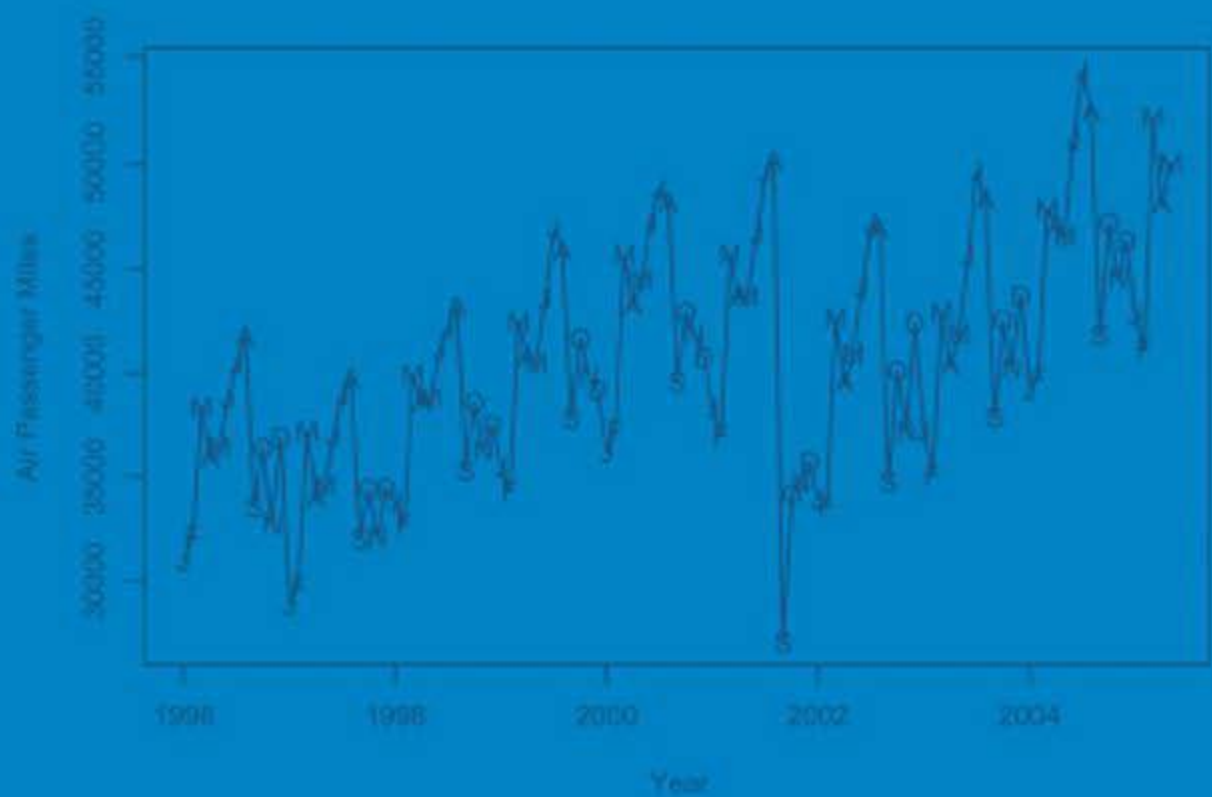


Springer Texts in Statistics

Jonathan D. Cryer
Kung-Sik Chan

Time Series Analysis

With Applications in R



Second Edition



Statistics Texts in Statistics

Series Editors:

