# OPTICS

## Fourth Edition

# INSTRUCTOR'S SOLUTIONS MANUAL

**Eugene Hecht**
*Adelphi University*

**Mark Coffey**
*University of Colorado*

**Paul Dolan**
*Northeastern Illinois University*

Addison Wesley

San Francisco Boston New York
Cape Town Hong Kong London Madrid Mexico City
Montreal Munich Paris Singapore Sydney Tokyo Toronto

ISBN 0-8053-8578-9



4 5 6 7 8 9 10 —BTP— 05 04 03 02 01

Addison Wesley

www.aw.com/physics

# Contents

# Chapter 2 Solutions

**2.1** $(0.003)(2.54 \times 10^{-2}/580 \times 10^{-9})$ = number of waves = 131, $c = \nu\lambda$, $\lambda = c/\nu = 3 \times 10^8/10^{10}$, $\lambda = 3$ cm. Waves extend 3.9 m.

**2.2** $\lambda = c/\nu = 3 \times 10^8/5 \times 10^{14} = 6 \times 10^{-7}$ m $= 0.6\mu$ m.
$\lambda = 3 \times 10^8/60 = 5 \times 10^6$ m $= 5 \times 10^3$ km.

**2.3** $v = \lambda\nu = 5 \times 10^{-7} \times 6 \times 10^8 = 300$ m/s.

**2.4** The time between the crests is the period, so $\tau = 1/2$ s; hence $\nu = 1/\tau = 2.0$ Hz. As for the speed $v = L/t = 4.5$ m/1.5 s = 3.0 m/s. We now know $\tau$, $\nu$, and $v$ and must determine $\lambda$. Thus, $\lambda = v/\nu = 3.0$ m/s/2.0 Hz = 1.5 m.

**2.5** $v = \nu\lambda = 3.5 \times 10^3$ m/s $= \nu(4.3$ m$)$; $\nu = 0.81$ kHz.

**2.6** $v = \nu\lambda = 1498$ m/s $= (440$ Hz$)\lambda$; $\lambda = 3.40$ m.

**2.7** $v = (10$ m$)/2.0$ s$) = 5.0$ m/s; $\nu = v/\lambda = (5.0$ m/s$)/(0.50$ m$) = 10$ Hz.

**2.8** $v = \nu\lambda = (\omega/2\pi)\lambda$ and so $\omega = (2\pi/\lambda)v$.

**2.9**

| $\theta$ | $-\pi/2$ | $-\pi/4$ | $0$ | $\pi/4$ | $\pi/2$ | $3\pi/4$ |
|---|---|---|---|---|---|---|
| $\sin\theta$ | $-1$ | $-\sqrt{2}/2$ | $0$ | $\sqrt{2}/2$ | $1$ | $\sqrt{2}/2$ |
| $\cos\theta$ | $0$ | $\sqrt{2}/2$ | $1$ | $\sqrt{2}/2$ | $0$ | $-\sqrt{2}/2$ |
| $\sin(\theta - \pi/4)$ | $-\sqrt{2}/2$ | $-1$ | $-\sqrt{2}/2$ | $0$ | $\sqrt{2}/2$ | $1$ |
| $\sin(\theta - \pi/2)$ | $0$ | $-\sqrt{2}/2$ | $-1$ | $-\sqrt{2}/2$ | $0$ | $\sqrt{2}/2$ |
| $\sin(\theta - 3\pi/4)$ | $\sqrt{2}/2$ | $0$ | $-\sqrt{2}/2$ | $-1$ | $-\sqrt{2}/2$ | $0$ |
| $\sin(\theta + \pi/2)$ | $0$ | $\sqrt{2}/2$ | $1$ | $\sqrt{2}/2$ | $0$ | $-\sqrt{2}/2$ |

