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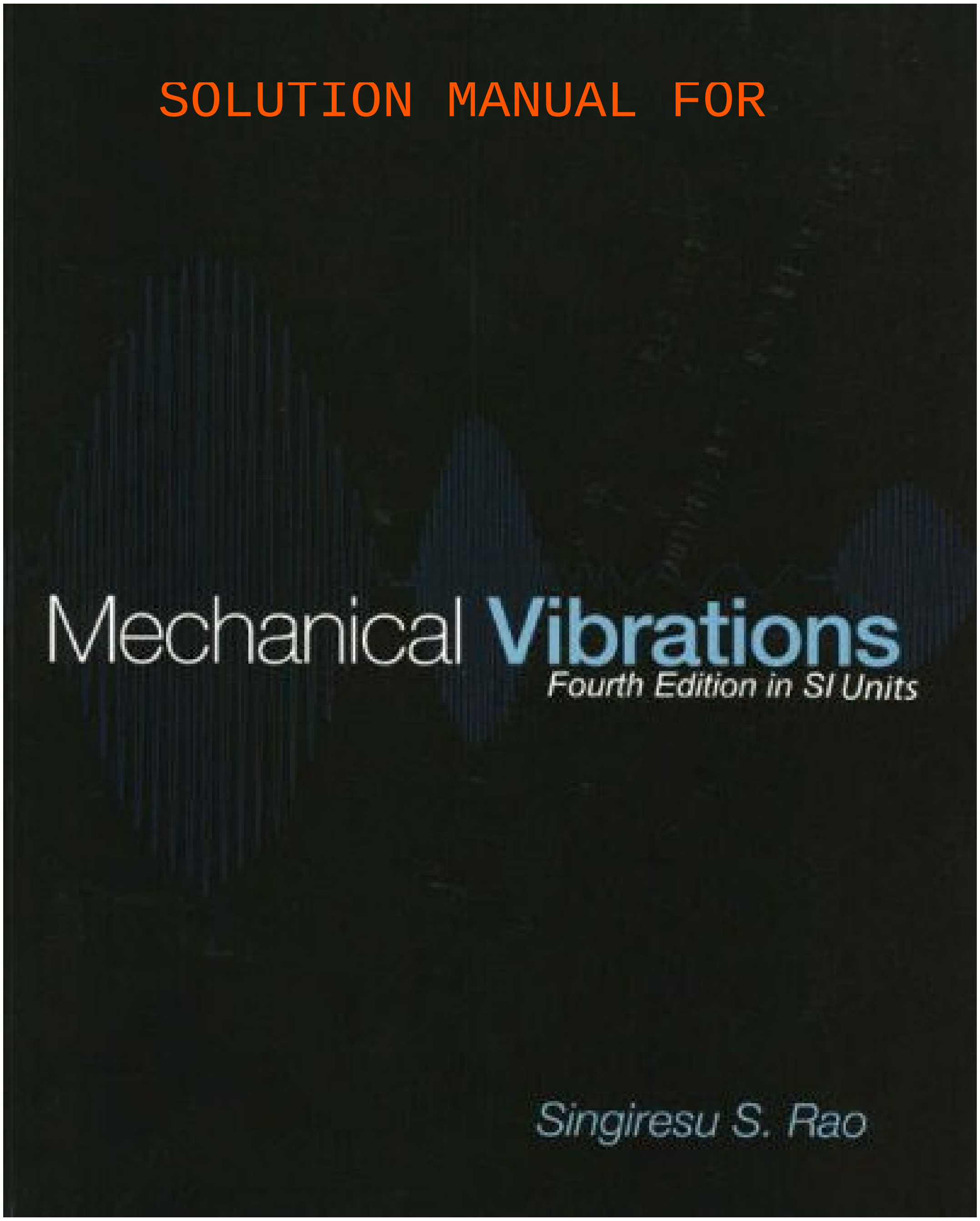