G. Casella

S. Fienberg

I. Olkin

Springer Texts in Statistics

Athreya/Lahiri: Measure Theory and Probability Theory
Bilodeau/Brenner: Theory of Multivariate Statistics
Brockwell/Davis: An Introduction to Time Series and Forecasting
Carmona: Statistical Analysis of Financial Data in S-PLUS
Chow/Teicher: Probability Theory: Independence, Interchangeability, Martingales, 3rd ed.
Christensen: Advanced Linear Modeling: Multivariate, Time Series, and Spatial Data;
Nonparametric Regression and Response Surface Maximization, 2nd ed.
Christensen: Log-Linear Models and Logistic Regression, 2nd ed.
Christensen: Plane Answers to Complex Questions: The Theory of Linear Models, 2nd ed.
Cryer/Chan: Time Series Analysis, Second Edition
Davis: Statistical Methods for the Analysis of Repeated Measurements
Dean/Voss: Design and Analysis of Experiments
Dekking/Kraaikamp/Lopuhaä/Meester: A Modern Introduction to Probability and Statistics
Durrett: Essential of Stochastic Processes
Edwards: Introduction to Graphical Modeling, 2nd ed.
Everitt: An R and S-PLUS Companion to Multivariate Analysis
Gentle: Matrix Algebra: Theory, Computations, and Applications in Statistics
Ghosh/Delampady/Samanta: An Introduction to Bayesian Analysis
Gut: Probability: A Graduate Course
Heiberger/Holland: Statistical Analysis and Data Display; An Intermediate Course with Examples
in S-PLUS, R, and SAS
Jobson: Applied Multivariate Data Analysis, Volume I: Regression and Experimental Design
Jobson: Applied Multivariate Data Analysis, Volume II: Categorical and Multivariate Methods
Karr: Probability
Kulkarni: Modeling, Analysis, Design, and Control of Stochastic Systems
Lange: Applied Probability
Lange: Optimization
Lehmann: Elements of Large Sample Theory
Lehmann/Romano: Testing Statistical Hypotheses, 3rd ed.
Lehmann/Casella: Theory of Point Estimation, 2nd ed.
Longford: Studying Human Populations: An Advanced Course in Statistics
Marin/Robert: Bayesian Core: A Practical Approach to Computational Bayesian Statistics
Nolan/Speed: Stat Labs: Mathematical Statistics Through Applications
Pitman: Probability
Rawlings/Pantula/Dickey: Applied Regression Analysis
Robert: The Bayesian Choice: From Decision-Theoretic Foundations to Computational
Implementation, 2nd ed.
Robert/Casella: Monte Carlo Statistical Methods, 2nd ed.
Rose/Smith: Mathematical Statistics with *Mathematica*
Ruppert: Statistics and Finance: An Introduction
Sen/Srivastava: Regression Analysis: Theory, Methods, and Applications.
Shao: Mathematical Statistics, 2nd ed.
Shorack: Probability for Statisticians
Shumway/Stoffer: Time Series Analysis and Its Applications, 2nd ed.
Simonoff: Analyzing Categorical Data
Terrell: Mathematical Statistics: A Unified Introduction
Timm: Applied Multivariate Analysis
Toutenberg: Statistical Analysis of Designed Experiments, 2nd ed.
Wasserman: All of Nonparametric Statistics
Wasserman: All of Statistics: A Concise Course in Statistical Inference
Weiss: Modeling Longitudinal Data
Whittle: Probability via Expectation, 4th ed.

Jonathan D. Cryer • Kung-Sik Chan

Time Series Analysis

With Applications in R

Second Edition



Jonathan D. Cryer
Department of Statistics & Actuarial Science
University of Iowa
Iowa City, Iowa 52242
USA
jon-cryer@uiowa.edu

Kung-Sik Chan
Department of Statistics & Actuarial Science
University of Iowa
Iowa City, Iowa 52242
USA
kung-sik-chan@uiowa.edu

Series Editors:

George Casella
Department of Statistics
University of Florida
Gainesville, FL 32611-8545
USA

Stephen Fienberg
Department of Statistics
Carnegie Mellon University
Pittsburgh, PA 15213-3890
USA

Ingram Okin
Department of Statistics
Stanford University
Stanford, CA 94305
USA

ISBN: 978-0-387-75958-6

e-ISBN: 978-0-387-75959-3

Library of Congress Control Number: 2008923058

©2008 Springer Science+Business Media, LLC

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed on acid-free paper.

9 8 7 6 5 4 3 2 1

springer.com

To our families

PREFACE

The theory and practice of time series analysis have developed rapidly since the appearance in 1970 of the seminal work of George E. P. Box and Gwilym M. Jenkins, *Time Series Analysis: Forecasting and Control*, now available in its third edition (1994) with co-author Gregory C. Reinsel. Many books on time series have appeared since then, but some of them give too little practical application, while others give too little theoretical background. This book attempts to present both application, and theory at a level accessible to a wide variety of students and practitioners. Our approach is to mix application and theory throughout the book as they are naturally needed.

