Chapter 1

The Schrödinger Equation

1.1 **(a)** F; **(b)** T; **(c)** T.

1.2 **(a)** $E_{\text{photon}} = h\nu = hc/\lambda =$(6.626 × 10^{-34} J s)(2.998 × 10^8 m/s)/(1064 × 10^{-9} m) = 1.867 × 10^{-19} J.

(b) E = (5 × 10^6 J/s)(2 × 10^{-8} s) = 0.1 J = n(1.867 × 10^{-19} J) and n = 5 × 10^{17}.

1.3 Use of $E_{\text{photon}} = hc/\lambda$ gives

$$E = \frac{(6.022\times10^{23})(6.626\times10^{-34}\text{ J s})(2.998\times10^{8}\text{ m/s})}{300\times10^{-9}\text{ m}} = 399\text{ kJ}$$

1.4 **(a)** $T_{\max} = h\nu - \Phi =$
(6.626 × 10^{-34} J s)(2.998 × 10^8 m/s)/(200 × 10^{-9} m) – (2.75 eV)(1.602 × 10^{-19} J/eV) = 5.53 × 10^{-19} J = 3.45 eV.

(b) The minimum photon energy needed to produce the photoelectric effect is (2.75 eV)(1.602 × 10^{-19} J/eV) = $h\nu = hc/\lambda$ = (6.626 × 10^{-34} J s)(2.998 × 10^8 m/s)/λ and λ = 4.51 × 10^{-7} m = 451 nm.

(c) Since the impure metal has a smaller work function, there will be more energy left over after the electron escapes and the maximum T is larger for impure Na.

1.5 **(a)** At high frequencies, we have $e^{b\nu/T} >> 1$ and the -1 in the denominator of Planck's formula can be neglected to give Wien's formula.

(b) The Taylor series for the exponential function is $e^x = 1 + x + x^2/2! + \cdots$. For $x << 1$, we can neglect x^2 and higher powers to give $e^x - 1 \approx x$. Taking $x \equiv h\nu/kT$, we have for Planck's formula at low frequencies

$$\frac{a\nu^3}{e^{b\nu/T}-1} = \frac{2\pi h\nu^3}{c^2(e^{h\nu/kT}-1)} \approx \frac{2\pi h\nu^3}{c^2(h\nu/kT)} = \frac{2\pi\nu^2 kT}{c^2}$$

1.6 $\lambda = h/m\text{v} = 137h/mc =$ 137(6.626 × 10^{-34} J s)/(9.109 × 10^{-31} kg)(2.998 × 10^8 m/s) = 3.32 × 10^{-10} m = 0.332 nm.

1.7 Integration gives $x = -\frac{1}{2}gt^2 + (gt_0 + \mathsf{v}_0)t + c_2$. If we know that the particle had position x_0 at time t_0, then $x_0 = -\frac{1}{2}gt_0^2 + (gt_0 + \mathsf{v}_0)t_0 + c_2$ and $c_2 = x_0 - \frac{1}{2}gt_0^2 - \mathsf{v}_0 t_0$. Substitution of the expression for c_2 into the equation for x gives $x = x_0 - \frac{1}{2}g(t-t_0)^2 + \mathsf{v}_0(t-t_0)$.

1.8 $-(\hbar/i)(\partial\Psi/\partial t) = -(\hbar^2/2m)(\partial^2\Psi/\partial x^2) + V\Psi$. For $\Psi = ae^{-ibt}e^{-bmx^2/\hbar}$, we find $\partial\Psi/\partial t = -ib\Psi$, $\partial\Psi/\partial x = -2bm\hbar^{-1}x\Psi$, and $\partial^2\Psi/\partial x^2 = -2bm\hbar^{-1}\Psi - 2bm\hbar^{-1}x(\partial\Psi/\partial x)$ $= -2bm\hbar^{-1}\Psi - 2bm\hbar^{-1}x(-2bm\hbar^{-1}x\Psi) = -2bm\hbar^{-1}\Psi + 4b^2m^2\hbar^{-2}x^2\Psi$. Substituting into the time-dependent Schrödinger equation and then dividing by Ψ, we get $-(\hbar/i)(-ib\Psi) = -(\hbar^2/2m)(-2bm\hbar^{-1} + 4b^2m^2\hbar^{-2}x^2)\Psi + V\Psi$ and $V = 2b^2mx^2$.

1.9 **(a)** F; **(b)** F. (These statements are valid only for stationary states.)

1.10 ψ satisfies the time-independent Schrödinger (1.19). $\partial\psi/\partial x = be^{-cx^2} - 2bcx^2e^{-cx^2}$; $\partial^2\psi/\partial x^2 = -2bcxe^{-cx^2} - 4bcxe^{-cx^2} + 4bc^2x^3e^{-cx^2} = -6bcxe^{-cx^2} + 4bc^2x^3e^{-cx^2}$. Equation (1.19) becomes $(-\hbar^2/2m)(-6bcxe^{-cx^2} + 4bc^2x^3e^{-cx^2}) + (2c^2\hbar^2x^2/m)bxe^{-cx^2} = Ebxe^{-cx^2}$. The x^3 terms cancel and $E = 3\hbar^2c/m =$
$3(6.626 \times 10^{-34}\text{ J s})^2 2.00(10^{-9}\text{ m})^{-2}/4\pi^2(1.00 \times 10^{-30}\text{ kg}) = 6.67 \times 10^{-20}\text{ J}$.

