

**SOLUTIONS MANUAL**  
For  
**INTRODUCTION TO  
OPERATIONS RESEARCH**

Ninth Edition

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## CHAPTER 1: INTRODUCTION

### 1.3-1.

Answers will vary.

### 1.3-2.

Answers will vary.

### 1.3-3.

By using operations research (OR), FedEx managed to survive crises that could drive it out of business. The new planning system provided more flexibility in choosing the destinations that it serves, the routes and the schedules. Improved schedules yielded into faster and more reliable service. OR applied to this complex system with a lot of interdependencies resulted in an efficient use of the assets. With the new system, FedEx maintained a high load factor while being able to service in a reliable, flexible and profitable manner. The model also enabled the company to foresee future risks and to take measures against undesirable outcomes. The systematic approach has been effective in convincing investors and employees about the benefits of the changes. Consequently, "today FedEx is one of the nation's largest integrated, multi-conveyance freight carriers" [p. 32].

## **CHAPTER 2: OVERVIEW OF THE OPERATIONS RESEARCH MODELING APPROACH**

### **2.1-1.**

- (a) The rise of electronic brokerage firms in the late 90s was a threat against full-service financial service firms like Merrill Lynch. Electronic trading offered very low costs, which were hard to compete with for full-service firms. With banks, discount brokers and electronic trading firms involved, the competition was fierce. Merrill Lynch needed an urgent response to these changes in order to survive.
- (b) "The group's mission is to aid strategic decision making in complex business situations through quantitative modeling and analysis" [p.8].
- (c) The data obtained for each client consisted of "data for six categories of revenue, four categories of account type, nine asset allocation categories, along with data on number of trades, mutual fund exchanges and redemptions, sales of zero coupon bonds, and purchases of new issues" [p. 10].
- (d) As a result of this study, two main pricing options, viz., an asset-based pricing option and a direct online pricing option were offered to the clients. The first targeted the clients who want advice from a financial advisor. The clients who would choose this option would be charged at a fixed rate of the value of their assets and would not pay for each trade. The latter pricing option was for the clients who want to invest online and who do not want advice. These self-directed investors would be charged for every trade.
- (e) "The benefits were significant and fell into four areas: seizing the marketplace initiative, finding the pricing sweet spot, improving financial performance, and adopting the approach in other strategic initiatives" [p.15].

### **2.1-2.**

- (a) This study arose from GM's efforts to survive the competition of the late 80s. Various factors, including the rise of foreign imports, the increase in customer expectations and the pricing constraints, forced GM to close plants and to incur large financial losses. While trying to copy Japanese production methods directly, GM was suffering from "missing production targets, working unscheduled overtime, experiencing high scrap costs, and executing throughput-improvement initiatives with disappointing results" [p. 7]. The real problems were not understood and the company was continuously losing money while the managers kept disagreeing about solutions.
- (b) The goal of this study was "to improve the throughput performance of existing and new manufacturing systems through coordinated efforts in three areas: modeling and algorithms, data collection, and throughput-improvement processes" [p. 7].
- (c) The data collection was automated by using programmable logic controllers (PLCs). The software kept track of the production events including "machine faults and blocking and starving events" [p. 13] and recorded their duration. The summary of this data was then transferred to a centralized database, which converted this to workstation-performance characteristics and used in validating the models, determining the bottleneck processes and enhancing throughput.
- (d) The improved production throughput resulted in more than \$2.1 billion in documented savings and increased revenue.

### 2.1-3.

(a) San Francisco Police Department has a total police force of 1900, with 850 officers on patrol. The total budget of SFPD in 1986 was \$176 million with patrol coverage cost of \$79 million. This brings out the importance of the problem.

Like most police departments, SFPD was also operated with manually designed schedules. It was impossible to know if the manual schedules were optimal in serving residents' needs. It was difficult to evaluate alternative policies for scheduling and deploying officers. There was also the problem of poor response time and low productivity, pressure of increasing demands for service with decreasing budgets. The scheduling system was facing the problem of providing the highest possible correlation between the number of officers needed and the number actually on duty during each hour. All these problems led the Task Force to search for a new system and thus undertake this study.

