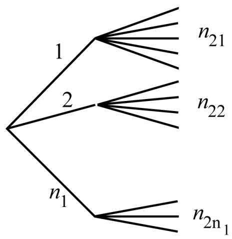
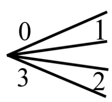
Chapter 1

1.1



(a) $\sum_{i=1}^{n_1} n_{2i}$

(b)



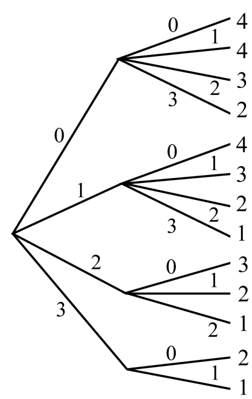
0 1 2 3
0 1 2 3
0 1 2
0 1

$\sum = 13$

1.2 $\sum_{i=1}^{n_1} n_2 i = \sum_{i=1}^{n_1} n_2 = n_1 n_2$

1.3 (a)

$n_{300} = 4$ $n_{320} = 3$
$n_{301} = 4$ $n_{321} = 2$
$n_{302} = 3$ $n_{322} = 1$
$n_{303} = 2$ $n_{330} = 2$
$n_{310} = 4$ $n_{331} = 1$
$n_{311} = 3$
$n_{312} = 2$
$n_{313} = 1$



(b) $\sum = 4 + 4 + 3 + \ldots + 2 + 1 = 32$

1.4 $\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} n_2 = n_1 n_2 n_3$

Chapter 3

3.1 **(a)** No, because $f(4)$ is negative; **(b)** Yes; **(c)** No, because $f(1) + f(2) + f(3) + f(4) = \frac{18}{19}$ is less than 1.

3.2 **(a)** No, because $f(1)$ is negative; **(b)** Yes; **(c)** No, because $f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$ is greater than 1.

3.3 $f(x) > 0$ for each value of x and

$$\sum_{x=1}^{k} f(x) = \frac{2}{k(k+1)}(1+2+\ldots+k) = \frac{2}{k(k+1)} \cdot \frac{k(k+1)}{2} = 1$$

3.4 **(a)** $c(1+2+3+\ldots 5) = 1$; thus $C = \frac{1}{15}$

(b) $c\left(5+\frac{5}{2}+\frac{5}{3}+\frac{5}{4}+1\right) = 1$**; thus,** $c = \frac{12}{137}$

(c) $\sum_{x=1}^{k} f(x) = c\sum_{x=1}^{k} x^2 = cS(k,2)$

From Theorem A.1 we obtain $S(k,2) = \frac{1}{6}k(k+1)(2k+1)$

Thus, for $f(x)$ to be a distribution function, $c = \frac{6}{k(k+1)(2k+1)}$, $k \neq 0$.

(d) $\sum_{x=1}^{\infty} f(x) = c\sum_{x=1}^{\infty}\left(\frac{1}{4}\right)^x$

The right-hand sum is a geometric progression with $a = 1$ and $r = 1/4$.
For $x = 1$ to n, this sum equals

$$S_n = \frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}} \to \frac{1/4}{3/4} = \frac{1}{3} \quad \text{as } n \to \infty.$$ Therefore, $c = 3$.

3.5 For $f(x) = (1-k)k^x$ to converge to 1, $0 < k < 1$.

3.6 For $c > 0$, $f(x)$ diverges. For $c = 0$, $f(x) = 0$ for all x, and it cannot be a density function

3.9 **(a)** No, because $F(4) > 1$; **(b)** No, because $F(2) < F(1)$; **(c)** Yes.

Chapter 7

7.1 $G(y) = P(Y \le y) = P(\ln X \le y) = P(X \le e^y)$

$$= \int_0^{e^y} \frac{1}{8} e^{-x/\theta} dx = -e^{-x/\theta} \Big|_0^{e^y} = 1 - e^{-(1/\theta)e^y}$$

$$g(y) = \frac{1}{8} e^y e^{-(1/\theta)e^y} \text{ for } -\infty < y < \infty$$

7.2 $G(y) = P(Y \le y) = P(X^2 \le y) = P(X \le \sqrt{y})$

$$= \int_0^{\sqrt{y}} 2xe^{-x^2} dx \qquad\qquad u = x^2 \qquad du = 2x\ dx$$

$$= \int_0^{y} e^{-u} du = -e^{-u} \Big|_0^{y} = 1 - e^{-y}$$

(a) $G(y) = \begin{cases} 1 - e^{-y} & y > 0 \\ 0 & \text{elsewhere} \end{cases}$

