Instructor's Solutions Manual
Part I

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to accompany

Thomas' Calculus, Early Transcendentals

Tenth Edition

Based on the original work by

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PRELIMINARY CHAPTER

P.1 LINES

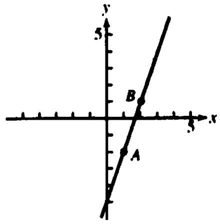
1. (a) $\Delta x = -1 - 1 = -2$
$\Delta y = -1 - 2 = -3$

(b) $\Delta x = -1 - (-3) = 2$
$\Delta y = -2 - 2 = -4$

2. (a) $\Delta x = -8 - (-3) = -5$
$\Delta y = 1 - 1 = 0$

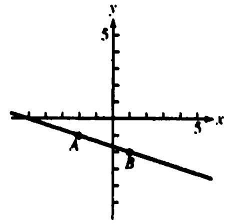
(b) $\Delta x = 0 - 0 = 0$
$\Delta y = -2 - 4 = -6$

3. (a)



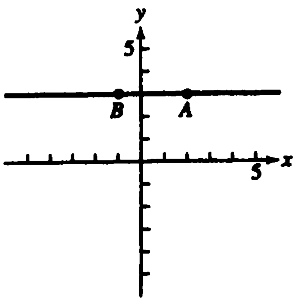
$m = \frac{1-(-2)}{2-1} = \frac{3}{1} = 3$

(b)



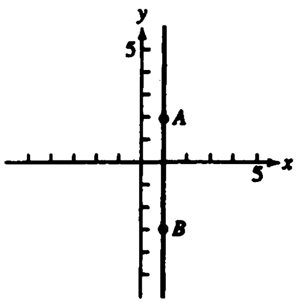
$m = \frac{-2-(-1)}{1-(-2)} = \frac{-1}{3} = -\frac{1}{3}$

4. (a)



$m = \frac{3-3}{-1-2} = \frac{0}{-3} = 0$

(b)



$m = \frac{-3-2}{1-1} = \frac{-5}{0}$ (undefined)

5. (a) $x = 2,\ y = 3$

(b) $x = -1,\ y = \frac{4}{3}$

6. (a) $x = 0,\ y = -\sqrt{2}$

(b) $x = -\pi,\ y = 0$

7. (a) $y = 1(x-1) + 1$

(b) $y = -1[x-(-1)] + 1 = -1(x+1) + 1$

8. (a) $y = 2(x-0) + 3$

(b) $y = -2[x-(-4)] + 0 = -2(x+4) + 0$

9. (a) $m = \frac{3-0}{2-0} = \frac{3}{2}$

$y = \frac{3}{2}(x-0) + 0$

$y = \frac{3}{2}x$

(b) $m = \frac{1-1}{2-1} = \frac{0}{1} = 0$

$y = 0(x-1) + 1$

$y = 1$

(d) $\lim\limits_{x\to 3} \sin\left(\frac{1}{x}-\frac{1}{2}\right) = \sin\left(\frac{1}{3}-\frac{1}{2}\right) = -\sin\left(\frac{1}{6}\right) \approx -0.1659$

14. (a) $\lim\limits_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} = \lim\limits_{x\to -1} \frac{\left(\sqrt{x^2+8}-3\right)\left(\sqrt{x^2+8}+3\right)}{(x+1)\left(\sqrt{x^2+8}+3\right)} = \lim\limits_{x\to -1} \frac{(x^2+8)-9}{(x+1)\left(\sqrt{x^2+8}+3\right)}$

$= \lim\limits_{x\to -1} \frac{(x+1)(x-1)}{(x+1)\left(\sqrt{x^2+8}+3\right)} = \lim\limits_{x\to -1} \frac{x-1}{\sqrt{x^2+8}+3} = \frac{-2}{3+3} = -\frac{1}{3}$

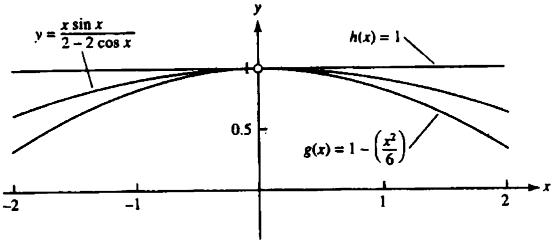
(b) $\lim\limits_{\theta\to 1} \frac{\theta^4-1}{\theta^3-1} = \lim\limits_{\theta\to 1} \frac{(\theta^2+1)(\theta+1)(\theta-1)}{(\theta^2+\theta+1)(\theta-1)} = \lim\limits_{\theta\to 1} \frac{(\theta^2+1)(\theta+1)}{\theta^2+\theta+1} = \frac{(1+1)(1+1)}{1+1+1} = \frac{4}{3}$