(b) After reviewing the manual system, the Task Force decided to search for a new system. The criteria it specified included the following six directives :

- the system must use the CAD (computer aided dispatching) system, which provides a large and rich data base on resident calls for service. The CAD system was used to dispatch patrol officers to call for service and to maintain operating statistics such as call types, waiting times, travel time and total time consumed in servicing calls. The directive was to use this data on calls for service and consumed times to establish work load by day of week and hour of day

- it must generate optimal and realistic integer schedules that meet management policy guidelines using a micro-computer

- it must allow easy adjustment of optimal schedules to accommodate human considerations without sacrificing productivity

- it must create schedules in less than 30 minutes and make changes in less than 60 seconds

- it must be able to perform both tactical scheduling and strategic policy testing in one integrated system

- the user interface must be flexible and easy, allowing the users (captains) to decide the sequence of functions to be executed instead of forcing them to follow a restrictive sequence.

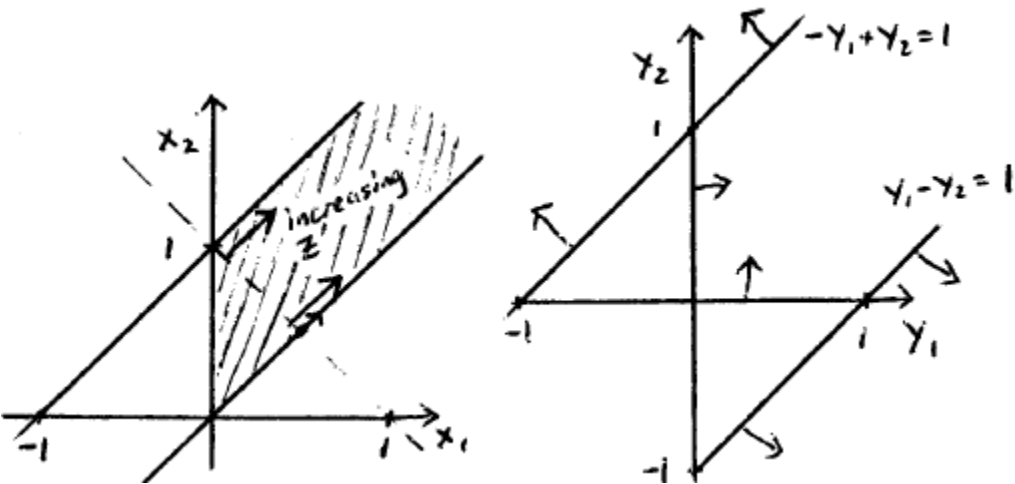
6.1-8.

Primal: maximize  $x_1 + x_2$   
subject to  $-x_1 + x_2 \leq 1$   
 $x_1 - x_2 \leq 0$   
 $x_1, x_2 \geq 0$

Let  $x_1 = x_2 = c \rightarrow \infty, Z = 2c$  is unbounded.

Dual: minimize  $y_1$   
subject to  $-y_1 + y_2 \geq 1$   
 $y_1 - y_2 \geq 1$   
 $y_1, y_2 \geq 0$

The dual problem is infeasible.



- g) For school 2, the allowable increase for school capacity is 36. This means the shadow price is only valid for a single additional portable classroom.

For school 3, the allowable increase for school capacity is 42. This means the shadow price is valid for up to two additional portable classrooms.

CHAPTER 11: INTEGER PROGRAMMING

11.1-1.

(a) 
$$x_j = \begin{cases} 1 & \text{if the decision is to build a factory in city } j, \\ 0 & \text{otherwise} \end{cases}$$
$$y_j = \begin{cases} 1 & \text{if the decision is to build a factory in city } j, \\ 0 & \text{otherwise} \end{cases}$$

for  $j = \text{LA, SF, SD}$ .

maximize  $\text{NPV} = 9x_{\text{LA}} + 5x_{\text{SF}} + 7x_{\text{SD}} + 6y_{\text{LA}} + 4y_{\text{SF}} + 5y_{\text{SD}}$   
subject to  $6x_{\text{LA}} + 3x_{\text{SF}} + 4x_{\text{SD}} + 5y_{\text{LA}} + 2y_{\text{SF}} + 3y_{\text{SD}} \leq 10$   
 $y_{\text{LA}} + y_{\text{SF}} + y_{\text{SD}} \leq 1$   
 $-x_{\text{LA}} + y_{\text{LA}} \leq 0$   
 $-x_{\text{SF}} + y_{\text{SF}} \leq 0$   
 $-x_{\text{SD}} + y_{\text{SD}} \leq 0$   
 $x_{\text{LA}}, x_{\text{SF}}, x_{\text{SD}}, y_{\text{LA}}, y_{\text{SF}}, y_{\text{SD}}$  binary

(b) - (c)

Constraint	Yes-or-No Question						Right-Hand Side	
	Warehouse in LA?	Factory in LA?	Warehouse in SD?	Factory in SD?	Warehouse in SF?	Factory in SF?	Totals	Side
Capital (\$millions)	5	6	3	4	2	3	10	≤ 10
≤ 1 Warehouse	1	0	1	0	1	0	1	≤ 1
NPV (\$millions)	6	9	5	7	4	5	42	
Solution	0	0	1	1	0	1		

11.1-2.

