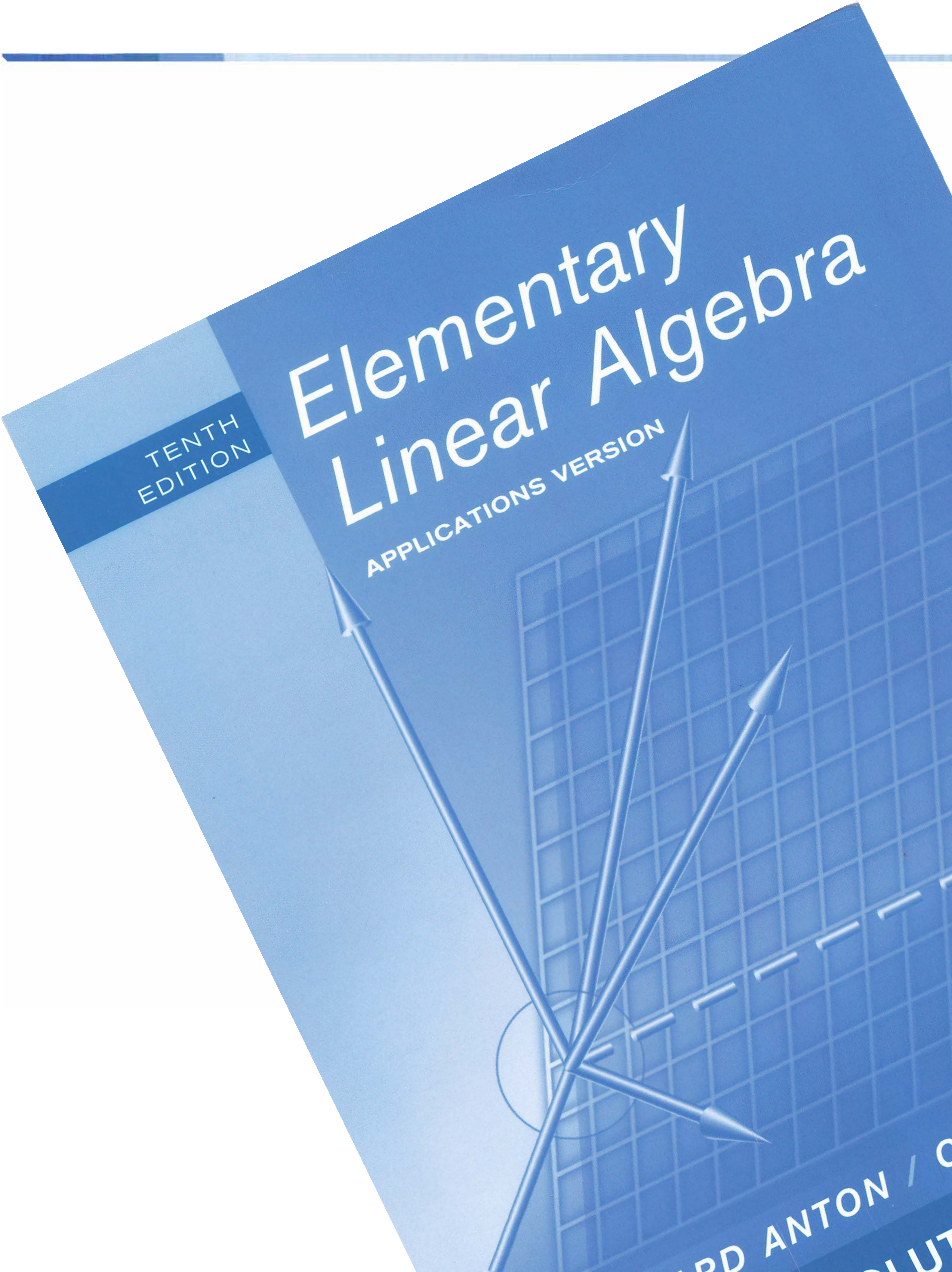
TENTH
EDITION
Elementary
Linear Algebra
APPLICATIONS VERSION
ANTON

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$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

Add 10 times the second row to the third.

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

Since there is a row of zeros on the left side,

$\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$ is not invertible.

17. $$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Add −1 times the first row to the third.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right]$$

Add −1 times the second row to the third.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right]$$

Multiply the third row by $-\frac{1}{2}$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

Add −1 times the third row to both the first and second rows.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

19. $$\left[\begin{array}{ccc|ccc} 2 & 6 & 6 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

Multiply the first row by $\frac{1}{2}$.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

Add −2 times the first row to both the second and third rows.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

Add −1 times the second row to the third.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

Add −3 times the third row to the first.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{1}{2} & 3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

Add −3 times the second row to the first.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{2} & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{7}{2} & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

21. $$\left[\begin{array}{cccc|cccc} 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

Interchange the first and second rows.