The book was developed for a one-semester course usually attended by students in statistics, economics, business, engineering, and quantitative social sciences. Basic applied statistics through multiple linear regression is assumed. Calculus is assumed only to the extent of minimizing sums of squares, but a calculus-based introduction to statistics is necessary for a thorough understanding of some of the theory. However, required facts concerning expectation, variance, covariance, and correlation are reviewed in appendices. Also, conditional expectation properties and minimum mean square error prediction are developed in appendices. Actual time series data drawn from various disciplines are used throughout the book to illustrate the methodology. The book contains additional topics of a more advanced nature that can be selected for inclusion in a course if the instructor so chooses.

All of the plots and numerical output displayed in the book have been produced with the R software, which is available from the R Project for Statistical Computing at www.r-project.org. Some of the numerical output has been edited for additional clarity or for simplicity. R is available as free software under the terms of the Free Software Foundation's GNU General Public License in source code form. It runs on a wide variety of UNIX platforms and similar systems, Windows, and MacOS.

R is a language and environment for statistical computing and graphics, provides a wide variety of statistical (e.g., time-series analysis, linear and nonlinear modeling, classical statistical tests) and graphical techniques, and is highly extensible. The extensive appendix An Introduction to R, provides an introduction to the R software specially designed to go with this book. One of the authors (KSC) has produced a large number of new or enhanced R functions specifically tailored to the methods described in this book. They are listed on page 468 and are available in the package named TSA on the R Project's Website at www.r-project.org. We have also constructed R command script files for each chapter. These are available for download at www.stat.uiowa.edu/~kchan/TSA.htm. We also show the required R code beneath nearly every table and graphical display in the book. The datasets required for the exercises are named in each exercise by an appropriate filename; for example, `larain` for the Los Angeles rainfall data. However, if you are using the TSA package, the datasets are part of the package and may be accessed through the R command `data(larain)`, for example.

All of the datasets are also available at the textbook website as ACSCII files with variable names in the first row. We believe that many of the plots and calculations

described in the book could also be obtained with other software, such as SAS[®], Splus[®], Statgraphics[®], SCA[®], EViews[®], RATS[®], Ox[®], and others.

This book is a second edition of the book *Time Series Analysis* by Jonathan Cryer, published in 1986 by PWS-Kent Publishing (Duxbury Press). This new edition contains nearly all of the well-received original in addition to considerable new material, numerous new datasets, and new exercises. Some of the new topics that are integrated with the original include unit root tests, extended autocorrelation functions, subset ARIMA models, and bootstrapping. Completely new chapters cover the topics of time series regression models, time series models of heteroscedasticity, spectral analysis, and threshold models. Although the level of difficulty in these new chapters is somewhat higher than in the more basic material, we believe that the discussion is presented in a way that will make the material accessible and quite useful to a broad audience of users. Chapter 15, Threshold Models, is placed last since it is the only chapter that deals with nonlinear time series models. It could be covered earlier, say after Chapter 12. Also, Chapters 13 and 14 on spectral analysis could be covered after Chapter 10.

We would like to thank John Kimmel, Executive Editor, Statistics, at Springer, for his continuing interest and guidance during the long preparation of the manuscript. Professor Howell Tong of the London School of Economics, Professor Henghsiu Tsai of Academia Sinica, Taipei, Professor Noelle Samia of Northwestern University, Professor W. K. Li and Professor Kai W. Ng, both of the University of Hong Kong, and Professor Nils Christian Stenseth of the University of Oslo kindly read parts of the manuscript, and Professor Jun Yan used a preliminary version of the text for a class at the University of Iowa. Their constructive comments are greatly appreciated. We would like to thank Samuel Hao who helped with the exercise solutions and read the appendix: An Introduction to R. We would also like to thank several anonymous reviewers who read the manuscript at various stages. Their reviews led to a much improved book. Finally, one of the authors (JDC) would like to thank Dan, Marian, and Gene for providing such a great place, *Casa de Artes*, Club Santiago, Mexico, for working on the first draft of much of this new edition.