1.11 Only the time-dependent equation.

1.12 **(a)** $|\Psi|^2\,dx = (2/b^3)x^2e^{-2|x|/b}\,dx =$
$2(3.0 \times 10^{-9}\text{ m})^{-3}(0.90 \times 10^{-9}\text{ m})^2e^{-2(0.90\text{ nm})/(3.0\text{ nm})}(0.0001 \times 10^{-9}\text{ m}) = 3.29 \times 10^{-6}$.

(b) For $x \ge 0$, we have $|x| = x$ and the probability is given by (1.23) and (A.7) as $\int_0^{2\text{ nm}} |\Psi|^2\,dx = (2/b^3)\int_0^{2\text{ nm}} x^2e^{-2x/b}\,dx = (2/b^3)e^{-2x/b}(-bx^2/2 - xb^2/2 - b^3/4)\Big|_0^{2\text{ nm}} =$
$-e^{-2x/b}(x^2/b^2 + x/b + 1/2)\Big|_0^{2\text{ nm}} = -e^{-4/3}(4/9 + 2/3 + 1/2) + 1/2 = 0.0753$.

(c) Ψ is zero at $x = 0$, and this is the minimum possible probability density.

(d) $\int_{-\infty}^{\infty}|\Psi|^2\,dx = (2/b^3)\int_{-\infty}^{0} x^2e^{2x/b}\,dx + (2/b^3)\int_0^{\infty} x^2e^{-2x/b}\,dx$. Let $w = -x$ in the first integral on the right. This integral becomes $\int_{\infty}^{0} w^2e^{-2w/b}(-dw) = \int_0^{\infty} w^2e^{-2w/b}\,dw$, which equals the second integral on the right [see Eq. (4.10)]. Hence $\int_{-\infty}^{\infty}|\Psi|^2\,dx = (4/b^3)\int_0^{\infty} x^2e^{-2x/b}\,dx = (4/b^3)[2!/(b/2)^3] = 1$, where (A.8) in the Appendix was used.

1.13 The interval is small enough to be considered infinitesimal (since Ψ changes negligibly within this interval). At $t = 0$, we have $|\Psi|^2\ dx = (32/\pi c^6)^{1/2} x^2 e^{-2x^2/c^2} dx =$ $[32/\pi(2.00\ \text{Å})^6]^{1/2}(2.00\ \text{Å})^2 e^{-2}(0.001\ \text{Å}) = 0.000216$.

1.14 $\int_a^b |\Psi|^2\, dx = \int_{1.5000\ \text{nm}}^{1.5001\ \text{nm}} a^{-1} e^{-2x/a}\, dx = -e^{-2x/a}/2\,\Big|_{1.5000\ \text{nm}}^{1.5001\ \text{nm}} = (-e^{-3.0002} + e^{-3.0000})/2 =$ 4.978×10^{-6}.

1.15 **(a)** This function is not real and cannot be a probability density.

(b) This function is negative when $x < 0$ and cannot be a probability density.

(c) This function is not normalized (unless $b = \pi$) and can't be a probability density.

1.16 **(a)** There are four equally probable cases for two children: BB, BG, GB, GG, where the first letter gives the gender of the older child. The BB possibility is eliminated by the given information. Of the remaining three possibilities BG, GB, GG, only one has two girls, so the probability that they have two girls is 1/3.

(b) The fact that the older child is a girl eliminates the BB and BG cases, leaving GB and GG, so the probability is 1/2 that the younger child is a girl.

1.17 The 138 peak arises from the case $^{12}C^{12}CF_6$, whose probability is $(0.9889)^2 = 0.9779$. The 139 peak arises from the cases $^{12}C^{13}CF_6$ and $^{13}C^{12}CF_6$, whose probability is $(0.9889)(0.0111) + (0.0111)(0.9889) = 0.02195$. The 140 peak arises from $^{13}C^{13}CF_6$, whose probability is $(0.0111)^2 = 0.000123$. (As a check, these add to 1.) The 139 peak height is $(0.02195/0.9779)100 = 2.24$. The 140 peak height is $(0.000123/0.9779)100 =$ 0.0126.

1.18 There are 26 cards, 2 spades and 24 nonspades, to be distributed between B and D. Imagine that 13 cards, picked at random from the 26, are dealt to B. The probability that every card dealt to B is a nonspade is $\frac{24}{26}\frac{23}{25}\frac{22}{24}\frac{21}{23}\cdots\frac{14}{16}\frac{13}{15}\frac{12}{14} = \frac{13(12)}{26(25)} = \frac{6}{25}$. Likewise, the probability that D gets 13 nonspades is $\frac{6}{25}$. If B does not get all nonspades and D does not get all nonspades, then each must get one of the two spades and the probability that each gets one spade is $1 - \frac{6}{25} - \frac{6}{25} = 13/25$. (A commonly given answer is: There are four possible outcomes, namely, both spades to B, both spades to D, spade 1 to B and spade 2 to D, spade 2 to B and spade 1 to D, so the probability that each gets one spade is 2/4 = 1/2. This answer is wrong, because the four outcomes are not all equally likely.)

