SOLUTIONS TO CHAPTER 1 EXERCISES: PARTICLE SIZE ANALYSIS

SOLUTION TO EXERCISE 1.1:

(a) volume of cuboid = 12 mm^3

If x_v is the equivalent volume sphere diameter, then $\frac{\pi}{6}x_v^3 = 12$

Hence, x_v = 2.840 mm.

(b) surface area of cuboid = $(6 \times 2 \times 2) + (6 \times 1 \times 2) + (1 \times 2 \times 2) = 40\ \text{mm}^2$

If x_s is the equivalent surface sphere diameter, then $\pi x_s^2 = 40$

Hence, x_s = 3.568mm.

(c) Surface to volume ratio of the cuboid $= \frac{40}{12} = 3.333\ \text{mm}^2/\text{mm}^3$

If x_{sv} is the surface-volume sphere diameter, then $\frac{6}{x_{sv}} = 3.333$ (surface-volume ratio for a sphere of diameter x is 6/x)

Hence, x_{sv} = 1.8 mm.

(d) Sieve diameter is the second smallest dimension, i.e. 2 mm.

(e) The cuboid has three stable resting positions and therefore has three projected areas:

projected area 1 = 6 mm^2

projected area 2 = 2 mm^2

projected area 3 = 12 mm^2

If x_p is the projected area diameter, then

$$\frac{\pi}{4}x_{p1}^2 = 6; \qquad \frac{\pi}{4}x_{p2}^2 = 2 \qquad \frac{\pi}{4}x_{p3}^2 = 12$$

Giving three projected area diameters:

x_{p1} = 2.76 mm; x_{p2} = 1.60 mm; x_{p3} = 3.91 mm.

Flux mm/s (x 10^3)	6.7	5.6	5.1	4.5	4.2	3.8	3.5	3.3	3.0	2.9

(b) Under the same flow conditions as above, the concentration in the feed increases to 110% of the limiting value. Estimate the solids concentration in the overflow, in the underflow, in the section of the thickener above the feed well and in the section below the feed well.

SOLUTION TO EXERCISE 3.12:

Part (a)

The batch flux data is first plotted with concentration as abscissa (see Solution Manual-Figure 3.12.1).

Feed rate, F = 0.03 m^3/s

Underflow rate, L = 0.015 m^3/s

Material balance gives, underflow rate, V = F - L = 0.015 m^3/s

Expressing these flows as fluxes based on the thickener area (A = 300 m^2):

$$\frac{F}{A} = 0.1\ \mathrm{mm/s}$$

$$\frac{L}{A} = 0.05\ \mathrm{mm/s}$$

$$\frac{V}{A} = 0.05\ \mathrm{mm/s}$$

The variations in fluxes with concentration of the suspension are then:

$$\text{Feed flux} = C_F\left(\frac{F}{A}\right)$$

$$\text{Flux in underflow} = C_L\left(\frac{L}{A}\right)$$

$$\text{Flux in overflow} = C_V\left(\frac{V}{A}\right)$$

Lines of slope F/A, L/A and -V/A drawn on the flux plot represent the fluxes in the feed, underflow and overflow respectively (Solution Manual-Figure 3.12.1). The total flux plot for the section below the feed well is found by adding batch flux plot to the underflow flux line. The total flux plot for the section above the feed well is found by adding the batch flux plot to the overflow flux line (which is negative since it is an upward flux). These plots are shown in Solution Manual-Figure 3.12.1.

(b) The force (F) is simply the negative of the derivative of the potential energy (V) versus particle surface to surface separation distance (D). $F = -\dfrac{dV}{dD}$

(c) Shear thinning, yield stress and viscoelasticity.

EXERCISE 5.3:

(a) Describe the mechanism responsible for shear thinning behaviour observed for concentrated suspensions of micron sized hard sphere suspensions.
(b) Consider the same suspension as in a) except instead of hard sphere interactions, the particles are interacting with a strong attraction such as when they are at their isoelectric point. In this case describe the mechanism for the shear thinning behaviour observed.
(c) Draw a schematic plot (log-log) of the relative viscosity as a function of shear rate comparing the behaviour of the two suspensions described in a) and b). Be sure to indicate the relative magnitude of the low shear rate viscosites.
(d) Consider two suspensions of particles. All factors are the same except for the particle shape. One suspension has spherical particles and the other rod-shaped particles like grains of rice.
i) Which suspension will have a higher viscosity?
ii) What two physical parameters does the shape of the particles influence that affect the suspension viscosity.