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

Add −2 times the first row to the second.

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & -8 & -24 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

Interchange the second and fourth rows.

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -8 & -24 & 0 & 1 & -2 & 0 & 0 \end{array}\right]$$

9. $\begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix} = (a-3)(a-2)-(5)(-3)$
$= a^2 - 5a + 6 + 15$
$= a^2 - 5a + 21$

11. $\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix} = \left|\begin{array}{ccc|cc} -2 & 1 & 4 & -2 & 1 \\ 3 & 5 & -7 & 3 & 5 \\ 1 & 6 & 2 & 1 & 6 \end{array}\right.$
$= [(-2)(5)(2)+(1)(-7)(1)+(4)(3)(6)]-[(4)(5)(1)+(-2)(-7)(6)+(1)(3)(2)]$
$= [-20-7+72]-[20+84+6]$
$= -65$

13. $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix} = \left|\begin{array}{ccc|cc} 3 & 0 & 0 & 3 & 0 \\ 2 & -1 & 5 & 2 & -1 \\ 1 & 9 & -4 & 1 & 9 \end{array}\right.$
$= [(3)(-1)(-4)+(0)(5)(1)+(0)(2)(9)]-[0(-1)(1)+(3)(5)(9)+(0)(2)(-4)]$
$= 12 - 135$
$= -123$

15. $\det(A) = (\lambda-2)(\lambda+4)-(-5)$
$= \lambda^2 + 2\lambda - 8 + 5$
$= \lambda^2 + 2\lambda - 3$
$= (\lambda-1)(\lambda+3)$
$\det(A) = 0$ for $\lambda = 1$ or -3.

17. $\det(A) = (\lambda-1)(\lambda+1) - 0 = (\lambda-1)(\lambda+1)$
$\det(A) = 0$ for $\lambda = 1$ or -1.

19. (a) $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix} = 3\begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix} - 0\begin{vmatrix} 2 & 5 \\ 1 & -4 \end{vmatrix} + 0\begin{vmatrix} 2 & -1 \\ 1 & 9 \end{vmatrix}$
$= 3(4-45)$
$= -123$

(b) $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix} = 3\begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix} - 2\begin{vmatrix} 0 & 0 \\ 9 & -4 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ -1 & 5 \end{vmatrix}$
$= 3(4-45) - 2(0-0) + (0-0)$
$= -123$

(c) $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix} = -2\begin{vmatrix} 0 & 0 \\ 9 & -4 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 0 \\ 1 & -4 \end{vmatrix} - 5\begin{vmatrix} 3 & 0 \\ 1 & 9 \end{vmatrix}$
$= -2(0-0) - (-12-0) - 5(27-0)$
$= 12 - 135$
$= -123$

(d) $\left[\begin{array}{rr|rr} 1 & -1 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{array}\right]$ reduces to $\left[\begin{array}{rr|rr} 1 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$

so $P_{E\to B'} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$.

$$[\mathbf{w}]_{B'} = P_{E\to B'}[\mathbf{w}]_E = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} -4 \\ -7 \end{bmatrix}$$

9. (a) $\left[\begin{array}{rrr|rrr} 3 & 1 & -1 & 2 & 2 & 1 \\ 1 & 1 & 0 & 1 & -1 & 2 \\ -5 & -3 & 2 & 1 & 1 & 1 \end{array}\right]$ reduces to

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 3 & 2 & \frac{5}{2} \\ 0 & 1 & 0 & -2 & -3 & -\frac{1}{2} \\ 0 & 0 & 1 & 5 & 1 & 6 \end{array}\right]$ so

$P_{B\to B'} = \begin{bmatrix} 3 & 2 & \frac{5}{2} \\ -2 & -3 & -\frac{1}{2} \\ 5 & 1 & 6 \end{bmatrix}$.

