Solutions Manual

Elasticity: Theory, Applications and Numerics
Second Edition

By

Martin H. Sadd
Professor
Department of Mechanical Engineering & Applied Mechanics
University of Rhode Island
Kingston, Rhode Island

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Foreword

Exercises found at the end of each chapter are an important ingredient of the text as they provide homework for student engagement, problems for examinations, and can be used in class to illustrate other features of the subject matter. This solutions manual is intended to aid the instructors in their own particular use of the exercises. Review of the solutions should help determine which problems would best serve the goals of homework, exams or be used in class.

The author is committed to continual improvement of engineering education and welcomes feedback from users of the text and solutions manual. Please feel free to send comments concerning suggested improvements or corrections to sadd@egr.uri.edu. Such feedback will be shared with the text user community via the publisher's web site.

Martin H. Sadd
January 2009

1-5.

$$\det(a_{ij}) = \varepsilon_{ijk}a_{1i}a_{2j}a_{3k} = \varepsilon_{123}a_{11}a_{22}a_{33} + \varepsilon_{231}a_{12}a_{23}a_{31} + \varepsilon_{312}a_{13}a_{21}a_{32}$$
$$+ \varepsilon_{321}a_{13}a_{22}a_{31} + \varepsilon_{132}a_{11}a_{23}a_{32} + \varepsilon_{213}a_{12}a_{21}a_{33}$$
$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$
$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$
$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

1-6.

$$45^{\circ} \text{ rotation about } x_1\text{-axis} \Rightarrow Q_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$\text{From Exercise 1-1(a)}: \; b_i' = Q_{ij}b_j = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix}$$

$$a_{ij}' = Q_{ip}Q_{jq}a_{pq} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}^T = \begin{bmatrix} 1 & \sqrt{2} & 0 \\ 0 & 4 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

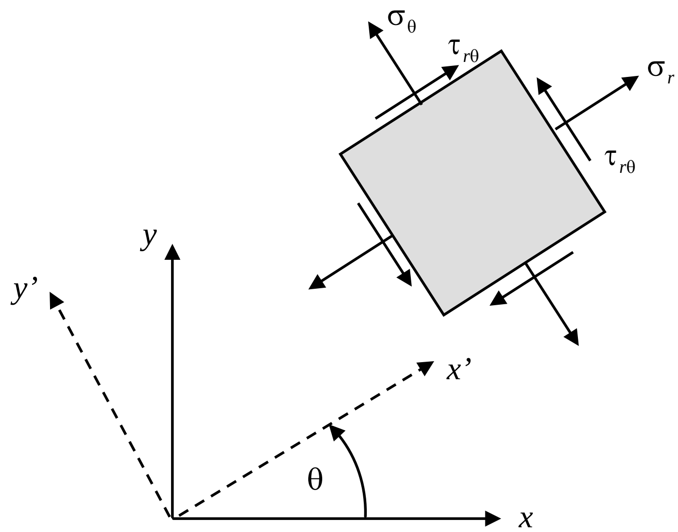
$$\text{From Exercise 1-1(b)}: \; b_i' = Q_{ij}b_j = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ \sqrt{2} \\ 0 \end{bmatrix}$$

$$a_{ij}' = Q_{ip}Q_{jq}a_{pq} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}^T = \begin{bmatrix} 1 & \sqrt{2} & -\sqrt{2} \\ 0 & 4.5 & -1.5 \\ 0 & 1.5 & -0.5 \end{bmatrix}$$

$$\text{From Exercise 1-1(c)}: \; b_i' = Q_{ij}b_j = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix}$$

$$a_{ij}' = Q_{ip}Q_{jq}a_{pq} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}^T = \begin{bmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{2}/2 & 3.5 & 2.5 \\ -\sqrt{2}/2 & 1.5 & 0.5 \end{bmatrix}$$

3-3.