(c) $\lim\limits_{t\to 9} \frac{3-\sqrt{t}}{9-t} = \lim\limits_{t\to 9} \frac{\sqrt{t}-3}{\left(\sqrt{t}-3\right)\left(\sqrt{t}+3\right)} = \lim\limits_{t\to 9} \frac{1}{\sqrt{t}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

(d) Let $\frac{\pi-s}{2} = u$ so that $u \to 0$ as $s \to \pi$, and then rewrite and evaluate the limit as $\lim\limits_{u\to 0} (\pi-2u)\cos(u) = \lim\limits_{u\to 0} (\pi-2u) \cdot \lim\limits_{u\to 0} \cos(u) = \pi \cdot 1 = \pi$

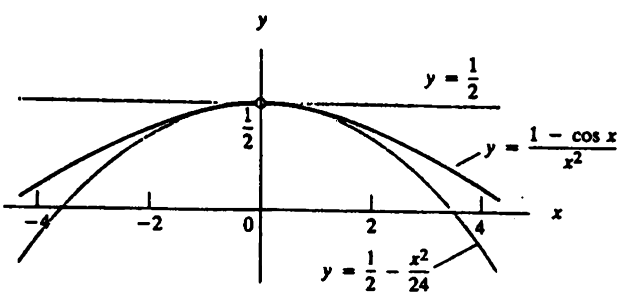
15. (a) $\lim\limits_{x\to 0} \left(1-\frac{x^2}{6}\right) = 1-\frac{0}{6} = 1$ and $\lim\limits_{x\to 0} 1 = 1$; by the sandwich theorem, $\lim\limits_{x\to 0} \frac{x\sin x}{2-2\cos x} = 1$

(b) For $x \neq 0$, $y = (x\sin x)/(2-2\cos x)$ lies between the other two graphs in the figure, and the graphs converge as $x \to 0$.



16. (a) $\lim\limits_{x\to 0} \left(\frac{1}{2}-\frac{x^2}{24}\right) = \lim\limits_{x\to 0} \frac{1}{2} - \lim\limits_{x\to 0} \frac{x^2}{24} = \frac{1}{2}-0 = \frac{1}{2}$ and $\lim\limits_{x\to 0} \frac{1}{2} = \frac{1}{2}$; by the sandwich theorem, $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$.

(b) For all $x \neq 0$, the graph of $f(x) = (1-\cos x)/x^2$ lies between the line $y = \frac{1}{2}$ and the parabola $y = \frac{1}{2} - x^2/24$, and the graphs converge as $x \to 0$.



39. $\frac{dy}{dx} = \frac{d}{dx} \log_{10} e^x = \frac{d}{dx}(x \log_{10} e) = \log_{10} e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10}$

40. $\frac{dy}{dx} = \frac{d}{dx} \ln 10^x = \frac{d}{dx}(x \ln 10) = \ln 10$

41. $y = x^{\ln x}, x > 0 \Rightarrow \ln y = \ln\left(x^{\ln x}\right) \Rightarrow \ln y = (\ln x)^2 \Rightarrow \frac{1}{y}\frac{dy}{dx} = 2(\ln x)\left(\frac{1}{x}\right) \Rightarrow \frac{dy}{dx} = \left(x^{\ln x}\right)\left(\frac{\ln x^2}{x}\right)$

42. $y = x^{(1/\ln x)} \Rightarrow \ln y = \ln\left(x^{(1/\ln x)}\right) \Rightarrow \ln y = \frac{\ln x}{\ln x} = 1 \Rightarrow \frac{d}{dx}(\ln y) = 0 \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(1) \Rightarrow \frac{dy}{dx} = 0$

43. $y = (\sin x)^x \Rightarrow \ln y = \ln(\sin x)^x \Rightarrow \ln y = x \ln(\sin x) \Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx}[x \ln(\sin x)]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (x)\left(\frac{1}{\sin x}\right)(\cos x) + \ln(\sin x)(1) \Rightarrow \frac{dy}{dx} = y[x \cot x + \ln(\sin x)] \Rightarrow \frac{dy}{dx} = (\sin x)^x[x \cot x + \ln(\sin x)]$

