Chapter 1

1. In case of an accident, there is a high chance of getting lost. The transportation cost is very high each time. However, if the infrastructure is set once, it will be very easy to use it repeatedly. Time for wireless transmission is negligible as signals travel at the speed of light.

2. Advantages of bursty data communication

   (a) Pulses are made very narrow, so multipaths are resolvable

   (b) The transmission device needs to be switched on for less time.

   Disadvantages

   (a) Bandwidth required is very high

   (b) Peak transmit power can be very high.

3. $P_b = 10^{-12}$
   $\frac{1}{2\overline{\gamma}} = 10^{-12}$
   $\overline{\gamma} = \frac{10^{12}}{2} = 5 \times 10^{11}$ (very high)

4. Geo: 35,786 Km above earth $\Rightarrow RTT = \frac{2\times35786\times10^3}{c} = 0.2386s$
   Meo: 8,000- 20,000 Km above earth $\Rightarrow RTT = \frac{2\times8000\times10^3}{c} = 0.0533s$
   Leo: 500- 2,000 Km above earth $\Rightarrow RTT = \frac{2\times500\times10^3}{c} = 0.0033s$
   Only Leo satellites as delay $= 3.3ms < 30ms$

5.

6. optimum no. of data user = d
   optimum no. of voice user = v
   Three different cases:
   Case 1: d=0, v=6
   $\Rightarrow revenue = 60.80.2 = 0.96$

   Case 2: d=1, v=3
   revenue = [prob. of having one data user]×(revenue of having one data user)
   + [prob. of having two data user]×(revenue of having two data user)
   + [prob. of having one voice user]×(revenue of having one voice user)
   + [prob. of having two voice user]×(revenue of having two voice user)
   + [prob. of having three or more voice user]×(revenue in this case)

$$\Rightarrow \quad 0.5^2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \times \$1 + 0.5^2 \times \$1 + \begin{pmatrix} 6 \\ 1 \end{pmatrix} 0.8 \times 0.2^5 \times \$0.2 + \begin{pmatrix} 6 \\ 2 \end{pmatrix} 0.8^2 \times 0.2^4 \times \$0.4 +$$

$$\left[1 - \begin{pmatrix} 6 \\ 1 \end{pmatrix} 0.8 \times 0.2^5 \times \$0.2 - \begin{pmatrix} 6 \\ 2 \end{pmatrix} 0.8^2 \times 0.2^4 \times \$0.4\right] \times \$0.6$$

$$\Rightarrow \$1.35$$

   Case 3: d=2, v=0
   revenue $=2 \times 0.5 = \$1$
   So the best case is case 2, which is to allocate 60kHz to data and 60kHz to voice.

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t2=.3*Q(a/sqrt(N0/2));
diff = abs(t1-t2);
[c,d] = min(diff);
a(d)
c
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3. $s(t) = \pm g(t)\cos 2\pi f_c t$
   $r = \hat{r}\cos\Delta\phi$
   where $\hat{r}$ is the signal after the sampler if there was no phase offset. Once again, the threshold that minimizes P_e is 0 as $(\cos\Delta\phi)$ acts as a scaling factor for both +1 and -1 levels. P_e however increases as numerator is reduced due to multiplication by $\cos\Delta\phi$

$$P_e = Q\left(\frac{d_{min}\cos\Delta\phi}{\sqrt{2N_0}}\right)$$

4.

$$\begin{aligned}
A_c^2\int_0^{T_b}\cos^2 2\pi f_c t dt &= A_c^2\int_0^{T_b}\frac{1+\cos 4\pi f_c t}{2} \\
&= A_c^2\left[\frac{T_b}{2}+\underbrace{\frac{\sin(4\pi f_c T_b)}{8\pi f_c}}_{\to 0 \text{ as } f_c\gg 1}\right] \\
&= \frac{A_c^2 T_b}{2} = 1 \\
x(t) &= 1 + n(t)
\end{aligned}$$

Let prob 1 sent $=p_1$ and prob 0 sent $=p_0$

$$\begin{aligned}
P_e &= \frac{1}{6}[1.p_1 + 0.p_0] + \frac{2}{6}[0.p_1 + 0.p_0] + \frac{2}{6}[0.p_1 + 0.p_0] + \\
&\quad \frac{1}{6}[0.p_1 + 1.p_0] \\
&= \frac{1}{6}[p_1 + p_0] = \frac{1}{6} \quad (\because p_1 + p_0 = 1 \text{ always })
\end{aligned}$$

5. We will use the approximation $P_e \sim$ (average number of nearest neighbors).$Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$
   where number of nearest neighbors = total number of points taht share decision boundary

   (a) 12 inner points have 5 neighbors
   4 outer points have 3 neighbors
   avg number of neighbors = 4.5
   $P_e = 4.5Q\left(\frac{2a}{\sqrt{2N_0}}\right)$

   (b) 16QAM, $P_e = 4\left(1-\frac{1}{4}\right)Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 3Q\left(\frac{2a}{\sqrt{2N_0}}\right)$

   (c) $P_e \sim \frac{2\times 3+3\times 2}{5}Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 2.4Q\left(\frac{2a}{\sqrt{2N_0}}\right)$

   (d) $P_e \sim \frac{1\times 4+4\times 3+4\times 2}{9}Q\left(\frac{3a}{\sqrt{2N_0}}\right) = 2.67Q\left(\frac{3a}{\sqrt{2N_0}}\right)$