Iowa City, Iowa
January 2008

Jonathan D. Cryer
Kung-Sik Chan

CONTENTS

- CHAPTER 1 INTRODUCTION 1
 - 1.1 Examples of Time Series 1
 - 1.2 A Model-Building Strategy 8
 - 1.3 Time Series Plots in History 8
 - 1.4 An Overview of the Book 9
 - Exercises 10
- CHAPTER 2 FUNDAMENTAL CONCEPTS 11
 - 2.1 Time Series and Stochastic Processes 11
 - 2.2 Means, Variances, and Covariances 11
 - 2.3 Stationarity 16
 - 2.4 Summary 19
 - Exercises 19
 - Appendix A: Expectation, Variance, Covariance, and Correlation . 24
- CHAPTER 3 TRENDS 27
 - 3.1 Deterministic Versus Stochastic Trends 27
 - 3.2 Estimation of a Constant Mean 28
 - 3.3 Regression Methods. 30
 - 3.4 Reliability and Efficiency of Regression Estimates. 36
 - 3.5 Interpreting Regression Output 40
 - 3.6 Residual Analysis 42
 - 3.7 Summary 50
 - Exercises 50
- CHAPTER 4 MODELS FOR STATIONARY TIME SERIES 55
 - 4.1 General Linear Processes 55
 - 4.2 Moving Average Processes 57
 - 4.3 Autoregressive Processes 66
 - 4.4 The Mixed Autoregressive Moving Average Model. 77
 - 4.5 Invertibility. 79
 - 4.6 Summary 80
 - Exercises 81
 - Appendix B: The Stationarity Region for an AR(2) Process 84
 - Appendix C: The Autocorrelation Function for ARMA(p, q). 85

CHAPTER 5 MODELS FOR NONSTATIONARY TIME SERIES .87

5.1 Stationarity Through Differencing88

5.2 ARIMA Models.....92

5.3 Constant Terms in ARIMA Models.....97

5.4 Other Transformations98

5.5 Summary102

Exercises.....103

Appendix D: The Backshift Operator.....106

CHAPTER 6 MODEL SPECIFICATION.....109

6.1 Properties of the Sample Autocorrelation Function109

6.2 The Partial and Extended Autocorrelation Functions112

6.3 Specification of Some Simulated Time Series.....117

6.4 Nonstationarity.....125

6.5 Other Specification Methods130

6.6 Specification of Some Actual Time Series.....133

6.7 Summary141

Exercises.....141

CHAPTER 7 PARAMETER ESTIMATION.....149

7.1 The Method of Moments149

7.2 Least Squares Estimation154

7.3 Maximum Likelihood and Unconditional Least Squares ...158

7.4 Properties of the Estimates160

7.5 Illustrations of Parameter Estimation.....163

7.6 Bootstrapping ARIMA Models167

7.7 Summary170

Exercises.....170

CHAPTER 8 MODEL DIAGNOSTICS175

8.1 Residual Analysis175

8.2 Overfitting and Parameter Redundancy.....185

8.3 Summary188

Exercises.....188

CHAPTER 9 FORECASTING.....	191
9.1 Minimum Mean Square Error Forecasting	191
9.2 Deterministic Trends.....	191
9.3 ARIMA Forecasting	193
9.4 Prediction Limits	203
9.5 Forecasting Illustrations	204
9.6 Updating ARIMA Forecasts	207
9.7 Forecast Weights and Exponentially Weighted Moving Averages	207
9.8 Forecasting Transformed Series.....	209
9.9 Summary of Forecasting with Certain ARIMA Models	211
9.10 Summary	213
Exercises	213
Appendix E: Conditional Expectation.....	218
Appendix F: Minimum Mean Square Error Prediction	218
Appendix G: The Truncated Linear Process	221
Appendix H: State Space Models	222
CHAPTER 10 SEASONAL MODELS.....	227
10.1 Seasonal ARIMA Models	228
10.2 Multiplicative Seasonal ARMA Models	230
10.3 Nonstationary Seasonal ARIMA Models	233
10.4 Model Specification, Fitting, and Checking.....	234
10.5 Forecasting Seasonal Models	241
10.6 Summary	246
Exercises	246
CHAPTER 11 TIME SERIES REGRESSION MODELS	249
11.1 Intervention Analysis	249
11.2 Outliers.....	257
11.3 Spurious Correlation.....	260
11.4 Prewhitening and Stochastic Regression.....	265
11.5 Summary	273
Exercises	274