44. $y = x^{\tan x} \Rightarrow \ln y = \ln(x^{\tan x}) \Rightarrow \ln y = (\tan x)(\ln x) \Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx}[(\tan x)(\ln x)]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (\tan x)\left(\frac{1}{x}\right) + (\ln x)(\sec^2 x) \Rightarrow \frac{dy}{dx} = y\left[\frac{\tan x}{x} + (\ln x)(\sec^2 x)\right] \Rightarrow \frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + (\ln x)(\sec^2 x)\right]$

45. $y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}} = \left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{1/5} \Rightarrow \ln y = \ln\left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{1/5}$

$\Rightarrow \ln y = \frac{1}{5} \ln \frac{(x-3)^4(x^2+1)}{(2x+5)^3} \Rightarrow \ln y = \frac{1}{5}\left[4 \ln(x-3) + \ln(x^2+1) - 3 \ln(2x+5)\right]$

$\Rightarrow \frac{d}{dx}(\ln y) = \frac{4}{5}\frac{d}{dx} \ln(x-3) + \frac{1}{5}\frac{d}{dx}(x^2+1) - \frac{3}{5}\frac{d}{dx} \ln(2x+5) \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{4}{5}\frac{1}{x-3} + \frac{1}{5}\frac{1}{x^2+1}(2x) - \frac{3}{5}\frac{1}{2x+5}(2)$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right) \Rightarrow \frac{dy}{dx} = \left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{1/5} \cdot \left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right)$

46. $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}} = \frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} \Rightarrow \ln y = \ln \frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} \Rightarrow \ln y = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$

$\Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx} \ln x + \frac{1}{2}\frac{d}{dx} \ln(x^2+1) - \frac{2}{3}\frac{d}{dx} \ln(x+1) \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\frac{1}{x^2+1}(2x) - \frac{2}{3}\frac{1}{x+1}(1)$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right) \Rightarrow \frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right)$

47. The line passes through (a, e^a) for some value of a and has slope $m = e^a$. Since the line also passes through the origin, the slope is also given by $m = \frac{e^a - 0}{a - 0}$ and we have $e^a = \frac{e^a}{a}$, so $a = 1$. Hence, the slope is e and the equation is $y = ex$.

48. For $y = xe^x$, we have $y' = (x)(e^x) + (e^x)(1) = (x+1)e^x$, so the normal line through the point (a, ae^a) has slope $m = -\frac{1}{(a+1)e^a}$ and its equation is $y = -\frac{1}{(a+1)e^a}(x-a) + ae^a$. The desired normal line includes the point

6. (a) $A(x) = \frac{\pi}{4}(\text{diameter})^2 = \frac{\pi}{4}\left(\frac{2}{\sqrt[4]{1-x^2}} - 0\right)^2 = \frac{\pi}{4}\left(\frac{4}{\sqrt{1-x^2}}\right) = \frac{\pi}{\sqrt{1-x^2}} \Rightarrow V = \int_a^b A(x)\,dx$

$$= \int_{-\sqrt{2}/2}^{\sqrt{2}/2} \frac{\pi}{\sqrt{1-x^2}}\,dx = \pi\left[\sin^{-1} x\right]_{-\sqrt{2}/2}^{\sqrt{2}/2} = \pi\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) - \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] = \pi\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi^2}{2}$$

(b) $A(x) = \frac{(\text{diagonal})^2}{2} = \frac{1}{2}\left(\frac{2}{\sqrt[4]{1-x^2}} - 0\right)^2 = \frac{2}{\sqrt{1-x^2}} \Rightarrow V = \int_a^b A(x)\,dx = \int_{-\sqrt{2}/2}^{\sqrt{2}/2} \frac{2}{\sqrt{1-x^2}}\,dx$

$$= 2\left[\sin^{-1} x\right]_{-\sqrt{2}/2}^{\sqrt{2}/2} = 2\left(\frac{\pi}{4}\cdot 2\right) = \pi$$

7. (a) STEP 1) $A(x) = \frac{1}{2}(\text{side})\cdot(\text{side})\cdot\left(\sin\frac{\pi}{3}\right) = \frac{1}{2}\cdot\left(2\sqrt{\sin x}\right)\cdot\left(2\sqrt{\sin x}\right)\left(\sin\frac{\pi}{3}\right) = \sqrt{3}\sin x$

