

**Solutions Manual**

for

**Microwave Engineering**  
**4<sup>th</sup> edition**

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## Chapter 1

1.1

This is an open-ended question where the focus of the answer may be largely chosen by the student or the instructor. Some of the relevant historical developments related to the early days of radio are listed here (as cited from T. S. Sarkar, R. J. Mailloux, A. A. Oliner, M. Salazar-Palma, and D. Sengupta, *History of Wireless*, Wiley, N.J., 2006):

1865: James Clerk Maxwell published his work on the unification of electric and magnetic phenomenon, including the introduction of the displacement current and the theoretical prediction of EM wave propagation.

1872: Mahlon Loomis, a dentist, was issued US Patent 129,971 for “aerial telegraphy by employing an ‘aerial’ used to radiate or receive pulsations caused by producing a disturbance in the electrical equilibrium of the atmosphere”. This sounds a lot like radio, but in fact Loomis was not using an RF source, instead relying on static electricity in the atmosphere. Strictly speaking this method does not involve a propagating EM wave. It was not a practical system.

1887-1888: Heinrich Hertz studied Maxwell’s equations and experimentally verified EM wave propagation using spark gap sources with dipole and loop antennas.

1893: Nikola Tesla demonstrated a wireless system with tuned circuits in the transmitter and receiver, with a spark gap source.

1895: Marconi transmitted and received a coded message over a distance of 1.75 miles in Italy.

1894: Oliver Lodge demonstrated wireless transmission of Morse code over a distance of 60 m, using coupled induction coils. This method relied on the inductive coupling between the two coils, and did not involve a propagating EM wave.

1897: Marconi was issued a British Patent 12,039 for wireless telegraphy.

1901: Marconi achieved the first trans-Atlantic wireless transmission.

1943: The US Supreme Court invalidated Marconi’s 1904 US patent on tuning using resonant circuits as being superseded by prior art of Tesla, Lodge, and Braun.

So it is clear that many workers contributed to the development of wireless technology during this time period, and that Marconi was not the first to develop a wireless system that relied on the propagation of electromagnetic waves. On the other hand, Marconi was very successful at making radio practical and commercially viable, for both shipping and land-based services.

1.2

$$E_y = E_0 \cos(\omega t - kx), \quad E_0 = 5 \text{ V/m}, \quad f = 2.4 \text{ GHz}.$$

$$\epsilon_r = 2.54, \quad \chi_1 = 0.1, \quad \chi_2 = 0.15$$

a)  $\eta = \eta_0 / \sqrt{\epsilon_r} = 236.6 \Omega$

$$H_z = E_y / \eta = 0.0211 \cos(\omega t - kx)$$

b)  $v_p = c / \sqrt{\epsilon_r} = 1.88 \times 10^8 \text{ m/sec}$

c)  $\lambda = v_p / f = 0.0784 \text{ m}, \quad k = 2\pi / \lambda = 80.11 \text{ m}^{-1}$

d)  $\Delta\phi = k(\chi_2 - \chi_1) = 80.11(0.15 - 0.1) = 4.00 \text{ rad} = 229.5^\circ$

1.3

$$\bar{E} = E_0 (a\hat{x} + b\hat{y}) e^{-jk_0 z} \quad ; \quad a, b \text{ real}$$

$$\text{Let } \bar{E} = A(\hat{x} - j\hat{y}) e^{-jk_0 z} + B(\hat{x} + j\hat{y}) e^{-jk_0 z}$$

where  $A, B$  are the amplitudes of the RHCP and LHCP components. Equating vector components gives

$$\hat{x}: A + B = aE_0$$

$$\hat{y}: -jA + jB = bE_0, \quad \text{or } A - B = j b E_0$$

so

$$A = E_0(a + jb)/2$$

$$B = E_0(a - jb)/2$$

check: if  $a=1, b=2$  then  $A = (\frac{1}{2} + j)E_0, B = (\frac{1}{2} - j)E_0$

(agrees with Problem 1.5 from 3rd ed.)

**3.15** The solution is similar to the TE mode case for the coax, but with  $E_z$  in place of  $H_z$ :

$$E_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) [C J_n(k_c \rho) + D Y_n(k_c \rho)]$$

Then the boundary condition that  $E_z = 0$  at  $\rho = a$  and at  $\rho = b$  yields two equations:

$$C J_n(k_c a) + D Y_n(k_c a) = 0$$

$$C J_n(k_c b) + D Y_n(k_c b) = 0$$

or,

$$J_n(k_c a) Y_n(k_c b) = J_n(k_c b) Y_n(k_c a)$$

For the  $TM_{01}$  mode,  $n=0$ . Let  $x = k_c a$ . Then for  $b=2a$ , we have that  $k_c b = 2x$ , and so the above equation can be written as,

$$f(x) = J_0(x) Y_0(2x) - J_0(2x) Y_0(x) = 0$$

We know that  $k_c$  should be greater than  $k_c$  for a circular waveguide of radius  $b$ , for which  $k_{c01} = P_{01}/b = 2.405/2a$ , which implies that  $x = 1.2$ . So we can begin the root search at  $x=1.2$ . Using a table of Bessel functions gives the following results in only a few minutes:

$x$	$J_0(x)$	$Y_0(x)$	$J_0(2x)$	$Y_0(2x)$	$f(x)$
1.2	.671	.228	.003	.510	.342
1.5	.512	.382	-.260	.377	.292
2.0	.224	.510	-.397	-.017	.198
3.1	-.292	.343	.202	-.248	.003
3.2	-.320	.307	.243	-.200	-.011

Linear interpolation between  $x=3.1$  and  $3.2$  gives a more accurate value for the root:

$$f(x) \approx .003 + \frac{.003 - (-0.011)}{3.1 - 3.2} (x - 3.1)$$

$$\approx .437 - .14x = 0$$

$$x = \frac{.437}{.14} = 3.12 = k_c a$$

**5.10** Analytic Solution: let  $t = \tan \beta d = \tan 135^\circ = -1.0$

From (5.22) the first stub susceptance is

$$b_1 = -b_L + \frac{1 \pm \sqrt{(1+t^2)g_L - g_L^2 t^2}}{t} = -3 \text{ or } -1.4 \checkmark$$

From (5.23) the second stub susceptance is

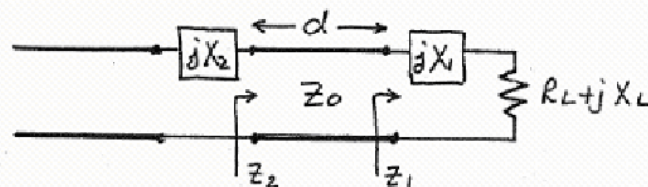
$$b_2 = \frac{\pm \sqrt{(1+t^2)g_L - g_L^2 t^2} + g_L}{g_L t} = -3 \text{ or } 1.0 \checkmark$$

The S.C. stub lengths are, from (5.24b),

$$l_1 = 0.051\lambda \text{ or } 0.0987\lambda$$

$$l_2 = 0.051\lambda \text{ or } 0.375\lambda$$

**5.11**



$$Z_1 = R_L + j(X_L + X_1)$$

$$Z_2 = Z_0 \frac{R_L + j(X_L + X_1 + Z_0 t)}{Z_0 + jt(R_L + jX_L + jX_1)} = Z_0 \quad t = \tan \beta d$$

Solving for  $R_L$ :

$$R_L = Z_0 \frac{1+t^2}{2t^2} \left[ 1 \pm \sqrt{\frac{1-4t^2(Z_0 - X_L t - X_1 t)^2}{Z_0(1+t^2)^2}} \right]$$

So we must have,

$$0 \leq R_L \leq Z_0 \frac{1+t^2}{2t^2} = \frac{Z_0}{\sin^2 \beta d}$$

The first stub reactance is,

$$X_1 = -X_L + \frac{Z_0 \pm \sqrt{(1+t^2)R_L Z_0 - R_L^2 t^2}}{t}$$

The second stub reactance is,

$$X_2 = \frac{\pm Z_0 \sqrt{Z_0 R_L (1+t^2) - R_L^2 t^2} + R_L Z_0}{R_L t}$$

The stub lengths are given by,

$$l_{oc} = \frac{1}{2\pi} \tan^{-1} \left( \frac{Z_0}{X} \right), \quad l_{sc} = \frac{1}{2\pi} \tan^{-1} \left( \frac{X}{Z_0} \right)$$

**8.18**  $f_0 = 2.45 \text{ GHz}$ ,  $BW = 10\%$ , EQUAL-RIPPLE (0.5 dB),  $N = 3$ ,  $Z_0 = 50 \Omega$

a) Use (8.71) to transform  $2.1 \text{ GHz}$  to normalized L.P. form:

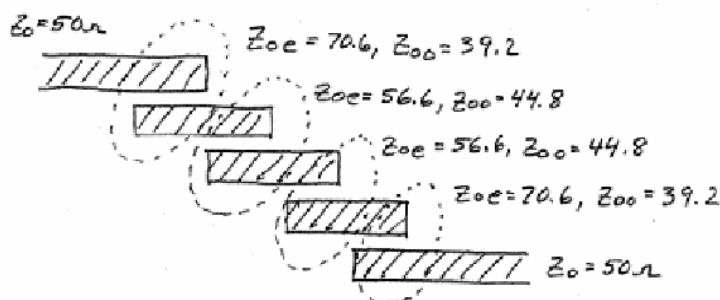
$$\omega \leftarrow \frac{1}{\Delta} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{0.1} \left( \frac{2.1}{2.45} - \frac{2.45}{2.1} \right) = -3.09$$