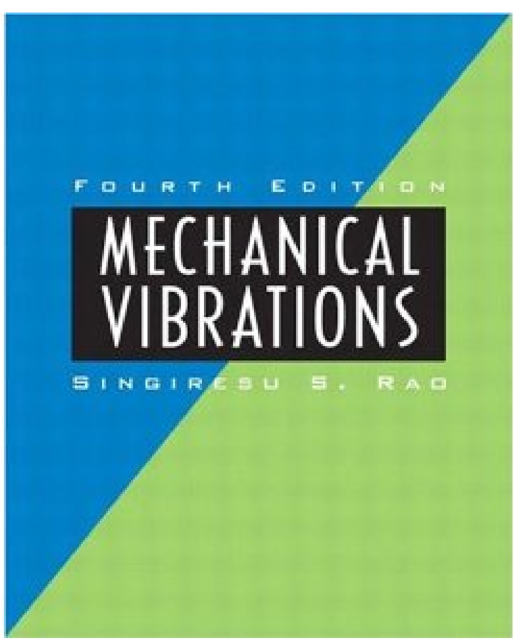

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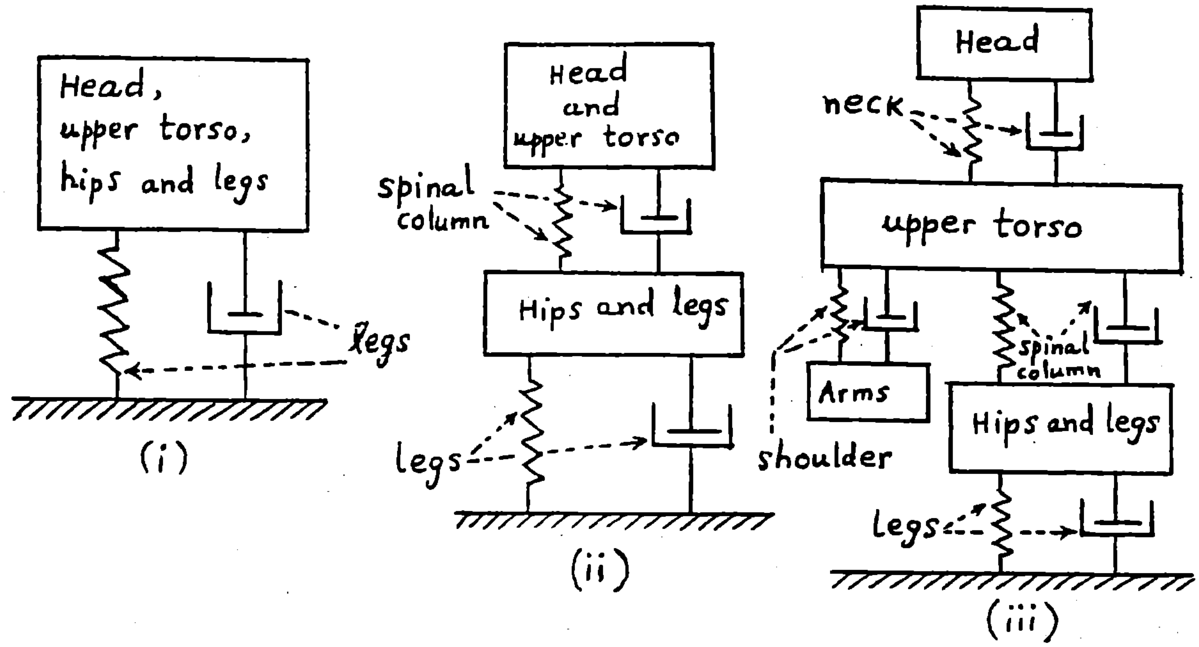
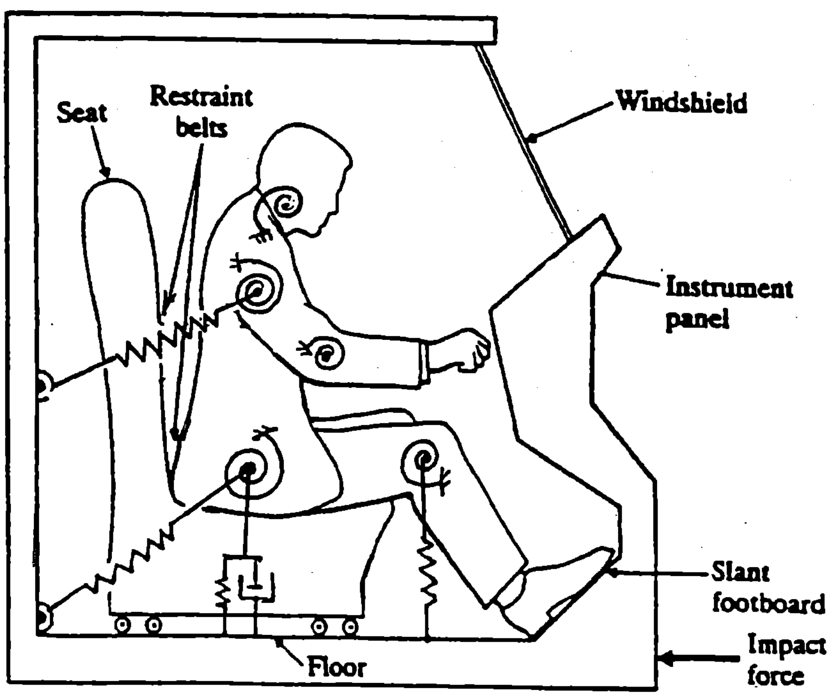
Chapter 1