STEP 2) $a = 0,\ b = \pi$

STEP 3) $V = \int_a^b A(x)\,dx = \sqrt{3}\int_0^{\pi} \sin x\,dx = \left[-\sqrt{3}\cos x\right]_0^{\pi} = \sqrt{3}(1+1) = 2\sqrt{3}$

(b) STEP 1) $A(x) = (\text{side})^2 = \left(2\sqrt{\sin x}\right)\left(2\sqrt{\sin x}\right) = 4\sin x$

STEP 2) $a = 0,\ b = \pi$

STEP 3) $V = \int_a^b A(x)\,dx = \int_0^{\pi} 4\sin x\,dx = [-4\cos x]_0^{\pi} = 8$

8. (a) STEP 1) $A(x) = \frac{\pi(\text{diameter})^2}{4} = \frac{\pi}{4}(\sec x - \tan x)^2 = \frac{\pi}{4}\left(\sec^2 x + \tan^2 x - 2\sec x\tan x\right)$

$$= \frac{\pi}{4}\left[\sec^2 x + \left(\sec^2 x - 1\right) - 2\,\frac{\sin x}{\cos^2 x}\right]$$

STEP 2) $a = -\frac{\pi}{3},\ b = \frac{\pi}{3}$

STEP 3) $V = \int_a^b A(x)\,dx = \int_{-\pi/3}^{\pi/3} \frac{\pi}{4}\left(2\sec^2 x - 1 - \frac{2\sin x}{\cos^2 x}\right)dx = \frac{\pi}{4}\left[2\tan x - x + 2\left(-\frac{1}{\cos x}\right)\right]_{-\pi/3}^{\pi/3}$

$$= \frac{\pi}{4}\left[2\sqrt{3} - \frac{\pi}{3} + 2\left(-\frac{1}{\left(\frac{1}{2}\right)}\right) - \left(-2\sqrt{3} + \frac{\pi}{3} + 2\left(-\frac{1}{\left(\frac{1}{2}\right)}\right)\right)\right] = \frac{\pi}{4}\left(4\sqrt{3} - \frac{2\pi}{3}\right)$$

(b) STEP 1) $A(x) = (\text{edge})^2 = (\sec x - \tan x)^2 = \left(2\sec^2 x - 1 - 2\,\frac{\sin x}{\cos^2 x}\right)$

STEP 2) $a = -\frac{\pi}{3},\ b = \frac{\pi}{3}$

STEP 3) $V = \int_a^b A(x)\,dx = \int_{-\pi/3}^{\pi/3} \left(2\sec^2 x - 1 - \frac{2\sin x}{\cos^2 x}\right)dx = 2\left(2\sqrt{3} - \frac{\pi}{3}\right) = 4\sqrt{3} - \frac{2\pi}{3}$

4. $(1-2x)^{1/2} = 1 + \frac{1}{2}(-2x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(-2x)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^3}{3!} + \ldots = 1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 - \ldots$

5. $\left(1+\frac{x}{2}\right)^{-2} = 1 - 2\left(\frac{x}{2}\right) + \frac{(-2)(-3)\left(\frac{x}{2}\right)^2}{2!} + \frac{(-2)(-3)(-4)\left(\frac{x}{2}\right)^3}{3!} + \ldots = 1 - x + \frac{3}{4}x^2 - \frac{1}{2}x^3$

6. $\left(1-\frac{x}{2}\right)^{-2} = 1 - 2\left(-\frac{x}{2}\right) + \frac{(-2)(-3)\left(-\frac{x}{2}\right)^2}{2!} + \frac{(-2)(-3)(-4)\left(-\frac{x}{2}\right)^3}{3!} + \ldots = 1 + x + \frac{3}{4}x^2 + \frac{1}{2}x^3 + \ldots$

7. $(1+x^3)^{-1/2} = 1 - \frac{1}{2}x^3 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(x^3)^2}{2!} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(x^3)^3}{3!} + \ldots = 1 - \frac{1}{2}x^3 + \frac{3}{8}x^6 - \frac{5}{16}x^9 + \ldots$

8. $(1+x^2)^{-1/3} = 1 - \frac{1}{3}x^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(x^2)^2}{2!} + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(x^2)^3}{3!} + \ldots = 1 - \frac{1}{3}x^2 + \frac{2}{9}x^4 - \frac{14}{81}x^6 + \ldots$

9. $\left(1+\frac{1}{x}\right)^{1/2} = 1 + \frac{1}{2}\left(\frac{1}{x}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{x}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{x}\right)^3}{3!} + \ldots = 1 + \frac{1}{2x} - \frac{1}{8x^2} + \frac{1}{16x^3}$