CHAPTER 12 TIME SERIES MODELS OF
HETEROSCEDASTICITY.....277

12.1 Some Common Features of Financial Time Series.....278

12.2 The ARCH(1) Model285

12.3 GARCH Models289

12.4 Maximum Likelihood Estimation298

12.5 Model Diagnostics301

12.6 Conditions for the Nonnegativity of the
Conditional Variances307

12.7 Some Extensions of the GARCH Model310

12.8 Another Example: The Daily USD/HKD Exchange Rates ..311

12.9 Summary.....315

Exercises.....316

Appendix I: Formulas for the Generalized Portmanteau Tests ...318

CHAPTER 13 INTRODUCTION TO SPECTRAL ANALYSIS....319

13.1 Introduction319

13.2 The Periodogram.....322

13.3 The Spectral Representation and Spectral Distribution....327

13.4 The Spectral Density330

13.5 Spectral Densities for ARMA Processes332

13.6 Sampling Properties of the Sample Spectral Density340

13.7 Summary.....346

Exercises.....346

Appendix J: Orthogonality of Cosine and Sine Sequences349

CHAPTER 14 ESTIMATING THE SPECTRUM351

14.1 Smoothing the Spectral Density351

14.2 Bias and Variance354

14.3 Bandwidth355

14.4 Confidence Intervals for the Spectrum356

14.5 Leakage and Tapering.....358

14.6 Autoregressive Spectrum Estimation.....363

14.7 Examples with Simulated Data364

14.8 Examples with Actual Data370

14.9 Other Methods of Spectral Estimation.....376

14.10 Summary.....378

Exercises.....378

Appendix K: Tapering and the Dirichlet Kernel381

CHAPTER 15 THRESHOLD MODELS 383

15.1 Graphically Exploring Nonlinearity 384

15.2 Tests for Nonlinearity 390

15.3 Polynomial Models Are Generally Explosive 393

15.4 First-Order Threshold Autoregressive Models 395

15.5 Threshold Models 399

15.6 Testing for Threshold Nonlinearity 400

15.7 Estimation of a TAR Model 402

15.8 Model Diagnostics 411

15.9 Prediction 415

15.10 Summary 420

Exercises 420

Appendix L: The Generalized Portmanteau Test for TAR 421

APPENDIX: AN INTRODUCTION TO R 423

Introduction 423

Chapter 1 R Commands 429

Chapter 2 R Commands 433

Chapter 3 R Commands 433

Chapter 4 R Commands 438

Chapter 5 R Commands 439

Chapter 6 R Commands 441

Chapter 7 R Commands 442

Chapter 8 R Commands 446

Chapter 9 R Commands 447

Chapter 10 R Commands 450

Chapter 11 R Commands 451

Chapter 12 R Commands 457

Chapter 13 R Commands 460

Chapter 14 R Commands 461

Chapter 15 R Commands 462

New or Enhanced Functions in the TSA Library 468

DATASET INFORMATION 471

BIBLIOGRAPHY 477

INDEX 487

CHAPTER 1

INTRODUCTION

Data obtained from observations collected sequentially over time are extremely common. In business, we observe weekly interest rates, daily closing stock prices, monthly price indices, yearly sales figures, and so forth. In meteorology, we observe daily high and low temperatures, annual precipitation and drought indices, and hourly wind speeds. In agriculture, we record annual figures for crop and livestock production, soil erosion, and export sales. In the biological sciences, we observe the electrical activity of the heart at millisecond intervals. In ecology, we record the abundance of an animal species. The list of areas in which time series are studied is virtually endless. The purpose of time series analysis is generally twofold: to understand or model the stochastic mechanism that gives rise to an observed series and to predict or forecast the future values of a series based on the history of that series and, possibly, other related series or factors.

This chapter will introduce a variety of examples of time series from diverse areas of application. A somewhat unique feature of time series and their models is that we usually cannot assume that the observations arise independently from a common population (or from populations with different means, for example). Studying models that incorporate dependence is the key concept in time series analysis.

1.1 Examples of Time Series

In this section, we introduce a number of examples that will be pursued in later chapters.