| $\theta$ | $\pi$ | $5\pi/4$ | $3\pi/2$ | $7\pi/4$ | $2\pi$ |
|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $-\sqrt{2}/2$ | $-1$ | $-\sqrt{2}/2$ | 0 |
| $\cos\theta$ | $-1$ | $-\sqrt{2}/2$ | 0 | $\sqrt{2}/2$ | 1 |
| $\sin(\theta-\pi/4)$ | $\sqrt{2}/2$ | 0 | $-\sqrt{2}/2$ | $-1$ | $-\sqrt{2}/2$ |
| $\sin(\theta-\pi/2)$ | 1 | $\sqrt{2}/2$ | 0 | $-\sqrt{2}/2$ | $-1$ |
| $\sin(\theta-3\pi/4)$ | $\sqrt{2}/2$ | 1 | $\sqrt{2}/2$ | 0 | $-\sqrt{2}/2$ |
| $\sin(\theta+\pi/2)$ | $-1$ | $-\sqrt{2}/2$ | 0 | $\sqrt{2}/2$ | 1 |

$\sin\theta$ leads $\sin(\theta-\pi/2)$.

**2.10**

| $x$ | $-\lambda/2$ | $-\lambda/4$ | 0 | $\lambda/4$ | $\lambda/2$ | $3\lambda/4$ | $\lambda$ |
|---|---|---|---|---|---|---|---|
| $\kappa x = 2\pi/\lambda x$ | $-\pi$ | $-\pi/2$ | 0 | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
| $\cos(\kappa x-\pi/2)$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ |
| $\cos(\kappa x+3\pi/4)$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ |

**2.11**

| $t$ | $-\tau/2$ | $-\tau/4$ | 0 | $\tau/4$ | $\tau/2$ | $3\tau/4$ | $\tau$ |
|---|---|---|---|---|---|---|---|
| $\omega t = 2\pi/\tau$ | $-\pi$ | $-\pi/2$ | 0 | $\pi/2$ | $\pi$ | $3\pi/2$ | $\pi$ |
| $\sin(\omega t+\pi/4)$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ |
| $\sin(\pi/4-\omega t)$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | $-\sqrt{2}/2$ | $-\sqrt{2}/2$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ |

**2.12** Comparing $y$ with Eq. (2.13) tells us that $A = 0.02$ m. Moreover, $2\pi/\lambda = 157\ \text{m}^{-1}$ and so $\lambda = 2\pi/(157\text{m}^{-1}) = 0.0400$ m. The relationship between frequency and wavelength is $v = \nu\lambda$, and so $\nu = v/\lambda = 1.2\ \text{m/s}/0.0400\ \text{m} = 30$ Hz. The period is the inverse of the frequency, and therefore $\tau = 1/\nu = 0.033$ s.

**2.13** (a) $\lambda = (4.0-0.0)\ \text{m} = 4.0$ m. (b) $v = \nu\lambda$, so $\nu = v/\lambda = (20.0\ \text{m/s})/(4.0\ \text{m}) = 5.0$ Hz. (c) Eq. (2.28) $\psi(x,t) = A\sin(kx-\omega t+\epsilon)$. From the figure, $A = 0.020$ m; $k = 2\pi/\lambda = 2\pi/(4.0\ \text{m}) = 0.5\pi\ \text{m}^{-1}$; $w = 2\pi\nu = 2\pi(5.0\ \text{Hz}) = 10.0\pi$ rad/s

At $t=0$, $x=0$, $\psi(0,0) = -0.020$ m;
$\psi(0,0) = (0.020\ \text{m})\sin(0.5\pi(0) - 10.0\pi(0)+\epsilon) = (0.020\ \text{m})\sin(\epsilon)$;
$\sin(\epsilon) = -1$; $\epsilon = -\pi/2$. $\psi(x,t) = (0.020\ \text{m})\sin(0.5\pi x - 10.0\pi t - \pi/2)$

**2.14** (a) $\lambda = (30.0-0.0)\ \text{cm} = 30.0$ cm. (c) $v = \nu\lambda$, so $\nu = v/\lambda = (100\ \text{cm/s})/(30.0\ \text{cm}) = 3.33$ Hz

**2.24** $\psi(0,t) = A\cos(kvt + \pi) = -A\cos(kvt) = -A\cos(\omega t)$, then
$\psi(0,\tau/2) = -A\cos(\omega\tau/2) = -A\cos(\pi) = A$,
$\psi(0,3\tau/4) = -A\cos(3\omega\tau/4) = -A\cos(3\pi/2) = 0$.

