Optical Networks: A Practical Perspective
Third Edition

Solutions Manual for Instructors

Rajiv Ramaswami, **Kumar N. Sivarajan** and **Galen Sasaki**

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Preface

In many cases, the problems in the book require further exploration of the topics in detail as opposed to simply plugging numbers into equations. Instructors may therefore want to review the solutions before assigning problems. See the book's web page *http://www.elsevierdirect.com/9780123740922* for the current errata of the book, as well as this solutions manual. If you discover an error that is not listed there, we would very much appreciate your letting us know about it. You can email *rajivramaswami@ieee.org*, or *kumar@tejasnetworks.com* or *galens@hawaii.edu*.

Note that all equation and figure numbers used in this manual refer to those in the third edition of the book.

Therefore,

$$T^{rms} = \sqrt{\frac{T_0^2}{2}} = \frac{T_0}{\sqrt{2}}.$$

2.11 From (2.13),

$$\frac{|T_z|}{T_0} = \sqrt{\left(1 + \frac{\kappa\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2}.$$

For positive κ and negative β_2,

$$\frac{|T_z|}{T_0} = \sqrt{\left(1 - \frac{\kappa z}{L_D}\right)^2 + \left(\frac{z}{L_D}\right)^2}.$$

(a) Differentiating the equation above, the minimum pulse width occurs for $z = z_{\min}$ which solves

$$-\kappa\left(1 - \frac{\kappa z}{L_D}\right) + \frac{z}{L_D} = 0.$$

This yields

$$z_{\min} = \frac{\kappa}{1+\kappa^2} L_D.$$

For $\kappa = 5$,

$$z_{\min} = \frac{5}{26} L_D = 0.192 L_D.$$

(b) The pulse width equals that of an unchirped pulse if

$$\left(1 - \frac{\kappa z}{L_D}\right)^2 + \left(\frac{z}{L_D}\right)^2 = 1 + \left(\frac{z}{L_D}\right)^2,$$

that is, if

$$z = \frac{2L_D}{\kappa}.$$

For $\kappa = 5$, we get $z = 0.4L_D$.

2.12 We leave this to the reader to go through the algebra and verify.

2.13 For a first order soliton,

$$N^2 = \frac{\gamma P_0}{|\beta_2|/T_0^2} = 1.$$

Using $\gamma = 1$ /W-km, $\beta_2 = 2$ ps^2/km, and $P_0 = 50$ mW,

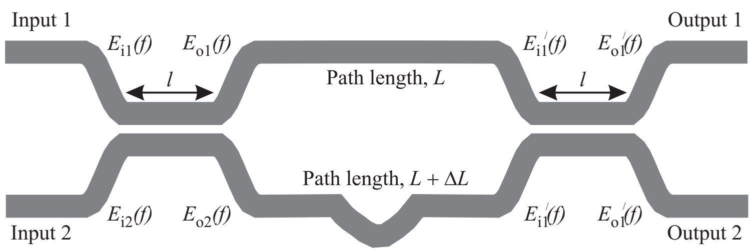
$$T_0 = \sqrt{\frac{|\beta_2|}{\gamma P_0}} = 6.32 \text{ ps}.$$

Recall that a soliton pulse is described by

$$\operatorname{sech}\left(\frac{t}{T_0}\right) = \frac{2}{e^{-t/T_0} + e^{t/T_0}}$$

refractive index dielectrics which are a quarter-wavelength thick at λ_0, acts as a highly reflective mirror at λ_0.

3.14



Since the directional couplers are 3-dB couplers, from (3.1), with $\kappa l = (2k+1)\pi/4$, for some integer k,

$$\begin{pmatrix} E'_{o1}(f) \\ E'_{o2}(f) \end{pmatrix} = \frac{e^{-i\beta l}}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E_{i1}(f) \\ E_{i2}(f) \end{pmatrix}.$$

$$\begin{pmatrix} E'_{i1}(f) \\ E'_{i2}(f) \end{pmatrix} = e^{-i\beta L} \begin{pmatrix} 1 & \\ & e^{-i\beta\Delta L} \end{pmatrix} \begin{pmatrix} E'_{o1}(f) \\ E'_{o2}(f) \end{pmatrix}.$$

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = \frac{e^{-i\beta l}}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E'_{i1}(f) \\ E'_{i2}(f) \end{pmatrix}.$$