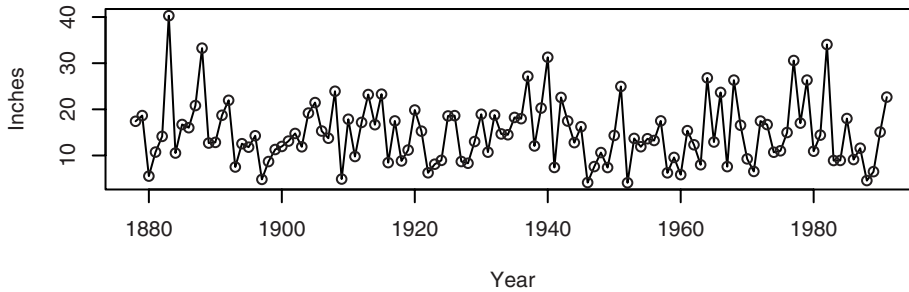
Annual Rainfall in Los Angeles

Exhibit 1.1 displays a time series plot of the annual rainfall amounts recorded in Los Angeles, California, over more than 100 years. The plot shows considerable variation in rainfall amount over the years—some years are low, some high, and many are in-between in value. The year 1883 was an exceptionally wet year for Los Angeles, while 1983 was quite dry. For analysis and modeling purposes we are interested in whether or not consecutive years are related in some way. If so, we might be able to use one year's rainfall value to help forecast next year's rainfall amount. One graphical way to investigate that question is to pair up consecutive rainfall values and plot the resulting scatterplot of pairs.

Exhibit 1.2 shows such a scatterplot for rainfall. For example, the point plotted near the lower right-hand corner shows that the year of extremely high rainfall, 40 inches in 1883, was followed by a middle of the road amount (about 12 inches) in 1884. The point

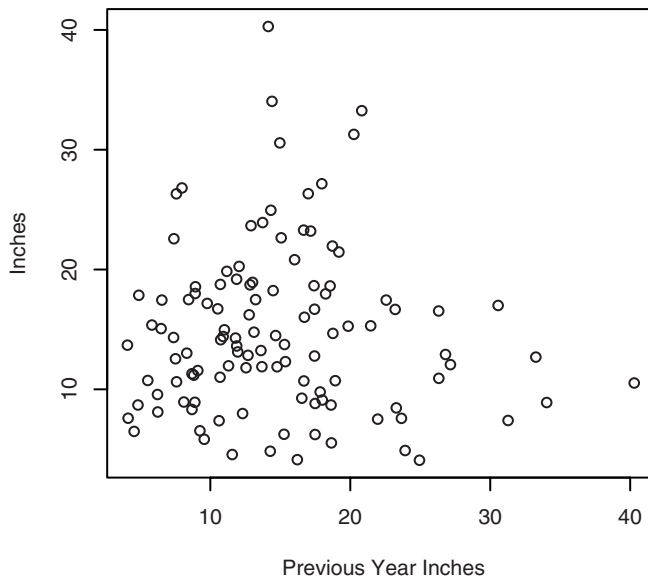
near the top of the display shows that the 40 inch year was preceded by a much more typical year of about 15 inches.

Exhibit 1.1 Time Series Plot of Los Angeles Annual Rainfall



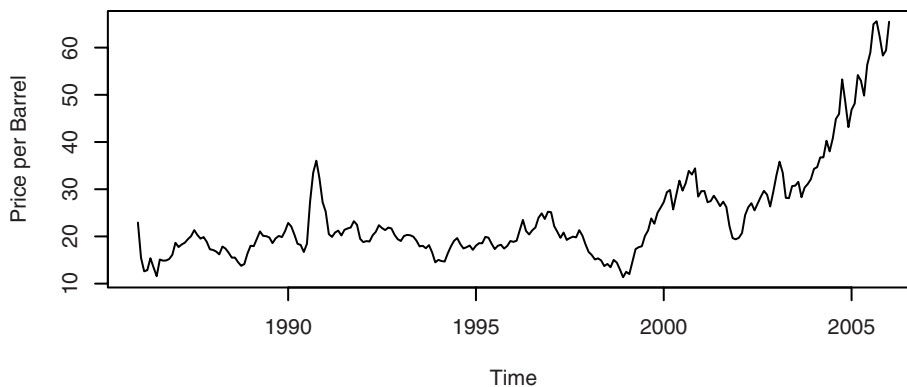
```
> library(TSA)
> win.graph(width=4.875, height=2.5,pointsize=8)
> data(larain); plot(larain,ylab='Inches',xlab='Year',type='o')
```