Fundamentals of Vibration

1.1



1.2



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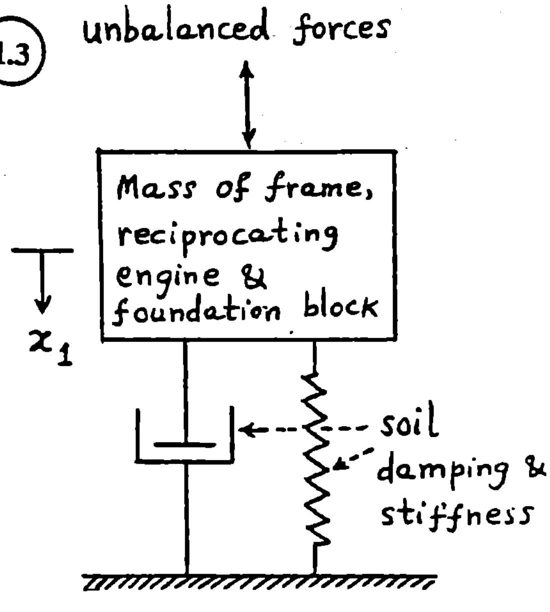
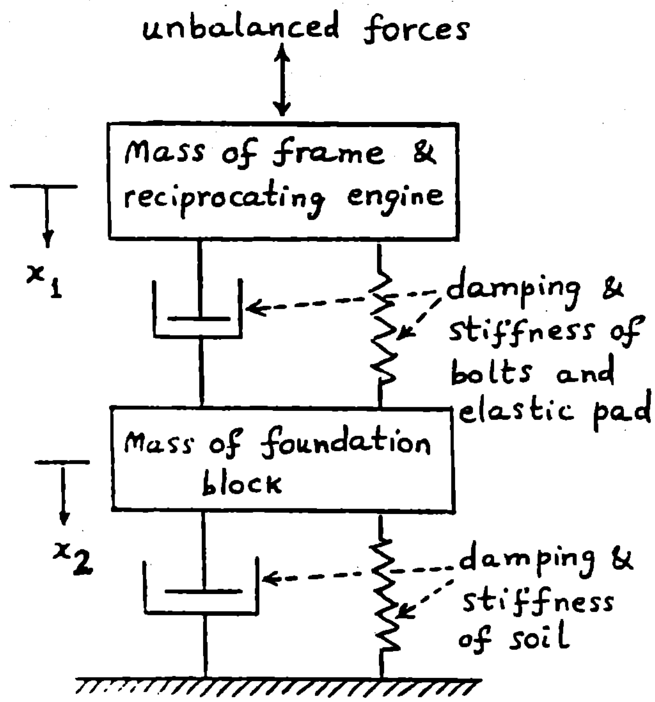
1.3

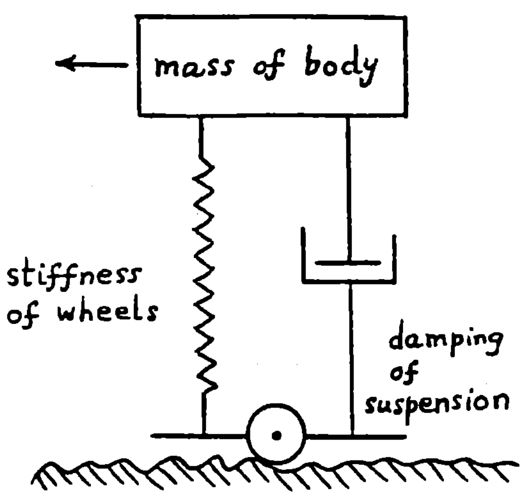


(a) one degree of freedom model

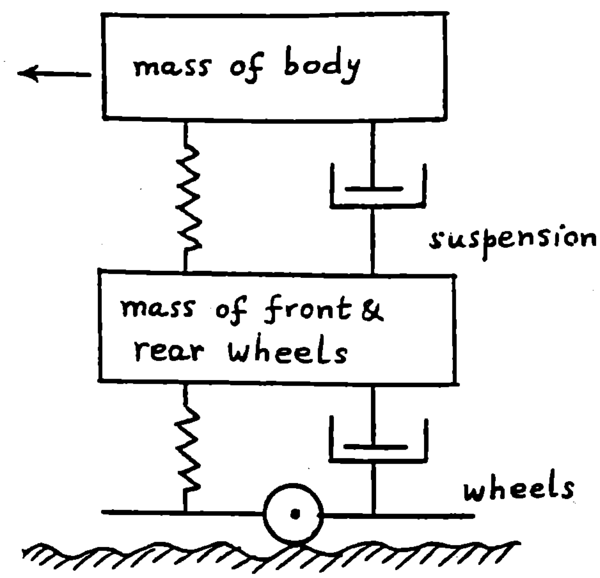


(b) Two degree of freedom model

1.4



(i)



(ii)

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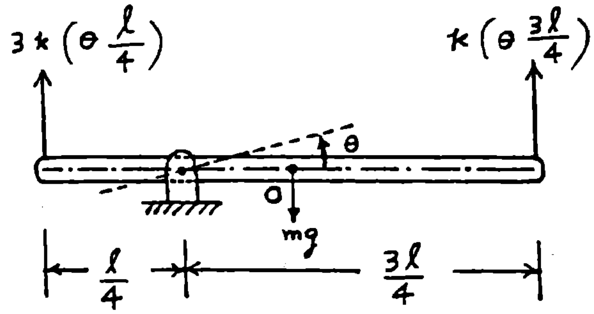
i.e., $b = \pm \dfrac{a}{\sqrt{2}}$

$$\omega_n \Big|_{b=+a/\sqrt{2}} = \sqrt{\frac{2g\,\frac{a}{\sqrt{2}}}{a^2 + 2(a^2/2)}} = \sqrt{\frac{g}{\sqrt{2}\,a}}$$

$b = -a/\sqrt{2}$ gives imaginary value for ω_n.