1.19 **(a)** The Maxwell distribution of molecular speeds; **(b)** the normal (or Gaussian) distribution.

1.20 **(a)** Real; **(b)** imaginary; **(c)** real; **(d)** imaginary; **(e)** imaginary; **(f)** real; **(g)** real; **(h)** real; **(i)** real.

1.21 **(a)** A point on the x axis three units to the right of the origin.

(b) A point on the y axis one unit below the origin.

(c) A point in the second quadrant with x coordinate –2 and y coordinate +3.

1.22 $\dfrac{1}{i} = \dfrac{1}{i}\dfrac{i}{i} = \dfrac{i}{i^2} = \dfrac{i}{-1} = -i$

1.23 **(a)** $i^2 = -1$. **(b)** $i^3 = ii^2 = i(-1) = -i$. **(c)** $i^4 = (i^2)^2 = (-1)^2 = 1$.

(d) $i^*i = (-i)i = 1$.

(e) $(1+5i)(2-3i) = 2+10i-3i-15i^2 = 17+7i$.

(f) $\dfrac{1-3i}{4+2i} = \dfrac{1-3i}{4+2i}\dfrac{4-2i}{4-2i} = \dfrac{4-14i-6}{16+8i-8i+4} = \dfrac{-2-14i}{20} = -0.1-0.7i$.

1.24 **(a)** –4 **(b)** $2i$; **(c)** $6-3i$; **(d)** $2e^{i\pi/5}$.

1.25 **(a)** 1, 90°; **(b)** 2, $\pi/3$;

(c) $z = -2e^{i\pi/3} = 2(-1)e^{i\pi/3}$. Since –1 has absolute value 1 and phase π, we have $z = 2e^{i\pi}e^{i\pi/3} = 2e^{i(4\pi/3)} = re^{i\theta}$, so the absolute value is 2 and the phase is 4π/3 radians.

(d) $|z| = (x^2+y^2)^{1/2} = [1^2+(-2)^2]^{1/2} = 5^{1/2}$; $\tan\theta = y/x = -2/1 = -2$ and θ = –63.4° = 296.6° = 5.176 radians.

1.26 On a circle of radius 5. On a line starting from the origin and making an angle of 45° with the positive x axis.

1.27 **(a)** $i = 1e^{i\pi/2}$; **(b)** $-1 = 1e^{i\pi}$;

(c) Using the answers to Prob. 1.25(d), we have $5^{1/2}e^{5.176i}$;

(d) $r = [(-1)^2+(-1)^2]^{1/2} = 2^{1/2}$; $\theta = 180° + 45° = 225° = 3.927$ rad; $2^{1/2}e^{3.927i}$.

1.28 **(a)** Using Eq. (1.36) with n = 3, we have $e^{i\cdot 0} = 1$, $e^{i(2\pi/3)} = \cos(2\pi/3) + i\sin(2\pi/3) = -0.5 + i\sqrt{3}/2$, and $e^{i(4\pi/3)} = -0.5 - i\sqrt{3}/2$.

(b) We see that ω in (1.36) satisfies $\omega\omega^* = e^0 = 1,$ so the nth roots of 1 all have absolute value 1. When k in (1.36) increases by 1, the phase increases by $2\pi/n$.

1.29 $$\frac{e^{i\theta} - e^{-i\theta}}{2i} = \frac{\cos\theta + i\sin\theta - [\cos(-\theta) + i\sin(-\theta)]}{2i} = \frac{\cos\theta + i\sin\theta - (\cos\theta - i\sin\theta)}{2i} = \sin\theta,$$

where (2.14) was used.

$$\frac{e^{i\theta} + e^{-i\theta}}{2} = \frac{\cos\theta + i\sin\theta + [\cos(-\theta) + i\sin(-\theta)]}{2} = \frac{\cos\theta + i\sin\theta + \cos\theta - i\sin\theta}{2} = \cos\theta.$$

1.30 **(a)** From $f = ma$, 1 N = 1 kg m/s^2.

(b) 1 J = 1 kg m^2/s^2.

1.31 $$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \frac{2(1.602\times10^{-19}\ \text{C})79(1.602\times10^{-19}\ \text{C})}{4\pi(8.854\times10^{-12}\ \text{C}^2/\text{N-m}^2)(3.00\times10^{-13}\ \text{m})^2} = 0.405\ \text{N},$$

where 2 and 79 are the atomic numbers of He and Au.

1.32 **(a)** $4x\sin(3x^4) + 2x^2(12x^3)\cos(3x^4) = 4x\sin(3x^4) + 24x^5\cos(3x^4)$.

(b) $(x^3 + x)\big|_1^2 = (8+2) - (1+1) = 8$.

1.33 **(a)** T; **(b)** F; **(c)** F; **(d)** T; **(e)** F; **(f)** T.