(a) 
$$M_j = \begin{cases} 1 & \text{if } j \text{ does marketing,} \\ 0 & \text{otherwise} \end{cases} \quad C_j = \begin{cases} 1 & \text{if } j \text{ does cooking,} \\ 0 & \text{otherwise} \end{cases}$$
$$D_j = \begin{cases} 1 & \text{if } j \text{ does dishwashing,} \\ 0 & \text{otherwise} \end{cases} \quad L_j = \begin{cases} 1 & \text{if } j \text{ does laundry,} \\ 0 & \text{otherwise} \end{cases}$$

for  $j = \text{E (Eve), S (Steven)}$ .

min  $T = 4.5M_E + 7.8C_E + 3.6D_E + 2.9L_E + 4.9M_S + 7.2C_S + 4.3D_S + 3.1L_S$   
st  $M_E + C_E + D_E + L_E = 2$   
 $M_S + C_S + D_S + L_S = 2$   
 $M_E + M_S = 1$   
 $C_E + C_S = 1$   
 $D_E + D_S = 1$   
 $L_E + L_S = 1$   
 $M_E, M_S, C_E, C_S, D_E, D_S, L_E, L_S$  binary



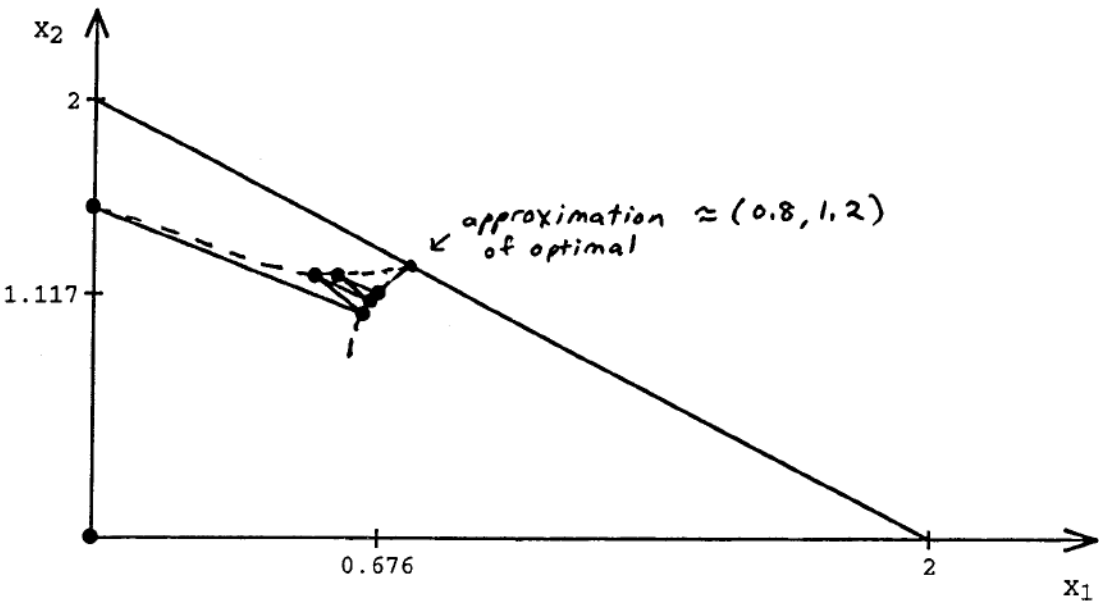
12.9-5.

(a)

k	$\mathbf{x}(k-1)$	$c_1$	$c_2$	$\mathbf{x}_{LP}(k)$	$t^*$	$\mathbf{x}(k)$
1	( 0, 0)	2	3	( 0, 2)	0.75	( 0, 1.5)
2	( 0, 1.5)	2	0	( 2, 0)	0.32	( 0.64, 1.02)
3	( 0.64, 1.02)	0.72	0.96	( 0, 2)	0.175	(0.528,1.192)
4	(0.528,1.192)	0.944	0.617	( 2, 0)	0.092	(0.663,1.082)
5	(0.663,1.082)	0.674	0.835	( 0, 2)	0.126	(0.579,1.198)
6	(0.579,1.198)	0.842	0.603	( 2, 0)	0.068	(0.676,1.117)

Final solution: ( 0.676,1.1166).