Since $\omega_n = 0$ when $b = 0$, we have $\omega_n\big|_{max}$ at $b = \dfrac{a}{\sqrt{2}}$.

2.73



Let θ be measured from static equilibrium position so that gravity force need not be considered.

(a) Newton's second law of motion:

$$J_0\,\ddot{\theta} = -3\,k\,(\theta\,\frac{\ell}{4})\,\frac{\ell}{4} - k\,(\theta\,\frac{3\,\ell}{4})\,(\frac{3\,\ell}{4}) \quad \text{or} \quad J_0\,\ddot{\theta} + \frac{3}{4}\,k\,\ell^2\,\theta = 0$$

(b) D'Alembert's principle:

$$M(t) - J_0\,\ddot{\theta} = 0 \quad \text{or} \quad -3\,k\,(\theta\,\frac{\ell}{4})\,(\frac{\ell}{4}) - k\,(\theta\,\frac{3\,\ell}{4})\,(\frac{3\,\ell}{4}) - J_0\,\ddot{\theta} = 0$$

$$\text{or} \quad J_0\,\ddot{\theta} + \frac{3}{4}\,k\,\ell^2\,\theta = 0$$

(c) Principle of virtual work:

Virtual work done by spring force:

$$\delta W_s = -3\,k\,(\theta\,\frac{\ell}{4})\,(\frac{\ell}{4}\,\delta\theta) - k\,(\theta\,\frac{3\,\ell}{4})\,(\frac{3\,\ell}{4}\,\delta\theta)$$

Virtual work done by inertia moment = - $(J_0\,\ddot{\theta})\,\delta\theta$

Setting total virtual work done by all forces/moments equal to zero, we obtain

$$J_0\,\ddot{\theta} + \frac{3}{4}\,k\,\ell^2\,\theta = 0$$

2.74 Let m_{eff} = effective part of mass of beam (m) at middle. Thus vibratory inertia force at middle is due to $(M + m_{eff})$. Assume a deflection shape: $y(x,t) = Y(x)\cos(\omega_n t - \phi)$ where $Y(x)$ = static deflection shape due to load at middle given by:

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3.7 $\delta_{st} = \frac{F_0}{k} = \frac{25}{2000} = 0.0125$ m

steady state solution at resonance $= x(t) = \frac{\delta_{st} \cdot \omega_n t}{2} \sin \omega_n t$

$= 0.00625\ \omega_n t\ \sin \omega_n t$ m

(a) At end of $\frac{1}{4}$ cycle, $\omega_n t = \frac{\pi}{2}$ and $x(t) = 0.00625\left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} = 0.009817$ m

(b) At end of $2\frac{1}{2}$ cycles, $\omega_n t = 5\pi$ and $x(t) = 0.00625(5\pi) \sin 5\pi = 0$

(c) At end of $5\frac{3}{4}$ cycles, $\omega_n t = 11\frac{1}{2}\pi$ and

$$x(t) = 0.00625\left(\frac{23}{2}\pi\right) \sin \frac{23}{2}\pi = -0.2258 \text{ m}$$

3.8 $\delta_{st} = \frac{F_0}{k} = \frac{100}{4000} = 0.025$ m

$X = \delta_{st} \cdot \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$, $\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right| = \frac{\delta_{st}}{X} = \frac{0.025}{20 \times 10^{-3}} = 1.25$

$\frac{\omega}{\omega_n} = \sqrt{1.25 + 1} = 1.5$

$\omega_n = \omega/1.5 = 5(2\pi)/1.5 = 20.944$ rad/sec

$m = k/\omega_n^2 = 4000/(20.944)^2 = 9.1189$ kg

3.9 $\omega_n = \sqrt{k/m} = \sqrt{5000/10} = 22.3607$ rad/sec

$\delta_{st} = F_0/k = 250/5000 = 0.05$ m

$$X = \delta_{st} \left\{ \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right\}$$

i.e., $\omega = \omega_n \left(1 - \frac{\delta_{st}}{X}\right)^{\frac{1}{2}} = 22.3607\left[1 - \frac{0.05}{0.10}\right]^{\frac{1}{2}}$