$$\sigma_r = \sigma'_x = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

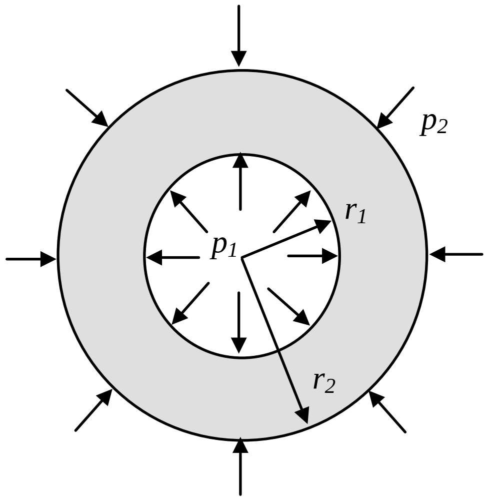
$$\sigma_\theta = \sigma'_y = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{r\theta} = \tau'_{xy} = -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

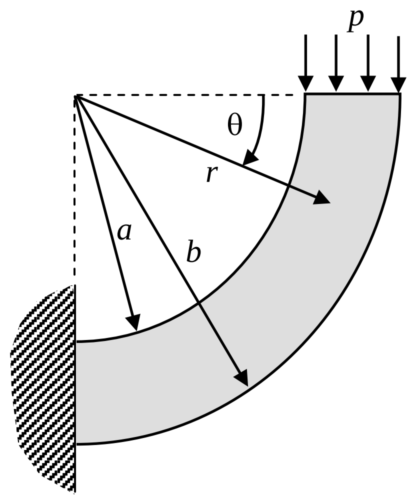
$$= \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

5-1. Continued



(e)

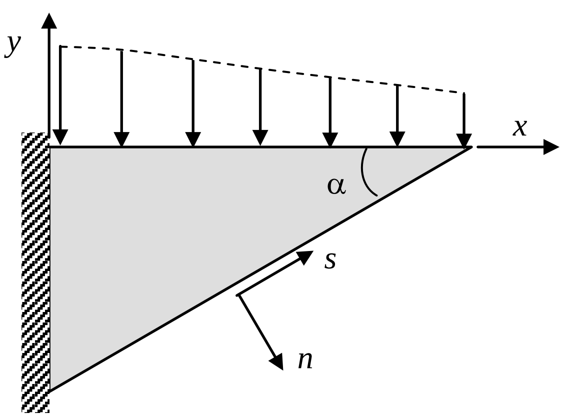
$$T_r(r_1,\theta) = p_1 \ , T_\theta(r_1,\theta) = 0$$
$$T_r(r_2,\theta) = -p_2 \ , T_\theta(r_2,\theta) = 0$$



(f)

$$T_r(a,\theta) = T_\theta(a,\theta) = 0$$
$$T_r(b,\theta) = T_\theta(b,\theta) = 0$$
$$T_r(r,0) = 0,\ T_\theta(r,0) = -p$$
$$u_r(r,\pi/2) = u_\theta(r,\pi/2) = 0$$

5-2.



$$\text{Bottom Surface}: n_x = \sin\alpha \ ,\ n_y = -\cos\alpha$$
$$T_x = \sigma_x n_x + \tau_{xy} n_y = 0 \ ,\ T_y = \tau_{xy} n_x + \sigma_y n_y = 0$$
$$T_n = T_x \sin\alpha - T_y \cos\alpha$$
$$T_s = T_x \cos\alpha + T_y \sin\alpha$$
$$\therefore \text{ If } T_x = T_y = 0 \Rightarrow T_n = T_s = 0$$

8-19.

Solutions from Exercise 8 - 18 :

$$\text{Material 1}: \sigma_r^{(1)} = \sigma_\theta^{(1)} = B^{(1)} \ , \ u_r^{(1)} = \frac{(1+\nu_1)(1-2\nu_1)}{E_1} B^{(1)} r$$

