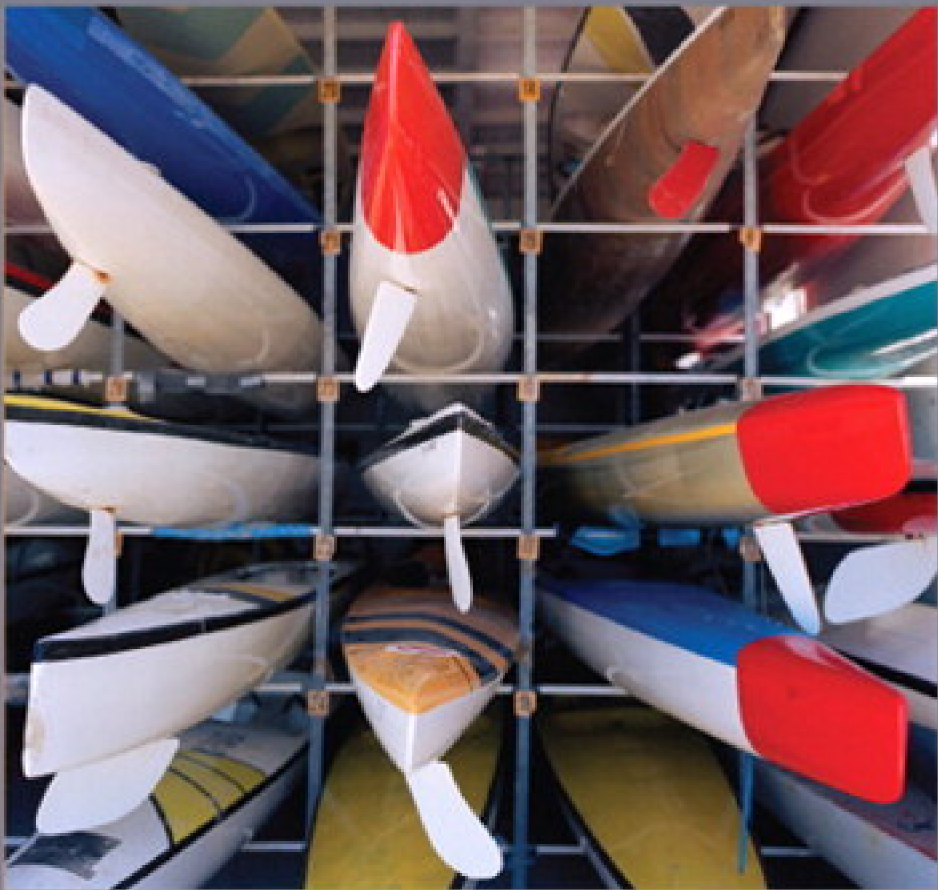
Linear Algebra

and its applications

FOURTH EDITION



INSTRUCTOR SOLUTIONS MANUAL

David C. Lay

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LINEAR ALGEBRA AND ITS APPLICATIONS
FOURTH EDITION

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b. False. See Theorem 6(b).

c. False. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then $ab - cd = 1 - 0 \neq 0$, but Theorem 4 shows that this matrix is not invertible, because $ad - bc = 0$.

d. True. This follows from Theorem 5, which also says that the solution of $A\mathbf{x} = \mathbf{b}$ is unique, for each $\mathbf{b}$.

e. True, by the box just before Example 6.

10. **a**. False. The last part of Theorem 7 is misstated here.

b. True, by Theorem 6(a).

c. False. The product matrix is invertible, but the product of inverses should be in the *reverse* order. See Theorem 6(b).

d. True. See the subsection "Another View of Matrix Inversion".

e. True, by Theorem 7.

11. (The proof can be modeled after the proof of Theorem 5.) The $n \times p$ matrix B is given (but is arbitrary). Since A is invertible, the matrix $A^{-1}B$ satisfies $AX = B$, because $A(A^{-1}B) = A\,A^{-1}B = IB = B$. To show this solution is unique, let X be any solution of $AX = B$. Then, left-multiplication of each side by A^{-1} shows that X must be $A^{-1}B$:

$$A^{-1}(AX) = A^{-1}B, \quad IX = A^{-1}B, \quad \text{and} \quad X = A^{-1}B.$$

12. Left-multiply each side of the equation $AD = I$ by A^{-1} to obtain

$$A^{-1}AD = A^{-1}I,\ ID = A^{-1}, \text{ and } D = A^{-1}.$$

Parentheses are routinely suppressed because of the associative property of matrix multiplication.

13. Left-multiply each side of the equation $AB = AC$ by A^{-1} to obtain

$$A^{-1}AB = A^{-1}AC, \quad IB = IC, \quad \text{and} \quad B = C.$$

This conclusion does not always follow when A is singular. Exercise 10 of Section 2.1 provides a counterexample.

14. Right-multiply each side of the equation $(B - C)D = 0$ by D^{-1} to obtain

$$(B - C)DD^{-1} = 0D^{-1}, \quad (B - C)I = 0, \quad B - C = 0, \quad \text{and} \quad B = C.$$

15. If you assign this exercise, consider giving the following *Hint*: Use elementary matrices and imitate the proof of Theorem 7. The solution in the Instructor's Edition follows this hint. Here is another solution, based on the idea at the end of Section 2.2.