6.

$$P_{s,\text{exact}} = 1 - \left(1 - \frac{2(\sqrt{M}-1)}{\sqrt{M}}Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)\right)^2$$

Chapter 10

1. (a)

$$\begin{aligned}
\overline{(AA^H)^T} &= \overline{(A^H)^T.A^T} \\
&= \overline{\overline{(A^T)}^T A^T} \\
&= AA^H \\
\therefore (AA^H)^H &= AA^H
\end{aligned}$$

For AA^H, $\quad \lambda = \overline{\lambda}$, i.e. eigen-values are real

$$AA^H = Q\Lambda Q^H$$

(b) $X^H AA^H X = (X^H A)(X^H A)^H = \|X^H A\| \geq 0$
$\therefore AA^H$ is positive semidefinite.

(c) $I_M + AA^H = I_M + Q\Lambda Q^H = Q(I + \Lambda)Q^H$
A^H positive semidefinite $\Rightarrow \lambda_i \geq 0 \forall i$
$\therefore 1 + \lambda_i > 0 \forall i$
$\therefore I_M + AA^H$ positive definite

(d)

$$\begin{aligned}
\det[I_M + AA^H] &= \det[I_M + Q\Lambda Q^H] \\
&= \det[Q(I_M + \Lambda_M)Q^H] \\
&= \det[I_M + \Lambda_M] \\
&= \Pi_{i=1}^{Rank(A)}(1 + \lambda_i)
\end{aligned}$$

$$\begin{aligned}
\det[I_N + A^H A] &= \det[I_N + \widetilde{Q}\Lambda \widetilde{Q}^H] \\
&= \det[\widetilde{Q}(I_N + \Lambda_N)\widetilde{Q}^H] \\
&= \det[I_N + \Lambda_N] \\
&= \Pi_{i=1}^{Rank(A)}(1 + \lambda_i)
\end{aligned}$$

$\because AA^H \quad$ and $\quad A^H A$ have the same eigen-value
$\therefore \det[I_M + AA^H] = \det[I_N + A^H A]$

2. $H = U\Sigma V^T$

$$U = \begin{bmatrix} -0.4793 & 0.8685 & -0.1298 \\ -0.5896 & -0.4272 & -0.6855 \\ -0.6508 & -0.2513 & 0.7164 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 1.7034 & 0 & 0 \\ 0 & 0.7152 & 0 \\ 0 & 0 & 0.1302 \end{bmatrix}$$

$$C_1 = C_2 = \log_2\left(1 + \frac{P_1}{nB}\right) = \log_2\left(1 + \frac{P - P_1}{nB + P_1}\right)$$
$$\Rightarrow \frac{P_1}{nB} = \frac{P - P_1}{nB + P_1}$$
$$\Rightarrow P_1^2 + 2P_1(nB) - PnB = 0$$
$$\Rightarrow P_1 = 2.889$$

$\therefore$ we get $C_1 = C_2 = 1.2925 \times 10^5$

15.
$$C_{MAC} = \left\{(R_1 R_2 \ldots R_k) : \sum_{k \in S} R_k \le B \log_2\left(1 + \frac{\sum_{k\in S} g_k P_k}{N_0 b}\right) \forall S \in \{1, \ldots k\}\right\}$$

Scale g_k by α , $\quad P_k$ by $1/\alpha$
Since capacity region depends on $g_k P_k$, it remains unchanged.

16. $B = 100KHz$
$P_1 = 3mW$
$P_2 = 1mW$
$N_0 = 0.001\mu W/Hz$

See Fig 4

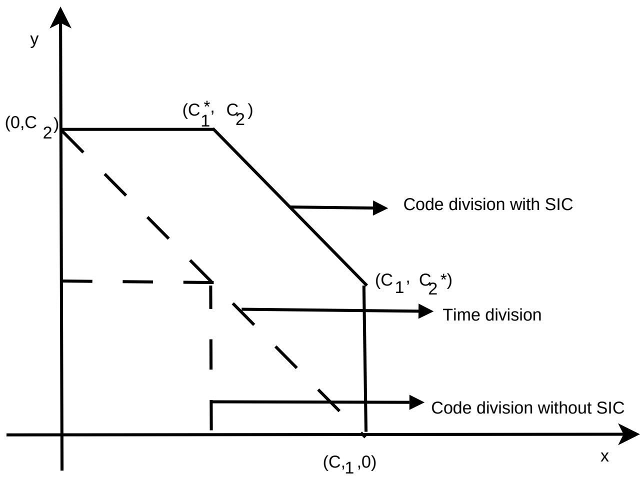

Figure 4: Problem 16

(a) $C_1 = B\log_2\left(1 + \frac{P_1}{N_0 B}\right) = 4.95B$
$C_2 = B\log_2\left(1 + \frac{P_2}{N_0 B}\right) = 3.46B$
$R_1 = 3B$ In TD rates lie on the straight line joining C_1 & C_2, so

$$\frac{R_1}{C_1} + \frac{R_2}{C_2} = 1$$
$$\frac{3B}{4.95B} + \frac{R_2}{3.46B} = 1$$
$$R_2 = 136.30Kbps$$

With superposition coding and successive interference cancellation we have,
$C_1^\star = B\log_2\left(1 + \frac{P_1}{N_0 B + P_2}\right) = 1.898B$