10. $\left(1-\frac{2}{x}\right)^{1/3} = 1 + \frac{1}{3}\left(-\frac{2}{x}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{x}\right)^2}{2!} + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{2}{x}\right)^3}{3!} + \ldots = 1 - \frac{2}{3x} - \frac{4}{9x^2} - \frac{40}{81x^3} - \ldots$

11. $(1+x)^4 = 1 + 4x + \frac{(4)(3)x^2}{2!} + \frac{(4)(3)(2)x^3}{3!} + \frac{(4)(3)(2)x^4}{4!} = 1 + 4x + 6x^2 + 4x^3 + x^4$

12. $\left(1+x^2\right)^3 = 1 + 3x^2 + \frac{(3)(2)\left(x^2\right)^2}{2!} + \frac{(3)(2)(1)\left(x^2\right)^3}{3!} = 1 + 3x^2 + 3x^4 + x^6$

13. $(1-2x)^3 = 1 + 3(-2x) + \frac{(3)(2)(-2x)^2}{2!} + \frac{(3)(2)(1)(-2x)^3}{3!} = 1 - 6x + 12x^2 - 8x^3$

14. $\left(1-\frac{x}{2}\right)^4 = 1 + 4\left(-\frac{x}{2}\right) + \frac{(4)(3)\left(-\frac{x}{2}\right)^2}{2!} + \frac{(4)(3)(2)\left(-\frac{x}{2}\right)^3}{3!} + \frac{(4)(3)(2)(1)\left(-\frac{x}{2}\right)^4}{4!} = 1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$

15. Assume the solution has the form $y = a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + a_nx^n + \ldots$

$\Rightarrow \frac{dy}{dx} = a_1 + 2a_2x + \ldots + na_nx^{n-1} + \ldots$

$\Rightarrow \frac{dy}{dx} + y = (a_1 + a_0) + (2a_2 + a_1)x + (3a_3 + a_2)x^2 + \ldots + (na_n + a_{n-1})x^{n-1} + \ldots = 0$

$\Rightarrow a_1 + a_0 = 0$, $2a_2 + a_1 = 0$, $3a_3 + a_2 = 0$ and in general $na_n + a_{n-1} = 0$. Since $y = 1$ when $x = 0$ we have $a_0 = 1$. Therefore $a_1 = -1$, $a_2 = \frac{-a_1}{2 \cdot 1} = \frac{1}{2}$, $a_3 = \frac{-a_2}{3} = -\frac{1}{3 \cdot 2}$, $\ldots$, $a_n = \frac{-a_{n-1}}{n} = \frac{(-1)^n}{n!}$

$\Rightarrow y = 1 - x + \frac{1}{2}x^2 - \frac{1}{3!}x^3 + \ldots + \frac{(-1)^n}{n!}x^n + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = e^{-x}$

$$= \frac{r^2 + r\left(\frac{d^2\theta}{dt^2}\right)(f')\left(\frac{dt}{d\theta}\right)^2 - rf'' - rf'\left(\frac{d^2\theta}{dt^2}\right)\left(\frac{dt}{d\theta}\right)^2 + 2(f')^2}{\left[(f')^2 + f^2\right]^{3/2}} = \frac{f^2 - ff'' + 2(f')^2}{\left[(f')^2 + f^2\right]^{3/2}}$$

23. (a) Let $r = 2 - t$ and $\theta = 3t \Rightarrow \frac{dr}{dt} = -1$ and $\frac{d\theta}{dt} = 3 \Rightarrow \frac{d^2r}{dt^2} = \frac{d^2\theta}{dt^2} = 0$. The halfway point is $(1,3) \Rightarrow t = 1$;

$$\mathbf{v} = \frac{dr}{dt}\mathbf{u}_r + r\frac{d\theta}{dt}\mathbf{u}_\theta \Rightarrow \mathbf{v}(1) = -\mathbf{u}_r + 3\mathbf{u}_\theta;\ \mathbf{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{u}_r + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]\mathbf{u}_\theta \Rightarrow \mathbf{a}(1) = -9\mathbf{u}_r - 6\mathbf{u}_\theta$$