$= 15.8114$ rad/sec

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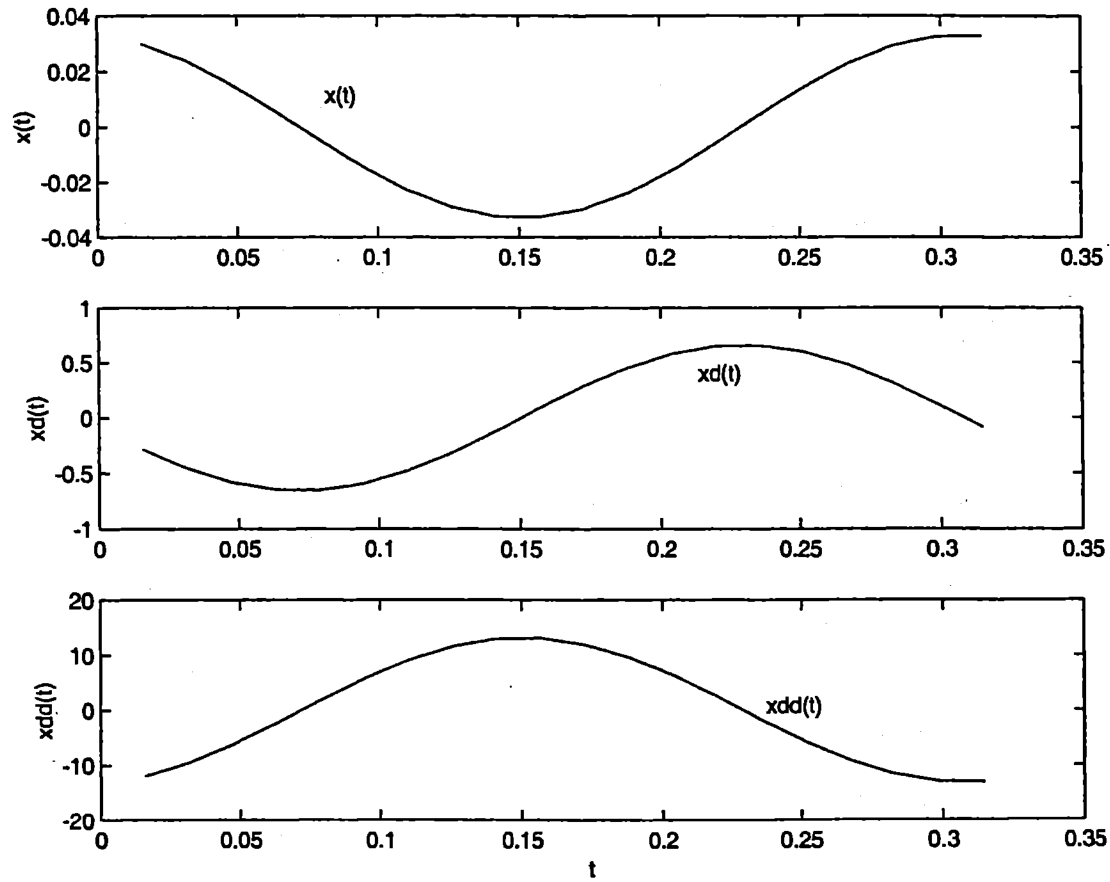
7	-2.27835571e-002	-4.78586496e-001	9.11342282e+000
8	-2.90630300e-002	-3.14352252e-001	1.16252120e+001
9	-3.24975978e-002	-1.19346877e-001	1.29990391e+001
10	-3.27510596e-002	8.73410566e-002	1.31004238e+001
11	-2.97986046e-002	2.85479399e-001	1.19194419e+001
12	-2.39292411e-002	4.55672899e-001	9.57169644e+000
13	-1.57175058e-002	5.81261754e-001	6.28700233e+000
14	-5.96722446e-003	6.49952395e-001	2.38688978e+000
15	4.36717313e-003	6.55020872e-001	-1.74686925e+000
16	1.42740794e-002	5.95971044e-001	-5.70963176e+000
17	2.27837329e-002	4.78583148e-001	-9.11349314e+000
18	2.90631454e-002	3.14347982e-001	-1.16252582e+001
19	3.24976417e-002	1.19342102e-001	-1.29990567e+001
20	3.27510275e-002	-8.73458687e-002	-1.31004110e+001



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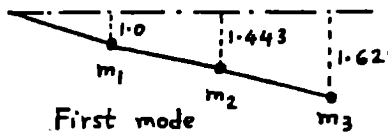
$$\vec{X}^{(j)} = \begin{Bmatrix} X_1^{(j)} \\ X_2^{(j)} \\ X_3^{(j)} \end{Bmatrix} = X_1^{(j)} \begin{Bmatrix} 1 \\ (-\omega_j^2 m + 3k)/(2k) \\ 3(-\omega_j^2 m + 3k)/[2(-3\omega_j^2 m + 3k)] \end{Bmatrix} \quad \text{--- } (E_5)$$

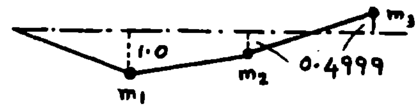
Thus

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1.0 \\ 1.443004 \\ 1.628659 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.49999 \\ -0.49998 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1.0 \\ -0.693 \\ 0.204666 \end{Bmatrix}$$

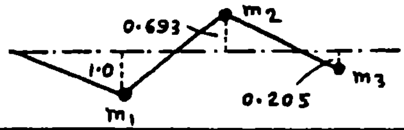
Mode shapes:



First mode



Second mode



Third mode

6.47 When $k_1 = 3k$, $k_2 = k_3 = k$, $m_1 = 3m$ and $m_2 = m_3 = m$, Eq. (E_2) of problem 6.46 gives the frequency equation

$$\begin{vmatrix} -3m\omega^2 + 4k & -k & 0 \\ -k & -m\omega^2 + 2k & -k \\ 0 & -k & -m\omega^2 + k \end{vmatrix} = 0$$

i.e. $(-3m\omega^2 + 4k)\left[(-m\omega^2 + 2k)(-m\omega^2 + k) - k^2\right] + k\left[-k(-m\omega^2 + k)\right] = 0$

i.e. $$3\alpha^3 - 13\alpha^2 + 14\alpha - 3 = 0 \quad \text{--- } (E_1)$$

where $\alpha = m\omega^2/k$. Roots of (E_1) are

$\alpha_1 = 0.284515$, $\quad \alpha_2 = 1.26053$, $\quad \alpha_3 = 2.78829$

$\omega_1 = 0.533399\sqrt{\frac{k}{m}}$, $\quad \omega_2 = 1.122733\sqrt{\frac{k}{m}}$, $\quad \omega_3 = 1.669817\sqrt{\frac{k}{m}}$

Eq. (E_5) of problem 6.46 gives the jth mode shape as

$$\vec{X}^{(j)} = X_1^{(j)} \begin{Bmatrix} 1.0 \\ (-3m\omega_j^2 + 4k)/k \\ (-3m\omega_j^2 + 4k)/(-m\omega_j^2 + k) \end{Bmatrix}$$

Thus

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1.0 \\ 3.146455 \\ 4.397653 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.21841 \\ -0.83833 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1.0 \\ -4.36487 \\ 2.44081 \end{Bmatrix}$$

<u>Orthogonality of normal modes:</u>

$$[m] = m \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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```
C ==================================================================
C
C SUBROUTINE DECOMP
C
C ==================================================================
      SUBROUTINE DECOMP (A,U,N)
      DIMENSION A(N,N),U(N,N)
      DO 10 I=1,N
      DO 10 J=1,N
10    U(I,J)=0.0
      U(1,1)=SQRT(A(1,1))
      DO 90 J=2,N
90    U(1,J)=A(1,J)/U(1,1)
      DO 40 I=2,N
      IM=I-1
      SUM=0.0
      DO 30 K=1,IM
30    SUM=SUM+U(K,I)**2
      U(I,I)=SQRT(A(I,I)-SUM)
      J=I+1
      SUM=0.0
      DO 50 K=1,IM
50    SUM=SUM+U(K,I)*U(K,J)
      U(I,J)=(A(I,J)-SUM)/U(I,I)
40    CONTINUE
      RETURN
      END
```

```
 UPPER TRIANGULAR MATRIX [U]:

   0.316228E+01  -0.126491E+01   0.000000E+00
   0.000000E+00   0.209762E+01  -0.953463E+00
   0.000000E+00   0.000000E+00   0.104447E+01

 INVERSE OF THE UPPER TRIANGULAR MATRIX, [UI],

   0.316228E+00   0.190693E+00   0.174078E+00
   0.000000E+00   0.476731E+00   0.435194E+00
   0.000000E+00   0.000000E+00   0.957427E+00

 MATRIX [UMU] = [UTI][M][UI]:

   0.300000E+00   0.180907E+00   0.165145E+00
   0.180907E+00   0.563636E+00   0.514527E+00
   0.165145E+00   0.514527E+00   0.138636E+01

 EIGENVALUES:

   0.167157E+01   0.383372E+00   0.195059E+00

 EIGENVECTORS (COLUMNWISE):

   0.289231E+00  -0.237705E+00   0.162815E+00
   0.593305E+00  -0.129231E+00  -0.218987E+00
   0.846513E+00   0.424805E+00   0.140081E+00
```

7.44