(b)



12.9-6.

k	$\mathbf{x}(k-1)$	$c_1$	$c_2$	$\mathbf{x}_{LP}(k)$	$t^*$	$\mathbf{x}(k)$
1	( 0, 0)	32	50	( 3, 2)	0.729	(2.188,1.458)
2	(2.188,1.458)	-9.87	14.81	( 0, 3.2)	0.131	(1.902,1.686)
3	(1.902,1.686)	4.499	5.634	( 3, 2)	0.111	(2.024,1.721)
4	(2.024,1.721)	-1.15	4.078	( 0, 3.2)	0.028	(1.966,1.763)

Final solution: (1.9662,1.7629).

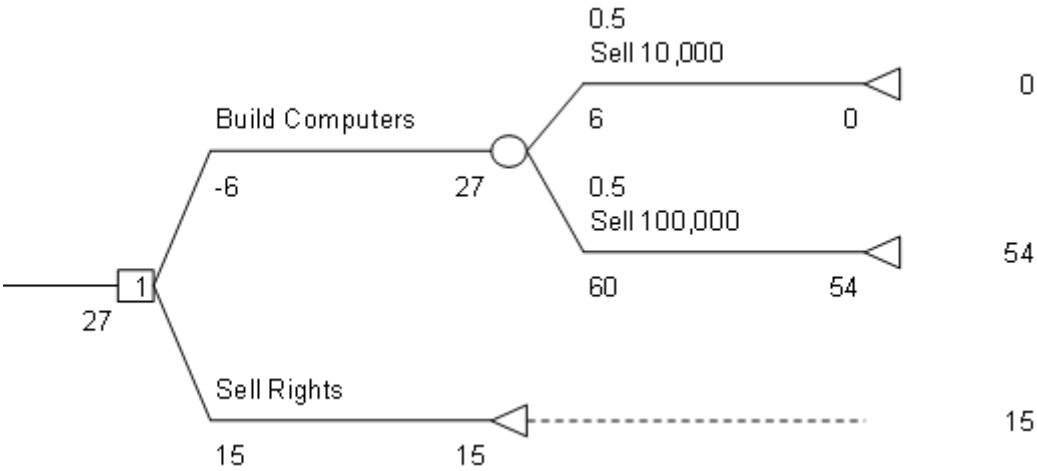
12.9-7.

k	$\mathbf{x}^{(k-1)}$	$c_1$	$c_2$	$\mathbf{x}_{LP}^{(k)}$	$t^*$	$\mathbf{x}^{(k)}$
1	( 0, 0)	40	30	( 3, 0)	0.616	(1.847, 0)
2	(1.847, 0)	0.001	35.54	( 0, 2)	0.406	(1.097,0.812)

15.5-1.

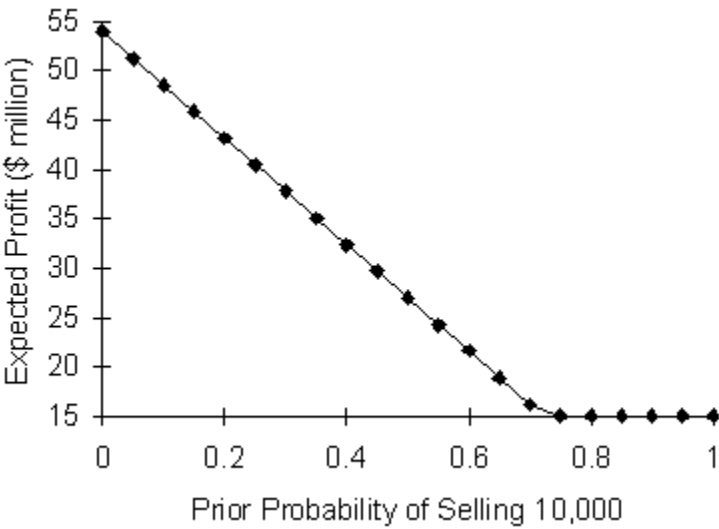
(a)

Alternative	State of Nature	
	Sell 10,000	Sell 100,000
Build Computers	0	54
Sell Rights	15	15



They should build computers with an expected payoff of \$27 million.

