

INSTRUCTOR'S SOLUTIONS MANUAL

INTRODUCTION TO MATHEMATICAL STATISTICS SEVENTH EDITION

Robert Hogg

University of Iowa

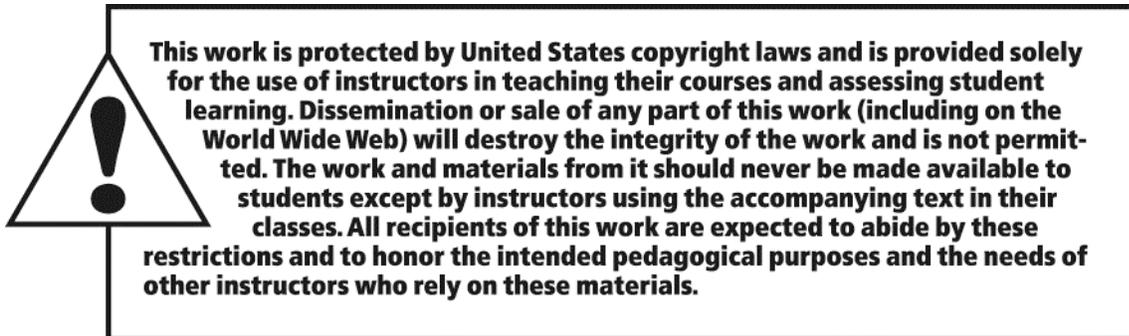
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ISBN-13: 978-0-321-79565-6
ISBN-10: 0-321-79565-2

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```

theta<-mean(x)
stan <- var(x)^.5
n = length(x)
teestar<-rep(0,b)
n<-length(x)
for(i in 1:b){xstar<-sample(x,n,replace=T)
              teestar[i] = (mean(xstar) - theta)/(var(xstar)^.5/sqrt(n))
              }
teestar<-sort(teestar)
pick<-round((alpha/2)*(b+1))
lower0<-teestar[pick]
upper0<-teestar[b-pick+1]
lower = theta - upper0*(stan/sqrt(n))
upper = theta - lower0*(stan/sqrt(n))
list(theta=theta,lower=lower,upper=upper,teestar=teestar)
#list(theta=theta,lower=lower,upper=upper)
}

```

The results for data in Example 4.9.3 based on 1000 bootstraps are:

```

> temp=prob595bs(x,1000,.10)
> temp$theta
[1] 90.59
> temp$lower
[1] 63.67547
> temp$upper
[1] 129.4924

```

4.9.7 Here are the results from a Minitab run on the data of Example 4.9.2:

```

TWO-SAMPLE T FOR C2 VS C1
      N      MEAN      STDEV   SE MEAN
C2  15    117.7    18.6      4.8
C1  15    111.1    20.4      5.3