Muliplying the above transfer functions,

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = \frac{e^{-2i\beta l}}{2} \begin{pmatrix} 1 - e^{-i\beta\Delta L} & i + ie^{-i\beta\Delta L} \\ i + ie^{-i\beta\Delta L} & -1 + e^{-i\beta\Delta L} \end{pmatrix} \begin{pmatrix} E_{i1}(f) \\ E_{i2}(f) \end{pmatrix}.$$

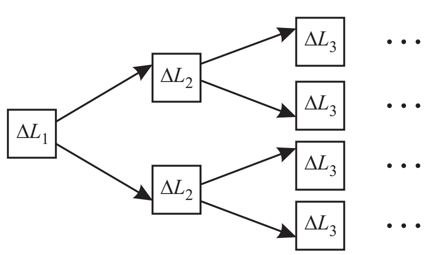
If only one input, say input 1, is active, then $E_{i2}(f) = 0$ and

$$\begin{pmatrix} E_{o1}(f) \\ E_{o2}(f) \end{pmatrix} = \frac{e^{-2i\beta l}}{2} \begin{pmatrix} 1 - e^{-i\beta\Delta L} \\ i + ie^{-i\beta\Delta L} \end{pmatrix} E_{i1}(f).$$

The power transfer function is

$$\begin{aligned} \begin{pmatrix} |E_{o1}(f)|^2/|E_{i1}(f)|^2 \\ |E_{o2}(f)|^2/|E_{i1}(f)|^2 \end{pmatrix} &= \frac{1}{4} \begin{pmatrix} (1-\cos\beta\Delta L)^2 + \sin^2\beta\Delta L \\ (1+\cos\beta\Delta L)^2 + \sin^2\beta\Delta L \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1 - \cos\beta\Delta L \\ 1 + \cos\beta\Delta L \end{pmatrix} = \begin{pmatrix} \sin^2\beta\Delta L/2 \\ \cos^2\beta\Delta L/2 \end{pmatrix}. \end{aligned}$$

3.15 (a)



5.23

$$\mathrm{PP}(dB) = \alpha \frac{\Delta\tau^2}{T^2}\epsilon(1-\epsilon).$$

Denote the probability density function of $\Delta\tau$ by $f_{\Delta\tau}(.)$ and its (cumulative) distribution function by $F_{\Delta\tau}(.)$.

$$\begin{aligned}
\Pr(\mathrm{PP} < p) &= \Pr\left(\Delta\tau^2\epsilon(1-\epsilon) \leq x = T^2 p/\alpha\right) \\
&= \int_{t=0}^{\infty} \Pr\left(\epsilon(1-\epsilon) \leq x/\Delta\tau^2 | \Delta\tau = t\right) f_{\Delta\tau}(t)\, dt \\
&= \int_{t=0}^{\infty} \Pr\left(\left(\epsilon - 0.5 + \sqrt{0.25 - x/t^2}\right)\left(\epsilon - 0.5 - \sqrt{0.25 - x/t^2}\right) > 0\right) f_{\Delta\tau}(t)\, dt
\end{aligned}$$

$$\begin{aligned}
\Pr\left(\left(\epsilon - 0.5 + \sqrt{0.25 - x/t^2}\right)\left(\epsilon - 0.5 - \sqrt{0.25 - x/t^2}\right) > 0\right) &= 1, \text{ for } t^2 < 4x, \\
&= 1 - \sqrt{1 - 4x/t^2}, \text{ for } t^2 \geq 4x.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Pr\left(\mathrm{PP} < p = x\alpha/T^2\right) &= F_{\Delta\tau}\left(2\sqrt{x}\right) + \int_{t=2\sqrt{x}}^{\infty} \left(1 - \sqrt{1 - 4x/t^2}\right) f_{\Delta\tau}(t)\, dt \\
&= 1 - \int_{t=2\sqrt{x}}^{\infty} \sqrt{1 - 4x/t^2}\, f_{\Delta\tau}(t)\, dt.
\end{aligned}$$

Using (see Appendix H.1.2),

$$f_{\Delta\tau}(x) = \frac{\sqrt{2}}{a^3\sqrt{\pi}} x^2 e^{-x^2/2a^2}, \qquad x \geq 0,$$

and the relation (use a symbolic integration package such as Mathematica™ or see a table of integrals),

