INSTRUCTOR'S SOLUTIONS MANUAL

INTRODUCTION TO MATHEMATICAL STATISTICS

SEVENTH EDITION

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PEARSON

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Contents

```
theta<-mean(x)
stan <- var(x)^.5
n = length(x)
teestar<-rep(0,b)
n<-length(x)
for(i in 1:b){xstar<-sample(x,n,replace=T)
              teestar[i] = (mean(xstar) - theta)/(var(xstar)^.5/sqrt(n))
             }
teestar<-sort(teestar)
pick<-round((alpha/2)*(b+1))
lower0<-teestar[pick]
upper0<-teestar[b-pick+1]
lower = theta - upper0*(stan/sqrt(n))
upper = theta - lower0*(stan/sqrt(n))
list(theta=theta,lower=lower,upper=upper,teestar=teestar)
#list(theta=theta,lower=lower,upper=upper)
}
```

The results for data in Example 4.9.3 based on 1000 bootstraps are:

```
> temp=prob595bs(x,1000,.10)
> temp$theta
[1] 90.59
> temp$lower
[1] 63.67547
> temp$upper
[1] 129.4924
```

4.9.7 Here are the results from a Minitab run on the data of Example 4.9.2:

```
TWOSAMPLE T FOR C2 VS C1
      N      MEAN     STDEV   SE MEAN
 C2  15     117.7      18.6       4.8
 C1  15     111.1      20.4       5.3

 95 PCT CI FOR MU C2 - MU C1: ( -8.0,  21.2)

 TTEST MU C2 = MU C1 (VS GT): T= 0.93  P=0.18  DF=  28

 POOLED STDEV =       19.5
```

where the data are in $C1$ and $C2$.

By the Central Limit Theorem, the terms on the right-side converge in distribution to $N(0, \sigma^2/\lambda_2)$ and $N(0, \sigma^2/\lambda_1)$ distributions, respectively. Using independence between the samples leads to the asymptotic distribution given in expression (10.4.28).

10.4.4 From the asymptotic distribution of U, we obtain the equation

$$\begin{aligned}\frac{\alpha}{2} &= P_\Delta[U(\Delta) \le c] = P_\Delta[U(\Delta) \le c + (1/2)] \\ &\doteq P\left[Z \le \left\{(c + (1/2) - (n_1 n_2/2))/\sqrt{n_1 n_2 (n+1)/12}\right\}\right].\end{aligned}$$

Setting the term in braces to $-z_{\alpha/2}$ yields the desired result.

10.4.5 Using $\Delta > 0$, we get the following implication which implies that $F_Y(y) \le F_X(y)$:

$$Y \le y \Leftrightarrow X + \Delta \le y \Leftrightarrow X \le y - \Delta \Rightarrow X \le y.$$

10.5.3 The value of s_a^2 for Wilcoxon scores is

$$\begin{aligned}s_a^2 &= 12\sum_{i=1}^n \left[\frac{i}{n+1} - \frac{1}{2}\right]^2 \\ &= \frac{12}{(n+1)^2}\left\{\sum_{i=1}^n i^2 - (n+1)\sum_{i=1}^n i + \frac{n(n+1)^2}{4}\right\} \\ &= \frac{n(n-1)}{n+1}.\end{aligned}$$

10.5.5 Use the change of variables $u = \Phi(x)$ to obtain

$$\begin{aligned}\int_0^1 \Phi^{-1}(u)\,du &= \int_{-\infty}^{\infty} x\phi(x)\,dx = 0 \\ \int_0^1 (\Phi^{-1}(u))^2\,du &= \int_{-\infty}^{\infty} x^2\phi(x)\,dx = 1.\end{aligned}$$

10.5.10 For this problem

$$\tau_\varphi^{-1} = \int_0^1 \Phi^{-1}(u)\left\{-\frac{f'(F^{-1}(u))}{f(F^{-1}(u))}\right\}du.$$

Without loss of generality assume that $\mu = 0$. Then $f(x) = (1/\sqrt{2\pi}\sigma)\exp\{-x^2/2\sigma^2\}$. It follows that

$$\frac{f'(x)}{f(x)} = -\frac{x}{\sigma^2}.$$

Furthermore, because $F(t) = \Phi(t/\sigma)$ we get $F^{-1}(u) = \sigma\Phi^{-1}(u)$. Substituting this into the expression which defines τ_φ^{-1}, we obtain $\tau_\varphi^{-1} = \sigma^{-1}$.

11.4.7 For this exercise a computer is not needed.

(a). The constant of proportionality K solves the equation

$$1 = K \int_0^1 y^{\alpha-1}(1-y)^{\beta-1} \left\{ \sum_{x=0}^{n} \binom{n}{x} y^x (1-y)^{n-x} \right\} dy,$$

which is easily determined to be $K = \Gamma(\alpha+\beta)/(\Gamma(\alpha)\Gamma(\beta))$.

(b) from the joint pdf, we have

$$f(x|y) \propto \binom{n}{x} y^x (1-y)^{n-x}.$$

Hence, $X|Y$ is binomial(n, Y). Likewise,

$$f(y|x) \propto y^{x+\alpha-1}(1-y)^{n-x+\beta-1};$$

so $Y|X$ is beta$(x+\alpha, n-x+\beta)$.

(c). The Gibbs sampler algorithm is: for $i = 1, 2, \ldots, m$

(1). Generate $Y_i|X_{i-1} \sim$ beta$(\alpha + X_{i-1}, n - X_{i-1} + \beta)$

(2). Generate $X_i|Y_i \sim$ binomial(n, Y_i) .

11.4.8 Here is R-code which runs the Gibbs sampler of the last exercise:

```
gibbser3 = function(alpha,beta,nt,m,n){
x0 = 1
yc = rep(0,m+n)
xc = c(x0,rep(0,m-1+n))
for(i in 2:(m+n)){yc[i] = rbeta(1,xc[i-1]+alpha,nt-xc[i-1]+beta)
              xc[i] = rbinom(1,nt,yc[i])}
y1=yc[1:m]
y2=yc[(m+1):(m+n)]
x1=xc[1:m]
x2=xc[(m+1):(m+n)]
list(y1 = y1,y2=y2,x1=x1,x2=x2)
}
```

To determine the mean of X, use the joint pdf to find that $E(X) = n(\alpha/(\alpha+\beta))$.

11.5.3 The Bayes model is

$$\begin{aligned} X|p &\sim \text{bin}(n,p), \quad 0 < p < 1 \\ p|\theta &\sim h(p|\theta) = \theta p^{\theta-1}, \quad \theta > 0 \\ \theta &\sim \Gamma(1,a), \quad a \text{ specified.} \end{aligned}$$