(b) $g(y) = \dfrac{dG(y)}{dy} = e^{-y}$ for $y > 0$ and 0 elsewhere

7.3 $G(y) = P(Y \le y) = P(\sqrt{X} \le y) = P(X \le y^2)$

$$= \int_0^{y^2} dx = y^2 \text{ for } 0 < y < 1$$

$g(y) = 2y$ for $0 < y < 1$ and 0 elsewhere

7.4 $G(z) = P(Z \le z) = P(X^2 + Y^2 + z^2)$

$$= \int_0^{z} \int_0^{\sqrt{z^2 - y^2}} 4xye^{-(x^2+y^2)} dx\ dy \qquad\qquad \text{let } u = x^2 \text{ and } v = y^2$$

$= 1 - (1 + z^2)e^{-z^2}$ for $z > 0$ and $G(z) = 0$ elsewhere

$g(z) = -(1 + z^2)e^{-z^2}(-2z) - 2z\ e^{-z^2}$

$= 2z^3 e^{-z^2}$ for $z > 0$ and elsewhere

Chapter 9

9.2 Let a_{ij} be element in ith row and jth column. Since saddle point is minimum of row and maximum of column

	j	l
i	a_{ij}	a_{il}
k	a_{kj}	a_{kl}

$a_{ij} \geq a_{kj} \geq a_{kl} \geq a_{il} \geq a_{ij}$

$\therefore$ must all be equal signs

$a_{ij} = a_{kj} = a_{kl} = a_{il}$ and both parts are proved

9.3 If we let $x = 0$ for n heads, $x = 1$ at least one tail
Only changes in risk functions are that
$R(d_1,\ \theta_2) = \dfrac{1}{2^n}$ and $R(d_4,\ \theta_2) = 1 - \dfrac{1}{2^n}$

dominance same as before

resulting risk functions given by

	d_1	d_2
θ_1	0	1
θ_2	$1/2^n$	0

9.4 $R(d_1,\ \theta) = \int_0^\theta c(kx-\theta)^2 1 \frac{1}{\theta}\, d\theta$

$$= \frac{c}{\theta}\left[\frac{(kx-\theta)^3}{3k}\right]\Bigg|_0^c$$

$$= \frac{c}{\theta}\left[\frac{(k\theta-\theta)^3}{3k} - \frac{\theta^3}{3k}\right] = \frac{c\theta^2}{3}(k^3 - 3k + 3)$$

9.5 $p(x<k) = \int_0^k \frac{2x}{\theta^2}\, dx = \frac{k^2}{\theta_2}$

	θ_1	θ_2
θ_1	0	C
θ_2	C	0

Probability	
$\frac{k^2}{\theta_1^2}$	$1-\frac{k^2}{\theta_1^2}$
$\frac{k^2}{\theta_2^2}$	$1-\frac{k^2}{\theta_2^2}$

$$R(d,\ \theta_1) = C\left(1-\frac{k^2}{\theta_1^2}\right),\ R\left(d,\ \theta_2\right) = C\cdot\frac{k^2}{\theta_2^2}$$

For mimimax solution $C\left(1-\dfrac{k^2}{\theta_1^2}\right) = C\cdot\dfrac{k^2}{\theta_2^2}$ $\qquad k = \dfrac{\theta_1\theta_2}{\sqrt{\theta_1^2+\theta_2^2}}$

Chapter 14

14.1 $$h(y)=\int_0^\infty xe^{-x(1+y)}dy=\frac{1}{(1+y)^2}$$

$$\phi(x|y)=xe^{-x(1+y)}(1+y)^2$$

$$E(x|y)=(1+y)^2\int_0^\infty x^2e^{-x(1+y)}dx \qquad z=x(1+y)$$

$$=\int_0^\infty z^2e^{-z}\frac{dz}{1+y}=\frac{\Gamma(3)}{1+y}=\frac{2}{1+y}$$

14.2 $$g(x)=\frac{2}{5}\int_0^1(2x+3y)dy=\frac{2}{5}\left(2x+\frac{3}{2}\right)$$

$$w(y|x)=\frac{\frac{2}{5}(2x+3y)}{\frac{2}{5}\left(2x+\frac{3}{2}\right)}=\frac{2x+3y}{2x+\frac{3}{2}}$$