95 PCT CI FOR MU C2 - MU C1: ( -8.0, 21.2)

TTEST MU C2 = MU C1 (VS GT): T= 0.93  P=0.18  DF= 28

POOLED STDEV =          19.5

```

where the data are in *C1* and *C2*.

By the Central Limit Theorem, the terms on the right-side converge in distribution to $N(0, \sigma^2/\lambda_2)$ and $N(0, \sigma^2/\lambda_1)$ distributions, respectively. Using independence between the samples leads to the asymptotic distribution given in expression (10.4.28).

10.4.4 From the asymptotic distribution of U , we obtain the equation

$$\begin{aligned}\frac{\alpha}{2} &= P_{\Delta}[U(\Delta) \leq c] = P_{\Delta}[U(\Delta) \leq c + (1/2)] \\ &\doteq P\left[Z \leq \left\{ (c + (1/2) - (n_1 n_2 / 2)) / \sqrt{n_1 n_2 (n + 1) / 12} \right\}\right].\end{aligned}$$

Setting the term in braces to $-z_{\alpha/2}$ yields the desired result.

10.4.5 Using $\Delta > 0$, we get the following implication which implies that $F_Y(y) \leq F_X(y)$:

$$Y \leq y \Leftrightarrow X + \Delta \leq y \Leftrightarrow X \leq y - \Delta \Rightarrow X \leq y.$$

10.5.3 The value of s_a^2 for Wilcoxon scores is

$$\begin{aligned}s_a^2 &= 12 \sum_{i=1}^n \left[\frac{i}{n+1} - \frac{1}{2} \right]^2 \\ &= \frac{12}{(n+1)^2} \left\{ \sum_{i=1}^n i^2 - (n+1) \sum_{i=1}^n i + \frac{n(n+1)^2}{4} \right\} \\ &= \frac{n(n-1)}{n+1}.\end{aligned}$$

10.5.5 Use the change of variables $u = \Phi(x)$ to obtain

$$\begin{aligned}\int_0^1 \Phi^{-1}(u) du &= \int_{-\infty}^{\infty} x \phi(x) dx = 0 \\ \int_0^1 (\Phi^{-1}(u))^2 du &= \int_{-\infty}^{\infty} x^2 \phi(x) dx = 1.\end{aligned}$$

10.5.10 For this problem

$$\tau_{\varphi}^{-1} = \int_0^1 \Phi^{-1}(u) \left\{ -\frac{f'(F^{-1}(u))}{f(F^{-1}(u))} \right\} du.$$

Without loss of generality assume that $\mu = 0$. Then $f(x) = (1/\sqrt{2\pi}\sigma) \exp\{-x^2/2\sigma^2\}$. It follows that

$$\frac{f'(x)}{f(x)} = -\frac{x}{\sigma^2}.$$

Furthermore, because $F(t) = \Phi(t/\sigma)$ we get $F^{-1}(u) = \sigma\Phi^{-1}(u)$. Substituting this into the expression which defines τ_{φ}^{-1} , we obtain $\tau_{\varphi}^{-1} = \sigma^{-1}$.

11.4.7 For this exercise a computer is not needed.

(a). The constant of proportionality K solves the equation

$$1 = K \int_0^1 y^{\alpha-1} (1-y)^{\beta-1} \left\{ \sum_{x=0}^n \binom{n}{x} y^x (1-y)^{n-x} \right\} dy,$$

which is easily determined to be $K = \Gamma(\alpha + \beta) / (\Gamma(\alpha)\Gamma(\beta))$.

(b) from the joint pdf, we have

$$f(x|y) \propto \binom{n}{x} y^x (1-y)^{n-x}.$$

Hence, $X|Y$ is binomial(n, Y). Likewise,

$$f(y|x) \propto y^{x+\alpha-1} (1-y)^{n-x+\beta-1};$$

so $Y|X$ is beta($x + \alpha, n - x + \beta$).

(c). The Gibbs sampler algorithm is: for $i = 1, 2, \dots, m$

- (1). Generate $Y_i | X_{i-1} \sim \text{beta}(\alpha + X_{i-1}, n - X_{i-1} + \beta)$
- (2). Generate $X_i | Y_i \sim \text{binomial}(n, Y_i)$.

11.4.8 Here is R-code which runs the Gibbs sampler of the last exercise:

```
gibbser3 = function(alpha,beta,nt,m,n){
  x0 = 1
  yc = rep(0,m+n)
  xc = c(x0,rep(0,m-1+n))
  for(i in 2:(m+n)){yc[i] = rbeta(1,xc[i-1]+alpha,nt-yc[i-1]+beta)
    xc[i] = rbinom(1,nt,yc[i])}
  y1=yc[1:m]
  y2=yc[(m+1):(m+n)]
  x1=xc[1:m]
  x2=xc[(m+1):(m+n)]
  list(y1 = y1,y2=y2,x1=x1,x2=x2)
}
```

To determine the mean of X , use the joint pdf to find that $E(X) = n(\alpha/(\alpha + \beta))$.

11.5.3 The Bayes model is

$$\begin{aligned} X|p &\sim \text{bin}(n, p), \quad 0 < p < 1 \\ p|\theta &\sim h(p|\theta) = \theta p^{\theta-1}, \quad \theta > 0 \\ \theta &\sim \Gamma(1, a), \quad a \text{ specified.} \end{aligned}$$