SOLUTION TO EXERCISE 5.3:

(a) Brownian motion dominates the behaviour of concentrated suspensions at rest and at low shear rate such that a random particle structure results that produces a viscosity dependent upon the particle volume fraction. As shown in Figure 5.14 there is typically a range of low shear rates over which the viscosity is independent of shear rate. This region is commonly referred to as the low shear rate Newtonian plateau. At high shear rates, hydrodynamic interactions are more significant than Brownian motion and preferred flow structures such as sheets and strings of particles develop as in Figure 5.14. The viscosity of suspensions with such preferred flow structures is much lower than the viscosity of the same volume fraction suspension with randomized structure. The preferred flow structure that minimizes the particle-particle hydrodynamic interaction develops naturally as the shear rate is increased. There is typically a range of high shear rates where the viscosity reaches a plateau. The shear thinning behaviour observed in concentrated hard sphere suspensions is due to the transition from the randomized structure of the low shear rate Newtonian

(b) *Calculate* the frictional pressure drop across the packing in the tower.

(c) *Discuss* how this pressure drop will vary with flow rate of the gas within ±10% of the quoted flow rate.

(d) *Discuss* how the pressure drop across the packing would vary with gas pressure and temperature.

SOLUTION TO EXERCISE 6.3:

(a) From Text-Equation 6.6:
$S_B = S(1-\varepsilon)$, where S surface area per unit volume of rings.

Therefore, $S = \dfrac{S_B}{(1-\varepsilon)} = \dfrac{190}{(1-0.71)} = 655.2\ \mathrm{m^2/m^3}$

If x_{sv} is the diameter of a sphere with the same surface-volume ratio as the rings,

$$\frac{\pi x_{sv}^2}{\frac{\pi}{6} x_{sv}^3} = 655.2\ \mathrm{m^2/m^3}$$

Hence, x_{sv} = 9.16 mm

(b) Superficial gas velocity, $U = \dfrac{Q}{\pi \frac{D^2}{4}} = \dfrac{6}{\pi \frac{2^2}{4}} = 1.91\ \mathrm{m/s}$

Using the Ergun equation (Text-Equation 6.15) to describe the relationship between gas velocity and pressure drop across the packed bed,

$$\frac{(-\Delta p)}{H} = 150 \frac{\mu U}{x_{sv}^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75 \frac{\rho_f U^2}{x_{sv}} \frac{(1-\varepsilon)}{\varepsilon^3}$$

With $\mu = 1.8 \times 10^{-5}$ Pa.s, $\rho_f = 1.2\ \mathrm{kg/m^3}$, $x_{sv} = 9.16 \times 10^{-3}$ m and H = 5 m,

$$\frac{(-\Delta p)}{5} = 150 \frac{1.8 \times 10^{-5} \times 1.91}{(9.16 \times 10^{-3})^2} \times \frac{(1-0.71)^2}{0.71^3} + 1.75 \frac{1.2 \times 1.91^2}{9.16 \times 10^{-3}} \times \frac{(1-0.71)}{0.71^3}$$

which gives $(-\Delta p) = 72.0 + 3388.4 = 3460.4\ \mathrm{Pa}$.

(c) We note that the turbulent component makes up 98% of the total. Hence, within ±10% of the quoted flow rate the pressure drop across the bed will increase with the square of the superficial velocity and hence with the square of the flow rate:

$$(-\Delta p) \propto Q^2$$

Solids flux, $G = M_p/A = \dfrac{2000}{3600} \times \dfrac{1}{\frac{\pi}{4}(0.05)^2} = 282.9 \text{ kg / m}^2\text{.s}$

Substituting Text-Equation 8.1 into Text-Equation 8.2 gives:

$$\rho_f^{0.77} = \frac{2250D\left(\varepsilon_{CH}^{-4.7} - 1\right)\rho_p^2\left(1-\varepsilon_{CH}\right)^2}{G^2}$$

which can be solved by trial and error to give $\varepsilon_{CH} = 0.9705$.
Substituting back into Text-Equation 8.1 gives choking velocity $U_{CH} = 5.446$ m/s.