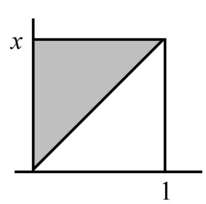
$$\mu_{Y|x}=\frac{1}{2x+\frac{3}{2}}\int_0^1 y(2x+3y)dy=\frac{x+1}{2x+\frac{3}{2}}=\frac{2(x+1)}{4x+3}$$

$$h(y)=\frac{2}{5}\int_0^1(2x+3y)dx=\frac{2}{5}(1+3y)$$

$$\phi(x|y)=\frac{\frac{2}{5}(2x+3y)}{\frac{2}{5}(1+3y)}=\frac{2x+3y}{1+3y}$$

$$\mu_{x|Y}=\frac{1}{1+3y}\int_0^1 x(2x+3y)dx=\frac{\frac{2}{3}+\frac{3}{2}y}{1+3y}=\frac{4+9y}{6(1+3y)}$$

14.3



$$g(x)=\int_x^1 6x\,dy=6x(1-x),\; w(y|x)=\frac{6x}{6x(1-x)}=\frac{1}{1-x}$$

$$E(Y|x)=\frac{1}{1-x}\int_x^1 y\,dy=\frac{1-x^2}{2(1-x)}=\frac{1+x}{2}$$

$$h(y)=\int_0^y 6x\,dx=3y^2 \qquad \phi(x|y)=\frac{2x}{y^2}$$

$$E(x|y)=\frac{2}{y^2}\int_0^y x^2\,dx=\frac{2}{y^2}\cdot\frac{y^3}{3}=\frac{2y}{3}$$

Chapter 16

16.1 **(a)** $$t = \frac{\bar{x}}{s/\sqrt{2}} = \frac{\sqrt{2}(x_1 + x_2)}{2\frac{\sqrt{(x_1 - x_2)^2 + (x_2 - x_1)^2}}{4}} = \frac{x_1 + x_2}{x_1 - x_2}$$

(b) Since the function $f(x) = \dfrac{x_1 + x_2}{x_1 - x_2}$ is decreasing for $x > x_2 > 0$

it follows that $\lim_{x \to \infty} f(x) = 1 < t' = f(10x_1) < t = f(x_1)$

16.2 When $T^+ = k$ then $T^- = \dfrac{n(n+1)}{2} - k$ and then

$$P(T^+ = k) = P\left(T^- = \frac{n(n+1)}{2} - k\right)$$
$$= P\left(T^+ = \frac{n(n+1)}{2} - k\right)$$

So that distribution is symmetrical about $\dfrac{n(n+1)}{4}$.

$$P\left(T^+ = \frac{n(n+1)}{4} + c\right) = P\left(T^- = \frac{n(n+1)}{4} - c\right)$$
$$= P\left(T^+ = \frac{n(n+1)}{4} - c\right)$$

16.3 $T^+ - T^- = T^+ - \left[\dfrac{n(n+1)}{2} - T^+\right] = 2T^+ - \dfrac{n(n+1)}{2} = X$

$E(X) = 2 \cdot \dfrac{n(n+1)}{4} - \dfrac{n(n+1)}{2} = 0$ by Theorem 16.1

$\text{var}(X) = 4 \cdot \dfrac{n(n+1)(2n+1)}{24}$ by Theorem 16.1

$$= \frac{n(n+1)(2n+1)}{6}$$

16.4 $n = 5$, $P(T = 0) = [P(x = 0)]^5 = (0.5)^5 = 0.031 > 0.02$, where x is a Bernoulli variable. Therefore, $T_{0.02}$ does not exist for $n = 5$.

16.5 **(a)** $$U_1 + U_2 = W_1 - \frac{n_1(n_1+1)}{2} + W_2 - \frac{n_2(n_2+1)}{2}$$
$$= \frac{(n_1+n_2)(n_1+n_2+1)}{2} - \frac{n_1(n_1+1)}{2} - \frac{n_2(n_2+1)}{2}$$
$$= n_1 n_2$$