{-1}\left\{\frac{e^{-\sqrt{s}\,x}}{s}\,e^{-s}\right\}
\end{aligned}$$

by entry 3 of Table 14.1.1 and the inverse form of the second translation theorem that:

$$u(x,t) = u_0\,\mathrm{erfc}\left(\frac{x}{2\sqrt{t}}\right) - u_0\,\mathrm{erfc}\left(\frac{x}{2\sqrt{t-1}}\right)\mathscr{U}(t-1)$$

or

$$u(x,t) = \begin{cases} u_0\,\mathrm{erfc}\left(\dfrac{x}{2\sqrt{t}}\right), & 0 < t < 1 \\ u_0\,\mathrm{erfc}\left(\dfrac{x}{2\sqrt{t}}\right) - u_0\,\mathrm{erfc}\left(\dfrac{x}{2\sqrt{t-1}}\right), & t > 1\,. \end{cases}$$

**18.** The Laplace transform with respect to $t$ of the partial differential equation gives

$$\frac{d^2U}{dx^2} - sU = -50 \quad \text{so} \quad U(x,s) = c_1e^{-\sqrt{s}x} + c_2e^{\sqrt{s}x} + \frac{50}{s}.$$

The boundary condition

$$\lim_{x\to\infty} u(x,t) = 50 \quad \text{implies} \quad \lim_{x\to\infty} U(x,s) = \frac{50}{s}$$

so we take $c_2 = 0$. Thus

$$U(x,s) = c_1e^{-\sqrt{s}x} + \frac{50}{s}.$$

The transform of the boundary condition at $x = 0$ is

$$U(0,s) = \frac{100}{s}e^{-5s} - \frac{100}{s}e^{-10s}.$$

Since

$$\frac{100}{s}e^{-5s} - \frac{100}{s}e^{-10s} = c_1 + \frac{50}{s}$$

we have

$$c_1 = -\frac{50}{s} + \frac{100}{s}e^{-5s} - \frac{100}{s}e^{-10s}$$

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