**2.25** Since $\psi(y,t) = (y - vt)A$ is only a function of $(y - vt)$, it does satisfy the conditions set down for a wave. Since $\partial^2\psi/\partial y^2 = \partial^2\psi/\partial t^2 = 0$, this function is a solution of the wave equation. However, $\psi(y,0) = Ay$ is unbounded, so cannot represent a localized wave profile.

**2.26** $k = \pi 3 \times 10^6\ \mathrm{m}^{-1}$, $\omega = \pi 9 \times 10^{14}$ Hz, $v = \omega/k = 3 \times 10^8$ m/s.

**2.27** $d\psi/dt = \partial\psi/\partial x dx/dt + (\partial\psi/\partial y)(dy/dt)$ and let $y = t$ whereupon $d\psi/dt = \partial\psi/\partial x(\pm v) + \partial\psi/\partial t = 0$ and the desired result follows immediately.

**2.28** $\varphi/dt = (\partial\varphi/\partial x)(dx/dt) + \partial\varphi/\partial t = 0 = k(dx/dt) - kv$ and this is zero provided $dx/dt = \pm v$, as it should be. For the particular wave of Problem 2.20, $\varphi/dt = \partial\varphi/\partial y(\pm v) + \partial\varphi/\partial t = \pi 3 \times 10^6(\pm v) + \pi 9 \times 10^{14} = 0$ and the speed is $-3 \times 10^8$ m/s.

**2.29** $-a(bx + ct)^2 = -ab^2(x + ct/b)^2 = g(x + vt)$ and so $v = c/b$ and the wave travels in the negative $x$-direction. Using Eq. (2.34) $(\partial\psi/\partial t)_x/(\partial\psi/\partial x)_t = -[A(-2a)(bx + ct)ce^{-a(bx+ct)^2}]/[A(-2a)(bx + ct)be{-}a(bx + ct)^2] = -c/b$; the minus sign tells us that the motion is in the negative $x$-direction.

**2.30** $\psi(z,0) = A\sin(kz + \epsilon)$; $\psi(-\lambda/12,0) = A\sin(-\pi/6 + \epsilon) = 0.866$;
$\psi(\lambda/6,0) = A\sin(\pi/3 + \epsilon) = 1/2$; $\psi(\lambda/4,0) = A\sin(\pi/2 + \epsilon) = 0$.
$A\sin(\pi/2 + \epsilon) = A(\sin\pi/2\cos\epsilon + \cos\pi/2\sin\epsilon) = A\cos\epsilon = 0$, $\epsilon = \pi/2$.
$A\sin(\pi/3 + \pi/2) = A\sin(5\pi/6) = 1/2$; therefore $A = 1$, hence
$\psi(z,0) = \sin(kz + \pi/2)$.

**2.31** Both (a) and (b) are waves since they are twice differentiable functions of $z - vt$ and $x + vt$, respectively. Thus for (a) $\psi = a^2(z - bt/a)^2$ and the velocity is $b/a$ in the positive $z$-direction. For (b) $\psi = a^2(x + bt/a + c/a)^2$ and the velocity is $b/a$ in the negative $x$-direction.

**6.13** For both, $-R_2 = R = R$, so (6.2) becomes

$$1/f = (n-1)[2/R + (n-1)d/nR^2];$$

$$1/f = (1.5-1)[2/50 + (1.5-1)(5.0)/1.5(50)^2]; \quad f = 49.2 \text{ cm.}$$

(6.1) $1/f_1 = 1/s_{o1} + 1/s_{i1}$, so $1/s_{i1} = 1/f_1 - 1/s_{o1} = 1/(49.2)$.