(b) It takes the beetle 2 min to crawl to the origin $\Rightarrow$ the rod has revolved 6 radians

$$\Rightarrow L = \int_0^6 \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_0^6 \sqrt{\left(2 - \frac{\theta}{3}\right)^2 + \left(-\frac{1}{3}\right)^2}\, d\theta = \int_0^6 \sqrt{4 - \frac{4\theta}{3} + \frac{\theta^2}{9} + \frac{1}{9}}\, d\theta$$

$$= \int_0^6 \sqrt{\frac{37 - 12\theta + \theta^2}{9}}\, d\theta = \frac{1}{3}\int_0^6 \sqrt{(\theta - 6)^2 + 1}\, d\theta = \frac{1}{3}\left[\frac{(\theta - 6)}{2}\sqrt{(\theta - 6)^2 + 1} + \frac{1}{2}\ln\left|\theta - 6 + \sqrt{(\theta - 6)^2 + 1}\right|\right]_0^6$$

$$= \sqrt{37} - \frac{1}{6}\ln\left(\sqrt{37} - 6\right) \approx 6.5 \text{ in.}$$

24. $\mathbf{L}(t) = \mathbf{r}(t) + m\mathbf{v}(t) \Rightarrow \frac{d\mathbf{L}}{dt} = \left(\frac{d\mathbf{r}}{dt} \times m\mathbf{v}\right) + \left(\mathbf{r} + m\frac{d^2\mathbf{r}}{dt^2}\right) \Rightarrow \frac{d\mathbf{L}}{dt} = (\mathbf{v} \times m\mathbf{v}) + (\mathbf{r} \times m\mathbf{a}) = \mathbf{r} \times m\mathbf{a}$; $\mathbf{F} = m\mathbf{a} \Rightarrow -\frac{c}{|\mathbf{r}|^3}\mathbf{r}$
$= m\mathbf{a} \Rightarrow \frac{d\mathbf{L}}{dt} = \mathbf{r} \times m\mathbf{a} = \mathbf{r} \times \left(-\frac{c}{|\mathbf{r}|^3}\mathbf{r}\right) = -\frac{c}{|\mathbf{r}|^3}(\mathbf{r} \times \mathbf{r}) = \mathbf{0} \Rightarrow \mathbf{L} =$ constant vector

Evaluate the integrals:

$$\int_1^2 \int_0^{x^2} \frac{1}{x+y}\, dy\, dx = \int_0^1 \int_1^2 \frac{1}{x+y}\, dx\, dy + \int_1^4 \int_{\sqrt{y}}^2 \frac{1}{x+y}\, dx\, dy = -1 + \ln\left(\frac{27}{4}\right) \approx 0.909543$$

76. Plot the region of integration with the following commands:

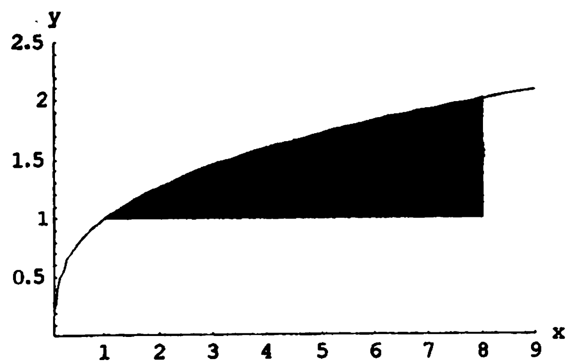
Mathematica:

```
<< Graphics`FilledPlot`;
p2 = Plot[{x^(1/3)}, {x,0,9}, PlotRange → {(0,9), (0,2.5)}, DisplayFunction → Identity];
p3 = FilledPlot[{1,x^(1/3)}, {x,1,8}, DisplayFunction → Identity];
Show[{p2,p3}, AxesLabel → {x,y}, DisplayFunction → $DisplayFunction];
```

Maple:

```
>plots[display]([plot([1,x^(1/3)], x=1..8,
  color=[white,blue], filled=true, labels=[x,y]),
  plot([x^(1/3)], x=0..9)], view=[0..9,0..2.5]);
```

The following graph was generated using Mathematica.



Evaluate the integrals:

$$\int_1^2 \int_{y^3}^8 \frac{1}{\sqrt{x^2+y^2}}\, dx\, dy = \int_1^8 \int_1^{\sqrt[3]{x}} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx \approx 0.866649$$

12.2 AREAS, MOMENTS, AND CENTERS OF MASS

1. $\int_0^2 \int_0^{2-x} dy\, dx = \int_0^2 (2-x)\, dx = \left[2x - \frac{x^2}{2}\right]_0^2 = 2,$

or $\int_0^2 \int_0^{2-y} dx\, dy = \int_0^2 (2-y)\, dy = 2$