Actual maximum volume flow rate available at the maximum pressure is 60 m^3/h, which in a 50 mm diameter pipe gives a superficial gas velocity of 8.49 m/s. Operating at this superficial gas velocity would give us a 56% safety margin over the predicted choking velocity ($U = U_{CH}$ x 1.56), which is acceptable.

The next step is to calculate the lift line pressure loss at this gas flow rate and compare it with the available blower pressure at this flow rate.

Starting with Text-Equation 8.15, the general pressure loss Equation, an expression for the total pressure loss in the vertical lift line may be derived. Initial acceleration of solids and gas must be taken into account and so terms 1 and 2 are included. The Fanning friction Equation is used to estimate the pressure loss due to gas-to-wall friction (term 3) assuming solids have negligible effect on this pressure loss. For term 4 the modified Konno and Saito correlation (Text-Equation 8.16) is used. For vertical transport θ is 90° in terms 5 and 6.

Thus, the pressure loss, Δp_v, in the vertical sections of the transport line is given by:

$$\Delta p_v = \frac{\rho_f \varepsilon_v U_{fv}^2}{2} + \frac{\rho_p(1-\varepsilon_v)U_{pv}^2}{2} + \frac{2 f_g \rho_f U^2 L_v}{D} + 0.057 G L_v \sqrt{\frac{g}{D}} + \rho_p\left(1-\varepsilon_v\right)g L_v + \rho_f \varepsilon_v g L_v$$

To use this Equation we need to calculate the voidage of the suspension in the vertical pipe line ε_v:

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, U_T.

i.e. $\quad U_{pv} = \dfrac{U}{\varepsilon_v} - U_T$

continuity gives particle mass flux, $G = \rho_p\left(1-\varepsilon_v\right)U_{pv}$

Table 12.3.2: Feed size distribution

Interval	1	2	3	4	5
Fraction	0.25	0.4	0.2	0.1	0.05

SOLUTION TO EXERCISE 12.3

Generally, from Equation 12.12:

$$\frac{dy_i}{dt} = \sum_{j=1}^{j=i-1}\{b(i,j).S_j.y_j\} - S_i y_i$$

For an increment in time equal to the time basis of the specific rate of breakage, S_i:

$$\Delta y_i = \sum_{j=1}^{j=i-1}\{b(i,j).S_j.y_j\} - S_i y_i$$

Change of fraction in interval 1:

Change in mass fraction in size interval one,

$$\Delta y_1 = 0 - S_1 y_1 = 0 - (0.65 \times 0.25)$$

$= -0.1625$

Hence, new y_1 = 0.25 - 0.1625 = 0.0875

Change of fraction in interval 2:

$$\Delta y_2 = b(2,1)\ S_1 y_1 - S_2 y_2$$

$$= (0.35 \times 0.65 \times 0.25) - (0.55 \times 0.4)$$

$= -0.1631$

Hence new y_2 = 0.4 - 0.1631 = 0.2369

Change in fraction in interval 3:

$$\Delta y_3 = \left[b(3,1)\ S_1 y_1 + b(3,2)\ S_2 y_2\right] - S_3 y_3$$

$$= \left[(0.25 \times 0.65 \times 0.25) + (0.45 \times 0.55 \times 0.4)\right] - (0.4 \times 0.2)$$

$= +0.05963$

Hence, new y_3 = 0.2 + 0.05963 = 0.2596

Change in fraction in interval 4:

$$\Delta y_4 = \left[b(4,1)\ S_1 y_1 + b(4,2)\ S_2 y_2 + b(4,3) S_3 y_3\right] - S_4 y_4$$

$$= \left[(0.2 \times 0.65 \times 0.25) + (0.3 \times 0.55 \times 0.4) + (0.6 \times 0.4 \times 0.2)\right] - (0.35 \times 0.1)$$

$= +0.1115$