**6.14** (6.8) $1/f = 1/f_1 + 1/f_2 - d/f_1f_2 = 1/(+20) + 1/(-20) - 10/(20)(-20)$;
$f = +40$ cm. The principal planes are found from (6.9) and (6.10).
(6.9) $\overline{H_{11}H_1} = fd/f_2 = (+40)(10)/(-20) = -20$ cm.
(6.10) $\overline{H_{22}H_2} = fd/f_1 = (+40)(10)/(20) = +20$ cm.

**6.15** $\mathcal{R}_1 = \begin{bmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{bmatrix}$ from (6.16) where
$\mathcal{D}_1 = (n-1)/R_1 = (1.5-1)/2.5 \text{ cm} = 0.2 \text{ cm}^{-1}$.

$$\mathcal{T}_{21} = \begin{bmatrix} 1 & 0 \\ d_{21}/n & 1 \end{bmatrix} \quad (6.24)$$

$$= \begin{bmatrix} 1 & 0 \\ 1.2/1.5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix}$$

$$\mathcal{R}_2 = \begin{bmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{bmatrix}$$

but $R_2 = \infty$, so $\mathcal{D}_2 = 0$.

$$(6.29) \quad A = \mathcal{R}_2\mathcal{T}_{21}\mathcal{R}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.2 \\ 0.8 & 1.16 \end{bmatrix}$$

Check: $|A| = 1(1.16) - 0.2(0.8) = 1$.

**6.16** Working in centimeters,

$$\mathcal{D}_1 = (2.4-1.9)/R_1 = 0.1 \text{ cm}^{-1}, \quad \mathcal{D}_2 = (1.9-2.4)/R_2 = -0.05 \text{ cm}^{-1}$$

therefore

# Chapter 13 Solutions

**13.1** $T = 673$ K, area of each face is $A = 10^{-2}$ m$^2$, $\sigma = 5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$, then $0.97AI_e = 0.97A\sigma T^4 = 110$ W.

**13.2** $0.97I_e = 0.97\sigma(T^4 - T_E^4) = 76.9$ W/m$^2$ with $T = 306$ K and $T_e = 293$ K is the temperature of the environment. Then $0.97AI_e = 108$ W for the radiated power.

**13.3** $I_e = 22.8 \times 10^4$ W/m$^2$, $T = (I_e/\sigma)^{1/4} = 1420$ K.

**13.4** $E \sim T^4$, so the energy radiated increases by a factor of $10^4$.

**13.5** $T = 306$ K, $\lambda_{\max} = 2.8978 \times 10^{-3}$ m$K/T = 9.45 \times 10^{-6}$ m $= 9.5\mu$ m (in the infrared).

**13.6** If the blackbody is at $T = 293$ K, then
$\lambda_{\max} = 2.8978 \times 10^{-3}$ m K/T $= 9.9\mu$ m (in the IR).

**13.7** $T = 4.0 \times 10^4$ K, $\nu_{\max} = c/\lambda_{\max} = cT/2.8978 \times 10^{-3}$ m K $= 4.1 \times 10^{15}$ Hz (in the UV).

**13.8** $T = 2.8978 \times 10^{-3}$ m K$/\lambda_{\max} = 2.8978 \times 10^{-3}$ mK$/4.65 \times 10^{-7}$ m $= 6230$ K.

**13.9** $T = 2.8978 \times 10^{-3}$ m K$/\lambda_{\max} = 4300$ K.

**13.10** We have for the total radiated power per unit area of the blackbody

$$P(T) = \int_0^\infty I_\lambda d\lambda = 2\pi hc^2 \int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda k_B T} - 1)}$$
$$= 2\pi hc^2 \left(\frac{k_B T}{hc}\right)^4 \int_0^\infty \frac{x^3 dx}{(e^x - 1)},$$