(b) $\left[\begin{array}{rrr|rrr} 2 & 2 & 1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$ reduces to

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & \frac{3}{2} & \frac{1}{2} & -\frac{5}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right]$ so

$P_{E\to B} = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -\frac{5}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ -1 & 0 & 2 \end{bmatrix}$ where E is the standard basis for R^3.

$$[\mathbf{w}]_B = P_{E\to B}[\mathbf{w}]_E = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -\frac{5}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 9 \\ -9 \\ -5 \end{bmatrix}$$

$$[\mathbf{w}]_{B'} = P_{B\to B'}[\mathbf{w}]_B = \begin{bmatrix} 3 & 2 & \frac{5}{2} \\ -2 & -3 & -\frac{1}{2} \\ 5 & 1 & 6 \end{bmatrix}\begin{bmatrix} 9 \\ -9 \\ -5 \end{bmatrix} = \begin{bmatrix} -\frac{7}{2} \\ \frac{23}{2} \\ 6 \end{bmatrix}$$

(c) $\left[\begin{array}{rrr|rrr} 3 & 1 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -5 & -3 & 2 & 0 & 0 & 1 \end{array}\right]$ reduces to

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & 2 & 1 \end{array}\right]$ so

$P_{E\to B'} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 2 & 1 \end{bmatrix}$.

$$[\mathbf{w}]_{B'} = P_{E\to B'}[\mathbf{w}]_E = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix} = \begin{bmatrix} -\frac{7}{2} \\ \frac{23}{2} \\ 6 \end{bmatrix}$$

11. (a) The span of $\mathbf{f}_1$ and $\mathbf{f}_2$ is the set of all linear combinations $a\mathbf{f}_1 + b\mathbf{f}_2 = a\sin x + b\cos x$ and this vector can be represented by (a, b). Since $\mathbf{g}_1 = 2\mathbf{f}_1 + \mathbf{f}_2$ and $\mathbf{g}_2 = 3\mathbf{f}_2$, it is sufficient to compute $\det\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = 6$. Since this determinant is nonzero, $\mathbf{g}_1$ and $\mathbf{g}_2$ form a basis for V.

(b) Since B can be represented as $\{(1, 0), (0, 1)\}$ $P_{B'\to B} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$.

(c) $\left[\begin{array}{rr|rr} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$ reduces to $\left[\begin{array}{rr|rr} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{3} \end{array}\right]$

so $P_{B\to B'} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$.

3. Since B is the standard basis for R^2,

$$[T]_B = \begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$ The matrices for $P_{B\to B'}$ and $P_{B'\leftrightarrow B}$ are the same as in Exercise 1, so

$$\begin{aligned}[T]_{B'} &= P_{B\to B'}[T]_B P_{B'\to B}\\ &= \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{2}{11} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}\\ &= \begin{bmatrix} \frac{13}{11\sqrt{2}} & -\frac{25}{11\sqrt{2}} \\ \frac{5}{11\sqrt{2}} & \frac{9}{11\sqrt{2}} \end{bmatrix}.\end{aligned}$$

5. $T(\mathbf{u}_1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $T(\mathbf{u}_2) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $T(\mathbf{u}_3) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, so

$$[T]_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

By inspection, $\mathbf{v}_1 = \mathbf{u}_1$, $\mathbf{v}_2 = \mathbf{u}_1 + \mathbf{u}_2$, and $\mathbf{v}_3 = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3$, so the transition matrix from B' to B is $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

Thus $P_{B\to B'} = P^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ and

$$\begin{aligned}[T]_{B'} &= P_{B\to B'}[T]_B P_{B'\to B}\\ &= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.\end{aligned}$$

7. $T(\mathbf{p}_1) = 6 + 3(x+1) = 9 + 3x = \frac{2}{3}\mathbf{p}_1 + \frac{1}{2}\mathbf{p}_2$, and

$T(\mathbf{p}_2) = 10 + 2(x+1) = 12 + 2x = -\frac{2}{9}\mathbf{p}_1 + \frac{4}{3}\mathbf{p}_2$,

so $[T]_B = \begin{bmatrix} \frac{2}{3} & -\frac{2}{9} \\ \frac{1}{2} & \frac{4}{3} \end{bmatrix}$.

$\mathbf{q}_1 = -\frac{2}{9}\mathbf{p}_1 + \frac{1}{3}\mathbf{p}_2$ and $\mathbf{q}_2 = \frac{7}{9}\mathbf{p}_1 - \frac{1}{6}\mathbf{p}_2$, so the transition matrix from B' to B is

$P = \begin{bmatrix} -\frac{2}{9} & \frac{7}{9} \\ \frac{1}{3} & -\frac{1}{6} \end{bmatrix}$. Thus $P_{B\to B'} = P^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{7}{2} \\ \frac{3}{2} & 1 \end{bmatrix}$

and
$$\begin{aligned}[T]_{B'} &= P_{B\to B'}[T]_B P_{B'\to B}\\ &= \begin{bmatrix} \frac{3}{4} & \frac{7}{2} \\ \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} \frac{2}{3} & -\frac{2}{9} \\ \frac{1}{2} & \frac{4}{3} \end{bmatrix}\begin{bmatrix} -\frac{2}{9} & \frac{7}{9} \\ \frac{1}{3} & -\frac{1}{6} \end{bmatrix}\\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.\end{aligned}$$