```
C ==================================================================
C
C PROGRAM 9.F
C MAIN PROGRAM FOR CALLING JACOBI
C
C ==================================================================
C FOLLOWING 3 LINES CONTAIN PROBLEM-DEPENDENT DATA
      DIMENSION D(3,3),E(3,3)
      DATA N,ITMAX,EPS/3,200,1.0E-05/
      DATA D/5.,-1.,1.,-1.,6.,-4.,1.,-4.,3./
C END OF PROBLEM-DEPENDENT DATA
      PRINT 50
50    FORMAT (//,37H EIGENVALUE SOLUTION BY JACOBI METHOD)
      PRINT 40
```

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9.39 Mass of engine = m = 500 kg
Force transmitted with out isolator = F_T = (18000 cos 300 t + 3600 cos 600 t) N
Maximum magnitude of force transmitted:

$$F_{01} = 18000 \text{ N at } \omega = 300 \text{ rad/sec}$$
$$F_{02} = 3600 \text{ N at } \omega = 600 \text{ rad/sec}$$

The maximum possible force transmitted will be the sum of the magnitudes of the two harmonics:

$$F_0 = F_{01} + F_{02} = 18000 + 3600 = 21600 \text{ N}$$

Since F_T = 12000 N, we use the relation

$$\frac{F_T}{F_0} = \frac{12000}{21600} = \left| \frac{1}{1-r^2} \right| \quad \text{or} \quad 0.5556 = \frac{1}{r^2-1} \quad \text{or} \quad r = 1.6733$$

At ω = 300 rad/sec:

$$\omega_n = \frac{\omega}{r} = \frac{300}{1.6733} = 179.2843 \text{ rad/sec} = \sqrt{\frac{k}{500}}$$

$$\text{or} \quad k = m\,(\omega_n^2) = (500)\,(179.2843^2) = 16.0714\,(10^6) \text{ N/m}$$

With this value of k, the value of $\frac{F_{T1}}{F_{01}}$ at 300 rad/sec is:

$$\frac{F_{T1}}{F_{01}} = \left| \frac{1}{1-r_1^2} \right| = \left| \frac{1}{1 - \left(\frac{300}{179.2843} \right)^2} \right| = 0.5556$$

$$\text{or} \quad F_{T1} = 0.5556\,(F_{01}) = 0.5556\,(18000) = 10000 \text{ N}$$

The value of $\frac{F_{T2}}{F_{02}}$ at ω = 600 rad/sec is:

$$\frac{F_{T2}}{F_{02}} = \left| \frac{1}{1-r_2^2} \right| = \left| \frac{1}{1 - \left(\frac{600}{179.2843} \right)^2} \right| = 0.0980$$

$$\text{or} \quad F_{T2} = 0.0980\,(F_{02}) = 0.0980\,(3600) = 352.8 \text{ N}$$

Since $F_{T1} + F_{T2}$ = 10000 + 352.8 = 10352.8 N < 12000 N (permitted value), the stiffness of the isolator can be taken as k = 16.0714 (10^6) N/m.

At ω = 600 rad/sec:

$$\omega_n = \frac{\omega}{r} = \frac{600}{1.6733} = 358.5729 \text{ rad/sec}$$

$$k = m\,\omega_n^2 = (500)\,(358.5729^2) = 64.2872\,(10^6) \text{ N/m} \tag{1}$$

With this value of k, the value of $\frac{F_{T1}}{F_{01}}$ at ω = 300 rad/sec is:

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```
      LA(IB)=LA(IB)+1
      IF (IA .EQ. IB) GO TO 190
      DO 160 I=1,N
      Z=A(IA,I)
      A(IA,I)=A(IB,I)
160   A(IB,I)=Z
      IF (IND .EQ. 0) GO TO 190
      Z=B(IA)
      B(IA)=B(IB)
      B(IB)=Z
190   LB(K,1)=IA
      LB(K,2)=IB
      S(K)=A(IB,IB)
      A(IB,IB)=1.0
      DO 200 I=1,N
200   A(IB,I)=A(IB,I)/S(K)
      IF (IND .EQ. 0) GO TO 220
      B(IB)=B(IB)/S(K)
220   DO 250 I=1,N
      IF(I .EQ. IB) GO TO 250
      Z=A(I,IB)
      A(I,IB)=0.0
      DO 230 J=1,N
230   A(I,J)=A(I,J)-A(IB,J)*Z
      IF (IND .EQ. 0) GO TO 250
      B(I)=B(I)-B(IB)*Z
250   CONTINUE
      DO 270 I=1,N
      J=N-I+1
      IF (LB(J,1) .EQ. LB(J,2)) GO TO 270
      IA=LB(J,1)
      IB=LB(J,2)
      DO 260 K=1,N
      Z=A(K,IA)
      A(K,IA)=A(K,IB)
      A(K,IB)=Z
260   CONTINUE
270   CONTINUE
300   RETURN
      END
```

SOLUTION BY CENTRAL DIFFERENCE METHOD

GIVEN DATA:
N= 2 NSTEP= 24 DELT= 0.24216267E+00

SOLUTION:

STEP	TIME	X(I,1)	XD(I,1)	XDD(I,1)	X(I,2)	XD(I,2)	XDD(I,2)
1	0.0000	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	0.2422	0.3867E+00	0.7984E+00	0.6594E+01	0.0000E+00	0.0000E+00	0.0000E+00
3	0.4843	0.3609E+00	0.7451E+00	-.7034E+01	0.2268E-01	0.4682E-01	0.3867E+00
4	0.7265	-.3707E+00	-.1564E+01	-.1203E+02	0.6119E-01	0.1263E+00	0.2702E+00
5	0.9687	-.1098E+01	-.3012E+01	0.7527E-01	0.6362E-01	0.8454E-01	-.6155E+00
6	1.2108	-.9436E+00	-.1183E+01	0.1503E+02	-.1326E-01	-.1537E+00	-.1352E+01
7	1.4530	0.1774E-01	0.2303E+01	0.1376E+02	-.1424E+00	-.4253E+00	-.8906E+00
8	1.6951	0.8044E+00	0.3609E+01	-.2979E+01	-.2370E+00	-.4620E+00	0.5872E+00
9	1.9373	0.6966E+00	0.1402E+01	-.1525E+02	-.2289E+00	-.1787E+00	0.1753E+01
10	2.1795	0.5749E-01	-.1542E+01	-.9061E+01	-.1263E+00	0.2287E+00	0.1612E+01
11	2.4216	-.2159E+00	-.1884E+01	0.6238E+01	0.9378E-02	0.4920E+00	0.5626E+00
12	2.6638	0.1297E+00	0.1490E+00	0.1055E+02	0.1302E+00	0.5295E+00	-.2534E+00
13	2.9060	0.4257E+00	0.1325E+01	-.8437E+00	0.2280E+00	0.4514E+00	-.3910E+00
14	3.1481	0.4341E-01	-.1781E+00	-.1157E+02	0.2974E+00	0.3452E+00	-.4864E+00
15	3.3903	-.6914E+00	-.2306E+01	-.6011E+01	0.2995E+00	0.1475E+00	-.1146E+01
16	3.6324	-.8546E+00	-.1854E+01	0.9746E+01	0.1908E+00	-.2200E+00	-.1889E+01
17	3.8746	-.1162E+00	0.1188E+01	0.1538E+02	-.1272E-01	-.6446E+00	-.1618E+01
18	4.1168	0.7762E+00	0.3367E+01	0.2624E+01	-.2201E+00	-.8484E+00	-.6526E-01
19	4.3589	0.8716E+00	0.2040E+01	-.1359E+02	-.3303E+00	-.6557E+00	0.1657E+01
20	4.6011	0.1564E+00	-.1280E+01	-.1383E+02	-.3119E+00	-.1897E+00	0.2193E+01

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Eq. (E_{15}) can be solved to obtain

$$A = -\frac{B}{2} \pm \frac{1}{2}\left\{\frac{16}{3k_2}(\omega^2 - k_1) - 7B^2 - 8C^2\right\}^{\frac{1}{2}} \qquad (E_{17})$$

This equation gives the condition for the existence of subharmonic of order 3 (nonzero A) as

$$\frac{16}{3k_2}(\omega^2 - k_1) - 7B^2 - 8C^2 > 0$$

i.e.,
$$\omega^2 > k_1 + \frac{3k_2}{16}(7B^2 + 8C^2) \qquad (E_{18})$$

13.15 Equation: $\ddot{x} + c\dot{x} + k_1 x + k_2 x^2 = a \cos 2\omega t \qquad (E_1)$

As in the solution of problem 13.14, we introduce

$$\theta = \omega t, \quad \dot{x} = \omega x' \text{ and } \ddot{x} = \omega^2 x'' \text{ with } x' = \frac{dx}{d\theta}.$$

Eq. (E_1) becomes

$$\omega^2 x'' + c\omega x' + k_1 x + k_2 x^2 = a \cos 2\theta \qquad (E_2)$$

The solution of (E_2) must contain terms involving $\cos\theta$ and/or $\sin\theta$ in order to include subharmonics of order 2. If the nonlinear term $k_2 x^2$ is assumed to be small with $k_2 > 0$, we can express c and a as $c = \epsilon k_2^2$ and $a = a_0 k_2$. Thus (E_2) becomes

$$\omega^2 x'' + \epsilon\omega k_2^2 x' + k_1 x + k_2 x^2 = k_2 a_0 \cos 2\theta \qquad (E_3)$$

Using perturbation method, we assume

$$x(t) = x_0(t) + k_2 x_1(t) + k_2^2 x_2(t) + \cdots \qquad (E_4)$$

$$\omega = \omega_0 + k_2\omega_1 + k_2^2\omega_2 + \cdots \qquad (E_5)$$

Substituting (E_4) and (E_5) into (E_3), we get

$$(\omega_0 + k_2\omega_1 + k_2^2\omega_2)^2(x_0'' + k_2 x_1'' + k_2^2 x_2'') + \epsilon k_2^2(\omega_0 + k_2\omega_1 + k_2^2\omega_2)(x_0' + k_2 x_1' + k_2^2 x_2') + k_1(x_0 + k_2 x_1 + k_2^2 x_2) + k_2(x_0 + k_2 x_1 + k_2^2 x_2)^2 = k_2 a_0 \cos 2\theta$$

i.e.,

$$k_2^0(x_0''\omega_0^2 + k_1 x_0) + k_2^1(x_1''\omega_0^2 + 2\omega_0\omega_1 x_0'' + k_1 x_1 + x_0^2 - a_0\cos 2\theta) + k_2^2(x_2''\omega_0^2 + x_0''\omega_1^2 + 2\omega_0\omega_1 x_1'' + 2\omega_0\omega_2 x_0'' + \epsilon\omega_0 x_0' + k_1 x_2 + 2x_0 x_1) + k_2^3(\cdots) + \cdots = 0 \qquad (E_6)$$

By setting the coefficients of various powers of k_2 equal to zero in (E_6), we get the following.

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