Solutions Manual for *Optimal Control Theory*

Applications to Management Science

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SOLUTIONS MANUAL

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Solutions for Chapter 1

1.1. (a) $\dot{I}(t) = 1000 - (900 + 10t) = 100 - 10t, \quad I(0) = 1000$

Integration gives $I(t) = 1000 + 100t - 5t^2$.

Since $I(10) = 1,500 > I_{min} = 800$, and since

$I(t) \geq 0$ for $0 \leq t \leq 10$, the control $P(t) = 1000 \in$

$[600,1200]$ is feasible.

$$J = \int_0^{10} - [10I(t) + 20P(t)]dt$$

$$= \int_0^{10} - [10,000 + 1000t - 50t^2 + 20,000]dt$$

$$= -333,333.$$

(b) $\dot{I}(t) = 800 - (900 + 10t) = -100 - 10t, \quad I(0) = 1000$

$I(t) = 1000 - 100t - 5t^2$

$I(10) = -500 < I_{min} = 800$, so that the terminal

constraint is violated.

(c) $$\dot{I}(t) = \begin{cases} 600 - (900 + 10t) = -300 - 10t & \text{for } 0 \leq t \leq 6 \\ 1200 - (900 + 10t) = 300 - 10t & \text{for } 6 < t \leq 10 \end{cases}$$

and $I(0) = 1000$. Integrating

$$I(t) = \begin{cases} 1000 - 300t - 5t^2 & \text{for } 0 \leq t \leq 6 \\ -2600 + 300t - 5t^2 & \text{for } 6 < t \leq 10 \end{cases}$$

Since $I(6) = -980 < 0$, the state constraint $I(t) \geq 0$

is violated.

1.2. $\dot{G}(t) = .8 - (.05)G(t)$

$\dot{G}(0) = .8 - (.05)(16) = 0$

$\Rightarrow \dot{G}(t) = 0$ for all t $\Rightarrow G(t) = 16$ for all t.

$$J = \int_0^{\infty}(2\sqrt{16} - .8)\, e^{-.2t}dt = 7.2 \int_0^{\infty} e^{-.2t}\, dt = 36.$$

The new Lagrangian replacing (4.17) is

$$L = H + \mu\dot{x} + \nu\dot{y} + \eta_1(u + U_2) + \eta_2(U_1 - u), \quad \text{where}$$

$$H = \lambda_1(r_1x-d+u) + \lambda_2(r_2y-u) + \mu(r_1x-d+u) + \nu(r_2y-u)$$

with additional complementary slackness conditions

$$\eta_1 \geq 0, \qquad \eta_1(u + U_2) = 0$$
$$\eta_2 \geq 0, \qquad \eta_2(U_1 - u) = 0 \quad .$$

(b) When $\alpha > 0$, we decompose $u = u_1 - u_2$ as in (4.9). The control constraints are

$$0 \leq u_1 \leq U_1 \quad \text{and} \quad 0 \leq u_2 \leq U_2 \quad .$$

The Lagrangian can be formulated as

$$\begin{aligned} L = {} & (\lambda_1+\mu)[r_1x-d+u_1-u_2-\alpha(u_1+u_2)] \\ & + (\lambda_2+\nu)[r_2y-u_1+u_2] + \eta_1u_1 + \beta_1(U_1-u_1) \\ & + \eta_2u_2 + \beta_2(U_2-u_2) \ . \end{aligned}$$

The additional complementary slackness conditions are

$$\eta_1 \geq 0, \quad \eta_1u_1 = 0, \quad \beta_1 \geq 0, \quad \beta_1(U_1-u_1) = 0$$
$$\eta_2 \geq 0, \quad \eta_2u_2 = 0, \quad \beta_2 \geq 0, \quad \beta_2(U_2-u_2) = 0.$$

4.3. At $\alpha = 0$, the optimal policy is to impulse-sell all the securities at $t = 5$. Because of the substantial difference in earnings between cash and securities in the interval (5,10], the above policy should continue to be optimal for small values of α. At a sufficiently high value of α,

$L_x = -2(x-2) + 2\lambda - 2\mu x = 0$

$L_y = -2(y-2) - \lambda - 2\mu y + \nu = 0$

along with (7.22)-(7.25) and $2x = y$.

Case 1. $\mu > 0$, $\nu = 0$; then $x^2 + y^2 = 1$, $2x = y$ give $x = \frac{1}{\sqrt{5}}$, $y = \frac{2}{\sqrt{5}}$. Solving $L_x = 0$ and $L_y = 0$ for μ gives $\mu > 0$; so $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ satisfies the K-T conditions.

Case 2. $\mu = 0$, $\nu = 0$. Solving $L_x = 0$, $L_y = 0$ and $2x = y$ gixes $x = 6/5 > 1$, which is infeasible.

Case 3. $\mu = 0$, $\nu > 0$; then $y = 0$ and $x = 0$. But then $\lambda = -2$ and $\nu = -6$, a contradiction.

Case 4. $\mu > 0$, $\nu > 0$; then $y = 0$ and $x^2 + y^2 = 1$ so that $x = 1$ which contradicts $2x = y$.

Conclusion: $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ is the closest point.

7.9. We set up the equations for all three parts using $h = x + ky$.

$$L = x + ky + \lambda[(2-y)^3 - x^2] + \mu y \quad (1)$$

$$L_x = 1 - 2\lambda x = 0 \quad (2)$$

$$L_y = k - 3\lambda(2-y)^2 + \mu = 0 \quad (3)$$

$$\lambda \geq 0, \quad \lambda[(2-y)^3 - x^2] = 0 \quad (4)$$

$$\mu \geq 0, \quad \mu y = 0 \quad . \quad (5)$$

From (2) and (4), $\lambda > 0$ always; also $x \neq 0$ and $x^2 = (2-y)^3$.