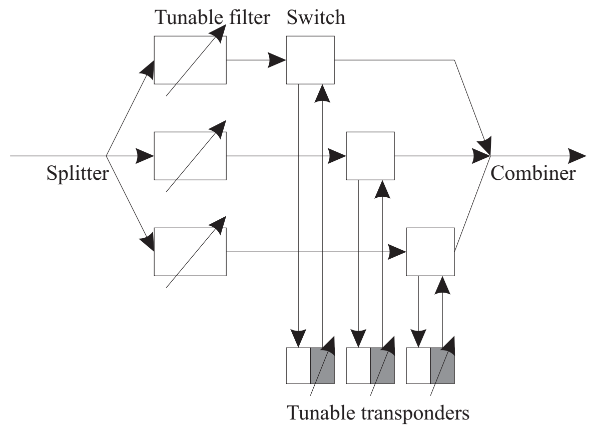
$$\int_{t=y}^{\infty} \frac{\sqrt{t^2 - y^2}}{t} f_{\Delta\tau}(t)\, dt = e^{-y^2/2a^2},$$

we get,

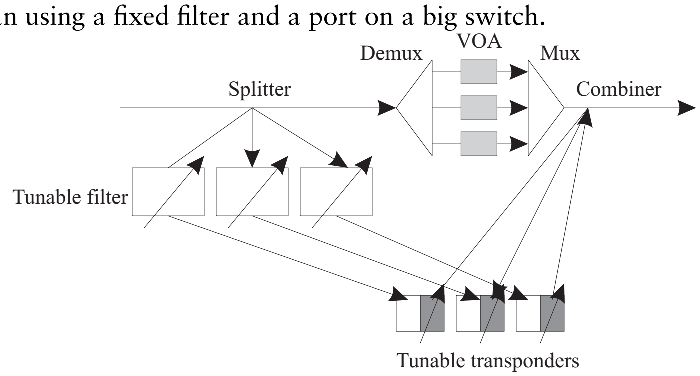
$$\Pr\left(\mathrm{PP} < p = x\alpha/T^2\right) = 1 - e^{-4x/2a^2} = 1 - e^{-4pT^2/2\alpha a^2}.$$

Therefore, PP is exponentially distributed with mean $\alpha a^2/2T^2$. Using $\langle\Delta\tau\rangle = 2a\sqrt{2/\pi}$ (Appendix H.1.2) or $a = \langle\Delta\tau\rangle\sqrt{\pi/8}$, the mean of PP is $\pi\alpha\langle\Delta\tau\rangle^2/16T^2$.

$$\Pr(\mathrm{PP} \geq 1) = e^{-\frac{16T^2}{\pi\alpha\langle\Delta\tau\rangle^2}}.$$



The main difference between this architecture and that of Figure 7.7(d) lies in the use of tunable filters and small switches instead of a mux/demux and a big switch. Since splitters and combiners are used, there is a minimum passthrough loss of $20 \log W$, where W is the number of channels. So a 32-channel OADM will have a minimum passthrough loss of 32 dB, which is quite high. Also now a tunable filter is required for each wavelength, which may or may not be more expensive than using a fixed filter and a port on a big switch.



Another plausible OADM architecture is shown above. Here a wavelength blocker device (a demux/mux combination with a per-channel variable optical attentuator) is used to either block the add/drop channels from passing through as well as equalize power levels for the passthrough channels. The loss in the passthrough path is low, but the loss in the add/drop path is high due to the splitters and combiners. However, tunable filters need be provided only for drop channels and not for all channels.

7.5 Each remote node drops and adds 2 wavelengths and 8 wavelengths are needed in total. Hub drops and adds all wavelengths.

System 1: Remote node needs 1 OADM and 2 regenerators for a cost of \$40,000. Hub node requires 2 OADMs for a cost of \$40,000, so total network cost is \$200,000.

System 2: Remote node needs 2 OADMs for a cost of \$20,000. Hub node needs 8 OADMs at \$80,000. Total network cost including amplifiers is \$220,000.

7.6 (a) For each WDM system, we require 24 line ports on the OXC and 16 trib ports, or 40 ports. Therefore a 256-port OXC can support 6 WDM systems.

(b) Out of the 24 lightpaths passing through, 6 of them need to be converted, taking up a

Lightpath	Wavelength
AB	1
AB	2
AC	3
AC	4
AC	5
AD	1
AE	2
AE	3
BC	1
BD	1
BD	2
BD	3
BD	4
BE	5
BE	6
CD	1
CD	2
CE	3
CE	4
CE	7
DE	2

(c) The most heavily loaded link is DE, with a total load of 7, which is also equal to the number of wavelengths.