(b)



(b) Optimal Order Quantity

Basic EOQ Model (Solver Version)				
Data			Results	
d =	600	(demand/year)	Reorder Point	0
K =	\$75	(setup cost)		
h =	\$48.00	(unit holding cost)	Annual Setup Cost	\$1,039.23
L =	0	(lead time in days)	Annual Holding Cost	\$1,039.23
WD =	365	(working days/year)	Total Variable Cost	\$2,078.46
Decision				
Q =	43.30127			

Current Order Quantity

Basic EOQ Model (Solver Version)				
Data			Results	
d =	600	(demand/year)	Reorder Point	0
K =	\$75	(setup cost)		
h =	\$48.00	(unit holding cost)	Annual Setup Cost	\$900.00
L =	0	(lead time in days)	Annual Holding Cost	\$1,200.00
WD =	365	(working days/year)	Total Variable Cost	\$2,100.00
Decision				
Q =	50			

(c)

Basic EOQ Model (Solver Version)				
Data			Results	
d =	600	(demand/year)	Reorder Point	10
K =	\$75	(setup cost)		
h =	\$48.00	(unit holding cost)	Annual Setup Cost	\$1,039.23
L =	5	(lead time in days)	Annual Holding Cost	\$1,039.23
WD =	300	(working days/year)	Total Variable Cost	\$2,078.46
Decision				
Q =	43.30127			

(d)  $ROP = 5 + (50)(5/25) = 15$  hammers, which adds  $5 \times \$4 = \$20$  to TVC every month, \$240 per year.

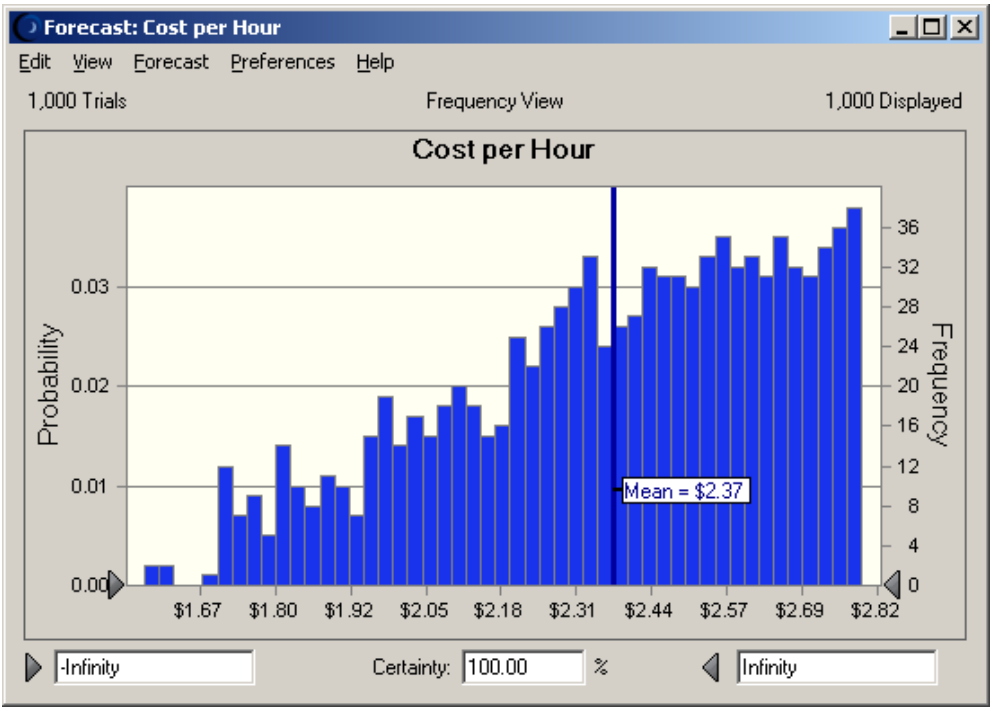
18.3-7.

$K = 12,000, h = 0.30, d = 8,000, p = 5$

$Q^* = \sqrt{\frac{2(8000)(12000)}{0.30}} \sqrt{\frac{5+0.30}{5}} = 26,046$

$S^* = \sqrt{\frac{2(8000)(12000)}{0.30}} \sqrt{\frac{5}{5+0.30}} = 24,572$

$t^* = Q^*/d = 3.26$  months



20.6-4.

The chance of negative clearance is approximately 18.4%.

	A	B	C	D	E	F
1	Shaft Radius	1.001	Triangular(min,likely,max)	1.000	1.001	1.002
2	Bushing Radius	1.002	Normal(mean,st.dev.)	1.002	0.001	
3						
4	Clearance	0.0010				