Exhibit 1.2 Scatterplot of LA Rainfall versus Last Year's LA Rainfall



```
> win.graph(width=3,height=3,pointsize=8)
> plot(y=larain,x=zl原因(larain),ylab='Inches',
      xlab='Previous Year Inches')
```

Exhibit 5.1 Monthly Price of Oil: January 1986–January 2006



```
> win.graph(width=4.875,height=3,pointsize=8)
> data(oil.price)
> plot(oil.price, ylab='Price per Barrel',type='l')
```

5.1 Stationarity Through Differencing

Consider again the AR(1) model

$$Y_t = \phi Y_{t-1} + e_t \quad (5.1.1)$$

We have seen that assuming e_t is a true “innovation” (that is, e_t is uncorrelated with Y_{t-1}, Y_{t-2}, \dots), we must have $|\phi| < 1$. What can we say about solutions to Equation (5.1.1) if $|\phi| \geq 1$? Consider in particular the equation

$$Y_t = 3Y_{t-1} + e_t \quad (5.1.2)$$

Iterating into the past as we have done before yields

$$Y_t = e_t + 3e_{t-1} + 3^2e_{t-2} + \cdots + 3^{t-1}e_1 + 3^tY_0 \quad (5.1.3)$$

We see that the influence of distant past values of Y_t and e_t does not die out—indeed, the weights applied to Y_0 and e_1 grow exponentially large. In Exhibit 5.2, we show the values for a very short simulation of such a series. Here the white noise sequence was generated as standard normal variables and we used $Y_0 = 0$ as an initial condition.

Exhibit 5.2 Simulation of the Explosive “AR(1) Model” $Y_t = 3Y_{t-1} + e_t$

t	1	2	3	4	5	6	7	8
e_t	0.63	−1.25	1.80	1.51	1.56	0.62	0.64	−0.98
Y_t	0.63	0.64	3.72	12.67	39.57	119.33	358.63	1074.91

- 8.9** The data file named `robot` contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series. Compare the fits of an $AR(1)$ model and an $IMA(1,1)$ model for these data in terms of the diagnostic tests discussed in this chapter.
- 8.10** The data file named `deere3` contains 57 consecutive values from a complex machine tool at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of deviation from target of the last item produced. Diagnose the fit of an $AR(1)$ model for these data in terms of the tests discussed in this chapter.
- 8.11** Exhibit 6.31 on page 139, suggested specifying either an $AR(1)$ or possibly an $AR(4)$ model for the difference of the logarithms of the oil price series. (The file-name is `oil.price`).
- (a) Estimate both of these models using maximum likelihood and compare the results using the diagnostic tests considered in this chapter.
 - (b) Exhibit 6.32 on page 140, suggested specifying an $MA(1)$ model for the difference of the logs. Estimate this model by maximum likelihood and perform the diagnostic tests considered in this chapter.
 - (c) Which of the three models $AR(1)$, $AR(4)$, or $MA(1)$ would you prefer given the results of parts (a) and (b)?

It is important to remember that the local polynomial approach assumes that the true lag 1 regression function is a smooth function. If the true lag 1 regression function is discontinuous, then the local polynomial approach may yield misleading estimates. However, a sharp turn in the estimated regression function may serve as a warning that the smoothness condition may not hold for the true lag 1 regression function.

15.2 Tests for Nonlinearity

Several tests have been proposed for assessing the need for nonlinear modeling in time series analysis. Some of these tests, such as those studied by Keenan (1985), Tsay (1986), and Luukkonen et al. (1988), can be interpreted as Lagrange multiplier tests for specific nonlinear alternatives.