Write $B = [\mathbf{b}_1 \ \cdots \ \mathbf{b}_p]$ and $X = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_p]$. By definition of matrix multiplication, $AX = [A\mathbf{u}_1 \ \cdots \ A\mathbf{u}_p]$. Thus, the equation $AX = B$ is equivalent to the p systems:

$$A\mathbf{u}_1 = \mathbf{b}_1, \ \ldots\ A\mathbf{u}_p = \mathbf{b}_p$$

Since A is the coefficient matrix in each system, these systems may be solved simultaneously, placing the augmented columns of these systems next to A to form $[A \ \ \mathbf{b}_1 \ \cdots \ \mathbf{b}_p] = [A \ \ B]$. Since A is invertible, the solutions $\mathbf{u}_1, \ldots, \mathbf{u}_p$ are uniquely determined, and $[A \ \ \mathbf{b}_1 \ \cdots \ \mathbf{b}_p]$ must row reduce to $[I \ \ \mathbf{u}_1 \ \cdots \ \mathbf{u}_p] = [I \ \ X]$. By Exercise 11, X is the unique solution $A^{-1}B$ of $AX = B$.

$$\det D = 3^{n-2}\begin{vmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2 & 2 & \dots & 2 \\ 1 & 2 & 3 & 3 & \dots & 3 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & 4 & \dots & n-2 \end{vmatrix} = 3^{n-2}(1) = 3^{n-2}$$

by Exercise 19. Thus the determinant of the matrix $\begin{bmatrix} A & B \\ O & D \end{bmatrix}$ is $2\det D = 2 \cdot 3^{n-2}$.

From $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ construct $P = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$. Then set $D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, where the eigenvalues in D correspond to $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ respectively.

19. Since A is triangular, its eigenvalues are 2, 3, and 5.

For $\lambda = 2$: $A - 2I = \begin{bmatrix} 3 & -3 & 0 & 9 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, and row reducing $\begin{bmatrix} A - 2I & \mathbf{0} \end{bmatrix}$ yields $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

The general solution is $x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, and a nice basis for the eigenspace is

$$\{\mathbf{v}_1, \mathbf{v}_2\} = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

For $\lambda = 3$: $A - 3I = \begin{bmatrix} 2 & -3 & 0 & 9 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$, and row reducing $\begin{bmatrix} A - 3I & \mathbf{0} \end{bmatrix}$ yields

$\begin{bmatrix} 1 & -3/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. The general solution is $x_2 \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, and a nice basis for the eigenspace is

$$\mathbf{v}_3 = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}.$$

For $\lambda = 5$: $A - 5I = \begin{bmatrix} 0 & -3 & 0 & 9 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$, and row reducing $\begin{bmatrix} A - 5I & \mathbf{0} \end{bmatrix}$ yields $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

The general solution is $x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and a basis for the eigenspace is $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

6.8 SOLUTIONS

Notes: The connections between this section and Section 6.7 are described in the notes for that section. For my junior-senior class, I spend three days on the following topics: Theorems 13 and 15 in Section 6.5, plus Examples 1, 3, and 5; Example 1 in Section 6.6; Examples 2 and 3 in Section 6.7, with the motivation for the definite integral; and Fourier series in Section 6.8.

1. The weighting matrix W, design matrix X, parameter vector β, and observation vector $\mathbf{y}$ are:

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 4 \\ 4 \end{bmatrix}$$

The design matrix X and the observation vector $\mathbf{y}$ are scaled by W:

$$WX = \begin{bmatrix} 1 & -2 \\ 2 & -2 \\ 2 & 0 \\ 2 & 2 \\ 1 & 2 \end{bmatrix}, W\mathbf{y} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 8 \\ 4 \end{bmatrix}$$

Further compute

$$(WX)^T WX = \begin{bmatrix} 14 & 0 \\ 0 & 16 \end{bmatrix}, (WX)^T W\mathbf{y} = \begin{bmatrix} 28 \\ 24 \end{bmatrix}$$

and find that

$$\hat{\boldsymbol{\beta}} = ((WX)^T WX)^{-1}(WX)^T W\mathbf{y} = \begin{bmatrix} 1/14 & 0 \\ 0 & 1/16 \end{bmatrix}\begin{bmatrix} 28 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ 3/2 \end{bmatrix}$$

Thus the weighted least-squares line is $y = 2 + (3/2)x$.

2. Let X be the original design matrix, and let $\mathbf{y}$ be the original observation vector. Let W be the weighting matrix for the first method. Then $2W$ is the weighting matrix for the second method. The weighted least-squares by the first method is equivalent to the ordinary least-squares for an equation whose normal equation is

$$(WX)^T WX\hat{\boldsymbol{\beta}} = (WX)^T W\mathbf{y} \tag{1}$$

while the second method is equivalent to the ordinary least-squares for an equation whose normal equation is

$$(2WX)^T(2W)X\hat{\boldsymbol{\beta}} = (2WX)^T(2W)\mathbf{y} \tag{2}$$

Since equation (2) can be written as $4(WX)^T WX\hat{\boldsymbol{\beta}} = 4(WX)^T W\mathbf{y}$, it has the same solutions as equation (1).