11. (a) The matrix for T relative to the standard basis B is $[T]_B = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$. The eigenvalues of $[T]_B$ are $\lambda = 2$ and $\lambda = 3$ with corresponding eigenvectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Then for $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$, we have

$P^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ and $P^{-1}[T]_B P = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

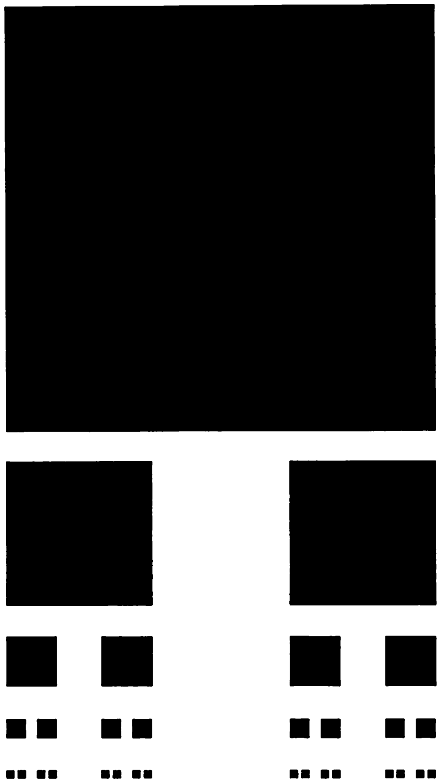
Since P represents the transition matrix from the basis B' to the standard basis B, then $B' = \left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}$ is a basis for which $[T]_{B'}$ is diagonal.

(b) The matrix for T relative to the standard basis B is $[T]_B = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$.

The eigenvalues of $[T]_B$ are $\lambda = \frac{5+\sqrt{21}}{2}$ and $\lambda = \frac{5-\sqrt{21}}{2}$ with corresponding eigenvectors $\begin{bmatrix} \frac{-3-\sqrt{21}}{6} \\ 1 \end{bmatrix}$ and $\begin{bmatrix} \frac{-3+\sqrt{21}}{6} \\ 1 \end{bmatrix}$.

Then for $P = \begin{bmatrix} \frac{-3-\sqrt{21}}{6} & \frac{-3+\sqrt{21}}{6} \\ 1 & 1 \end{bmatrix}$, we have

$P^{-1} = \begin{bmatrix} -\frac{3}{\sqrt{21}} & \frac{-3+\sqrt{21}}{2\sqrt{21}} \\ \frac{3}{\sqrt{21}} & \frac{3+\sqrt{21}}{2\sqrt{21}} \end{bmatrix}$ and



12. The area of the unit square S_0 is, of course, 1. Each of the eight similitudes $T_1, T_2, \ldots, T_8$ given in Equation (8) of the text has scale factor $s = \frac{1}{3}$, and so each maps the unit square onto a smaller square of area $\frac{1}{9}$. Because these eight smaller squares are nonoverlapping, their total area is $\frac{8}{9}$, which is then the area of the set S_1. By a similar argument, the area of the set S_2 is $\frac{8}{9}$-th the area of the set S_1. Continuing the argument further, we find that the areas of $S_0, S_1, S_2, S_3, S_4, \ldots$, form the geometric sequence $1, \frac{8}{9}, \left(\frac{8}{9}\right)^2, \left(\frac{8}{9}\right)^3, \left(\frac{8}{9}\right)^4, \ldots$. (Notice that this implies that the area of the Sierpinski carpet is 0, since the limit of $\left(\frac{8}{9}\right)^n$ as n tends to infinity is 0.)

Section 10.14

Exercise Set 10.14

1. Because $250 = 2 \cdot 5^3$ it follows from (i) that $\Pi(250) = 3 \cdot 250 = 750$.
Because $25 = 5^2$ it follows from (ii) that $\Pi(25) = 2 \cdot 25 = 50$.
Because $125 = 5^3$ it follows from (ii) that $\Pi(125) = 2 \cdot 125 = 250$.
Because $30 = 6 \cdot 5$ it follows from (ii) that $\Pi(30) = 2 \cdot 30 = 60$.
Because $10 = 2 \cdot 5$ it follows from (i) that $\Pi(10) = 3 \cdot 10 = 30$.