Keenan (1985) derived a test for nonlinearity analogous to Tukey's one degree of freedom for nonadditivity test (see Tukey, 1949). Keenan's test is motivated by approximating a nonlinear stationary time series by a second-order Volterra expansion (Wiener, 1958)

$$Y_t = \mu + \sum_{\mu=-\infty}^{\infty} \theta_{\mu} \varepsilon_{t-\mu} + \sum_{\nu=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \theta_{\mu\nu} \varepsilon_{t-\mu} \varepsilon_{t-\nu} \quad (15.2.1)$$

where $\{\varepsilon_t, -\infty < t < \infty\}$ is a sequence of independent and identically distributed zero-mean random variables. The process $\{Y_t\}$ is linear if the double sum on the right-hand side of (15.2.1) vanishes. Thus, we can test the linearity of the time series by testing whether or not the double sum vanishes. In practice, the infinite series expansion has to be truncated to a finite sum. Let Y_1, \dots, Y_n denote the observations. Keenan's test can be implemented as follows:

- (i) Regress Y_t on Y_{t-1}, \dots, Y_{t-m} , including an intercept term, where m is some pre-specified positive integer; calculate the fitted values $\{\hat{Y}_t\}$ and the residuals $\{\hat{\varepsilon}_t\}$, for $t = m+1, \dots, n$; and set $RSS = \sum \hat{\varepsilon}_t^2$, the residual sum of squares.
- (ii) Regress \hat{Y}_t^2 on Y_{t-1}, \dots, Y_{t-m} , including an intercept term, and calculate the residuals $\{\hat{\xi}_t\}$ for $t = m+1, \dots, n$.
- (iii) Regress $\hat{\varepsilon}_t$ on the residuals $\hat{\xi}_t$ without an intercept for $t = m+1, \dots, n$, and Keenan's test statistic, denoted by \hat{F} , is obtained by multiplying $(n-2m-2)/(n-m-1)$ to the F -statistic for testing that the last regression function is identically zero. Specifically, let

$$\eta = \eta_0 \sqrt{\sum_{t=m+1}^n \hat{\xi}_t^2} \quad (15.2.2)$$

where η_0 is the regression coefficient. Form the test statistic

$$\hat{F} = \frac{\eta^2 (n-2m-2)}{RSS - \eta^2} \quad (15.2.3)$$

	ma1	ma2	sma1	intercept	A0	I0-63	I0-106
	0.2432	0.1729	0.0899	27.7658	37.7247	28.1777	23.2698
s.e.	0.0767	0.0698	0.0713	0.7544	5.4619	5.5982	5.5740

sigma² estimated as 30.67: log likelihood = -407.06, aic = 828.13

Exercise 11.19

Let us see the plot of the log-transformed weekly unit sales of lite potato chips and the weekly average price over a period of 104 weeks.

```
> library(TSA)
> data(bluebirdlite)
> ts.bluebirdlite=ts(bluebirdlite)
> plot(ts.bluebirdlite,yax.flip=T)
```

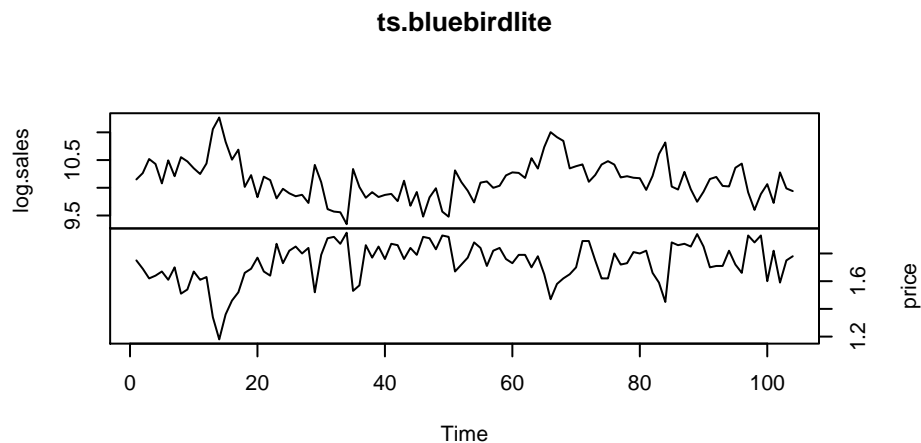


Figure 17: Weekly Log(Sales) and Price Series for Bluebirdlite Potato Chips

Next, after differencing and using prewhitened data, we draw the plot for CCF, which is significant only at lag 0, suggesting a strong contemporaneous negative relationship between lag 1 of price and sales. Higher prices are associated with lower sales.

```
> prewhiten(y=diff(ts.bluebirdlite)[,1],x=diff(ts.bluebirdlite)
+ [,2],ylab='CCF')
```