Then,  $\left| \frac{\omega}{\omega_c} \right| - 1 = 2.09 \Rightarrow \text{Fig 8.27 gives } \alpha \approx 30 \text{ dB}$

The L.P. prototype values are given in Table 8.4

$Z_0 J_n$  is found from (8.121), and  $Z_{0e}, Z_{0o}$  from (8.108):

$n$	$g_n$	$Z_0 J_n$	$Z_{0e}(\Omega)$	$Z_{0o}(\Omega)$
1	1.5963	0.3137	70.6	39.2
2	1.0967	0.1187	56.6	44.8
3	1.5963	0.1187	56.6	44.8
4	1.0000	0.3137	70.6	39.2



all lines are  $\lambda/4$  long at  $2.45 \text{ GHz}$ .

b)  $d = 0.158 \text{ cm}$ ,  $\epsilon_r = 4.2$ ,  $\tan \delta = 0.01$ , Cu,  $t = 0.5 \text{ mil}$

From Serenade,

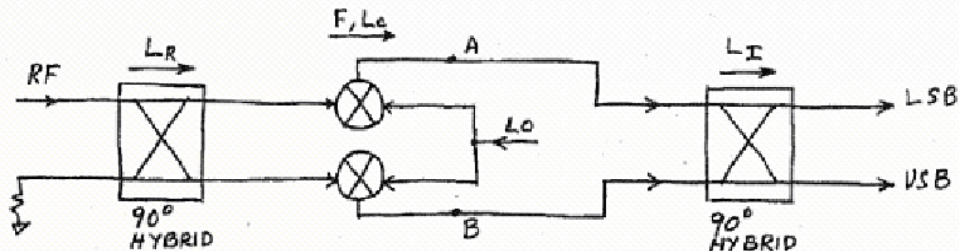
$$Z_{0e} = 70.6, Z_{0o} = 39.2 \Rightarrow W = 2.484 \text{ mm}, S = 0.415 \text{ mm}, l = 1.74 \text{ cm}$$

$$Z_{0e} = 56.6, Z_{0o} = 44.8 \Rightarrow W = 3.026 \text{ mm}, S = 1.723 \text{ mm}, l = 1.71 \text{ cm}$$

The calculated response, from Serenade, is shown on the following page.



13.23



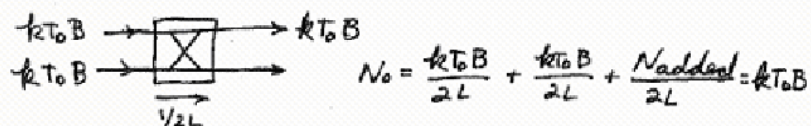
The noise power due to the RF hybrid and the mixer, ref. to IF output of mixer, is

$$N_A = N_B = \frac{kB}{L_c} [T_0 + (F-1)T_0] = \frac{kBT_0F}{L_c},$$

since the noise power output of the matched hybrid is  $kT_0B$ . The total noise power output is (at either LSB or USB),

$$N_o = \frac{N_A}{2L_I} + \frac{N_B}{2L_I} + \frac{N_{added}}{2L_I} = \frac{kBT_0F}{L_I L_c} + \frac{N_{added}}{2L_I}$$

$N_{added}$  is the output noise power of the IF hybrid when not terminated at second input port:



$$\text{Thus } N_{added} = 2kT_0B(L-1)$$

$$\text{So, } N_o = \frac{kBT_0F}{L_I L_c} + kT_0B\left(1 - \frac{1}{L_I}\right) \quad ; \quad S_o = \frac{4S_i}{L_c} \frac{1}{4L_I L_R} = \frac{S_i}{L_c L_I L_R}$$

$$N_i = kT_0B$$

And then,

$$F_{TOT} = \frac{S_i N_o}{S_o N_i} = \frac{L_c L_I L_R}{kT_0B} \left[ \frac{kBT_0F}{L_I L_c} + kT_0B\left(1 - \frac{1}{L_I}\right) \right] = \frac{FL_R + L_c L_R L_I - L_c L_R}{L_c L_I L_R}$$

CHECK: if  $L_R = L_I = 1$ ,  $F_{TOT} = F + 2L_c - 2L_c = F$  ✓ (mixer noise only)

CHECK: if  $F = L_c$  (passive mixer loss only),  $F_{TOT} = L_c L_I L_R$  ✓

(The cascade noise figure formula can be used to obtain the same result if we set  $F_R = L_R$ ,  $F_I = L_I$ .)