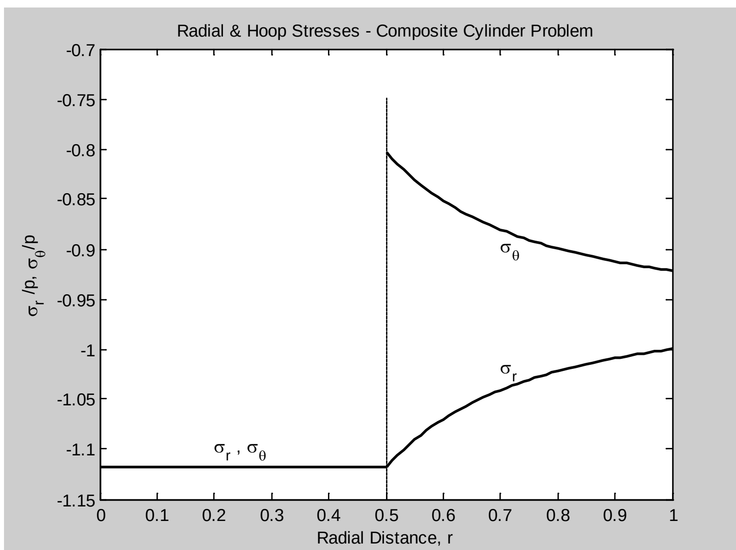
$$\text{Material 2}: \ \sigma_r^{(2)} = \frac{A^{(2)}}{r^2} - \frac{A^{(2)}}{r_2^2} - p \ , \ \sigma_\theta^{(2)} = -\frac{A^{(2)}}{r^2} - \frac{A^{(2)}}{r_2^2} - p$$

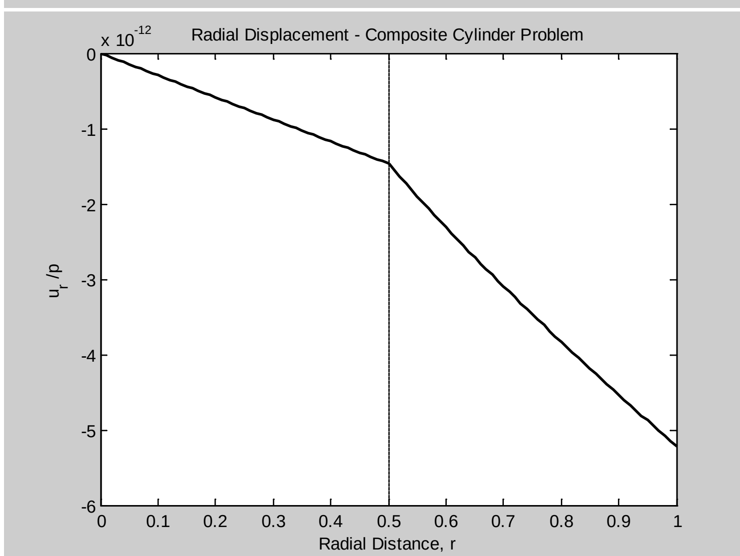
$$u_r^{(2)} = \frac{1+\nu_2}{E_2}\left[-\frac{A^{(2)}}{r^2} - (1-2\nu_2)\left(p + \frac{A^{(2)}}{r_2^2} \right) \right] r$$

Two Equations From Matching Conditions @ $r = r_1$:

$$\left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right) A^{(2)} - B^{(1)} = p \ , \ \left(\frac{1}{r_1^2} + \frac{(1-2\nu_2)}{r_2^2} \right) A^{(2)} - \left(\frac{E_2(1+\nu_1)(1-2\nu_1)}{E_1(1+\nu_2)} \right) B^{(1)} = -(1-2\nu_2) p$$

Using MATLAB to Solve Matching Relations and Then Calculate and Plot Stresses and Displacements for the Case $r_1 = 0.5$, $r_2 = 1.0$ Note that σ_θ is not continuous across the interface.





12-20*.

From Example 12 - 5, the non - dimensional hoop stress around hole boundary is gvien by equation $(12.8.32)_2$ with $\rho = 1$

$$\overline{\sigma}_\theta = \frac{\sigma_\theta}{E\alpha qa/k} = -\frac{(1+m)\{[(1+(1+m)^2+m^2]\sin\theta - 2m\sin 3\theta\}}{2(1-2m\cos 2\theta + m^2)^2}$$

$$= -\frac{(1+m)[(1+m+m^2)\sin\theta - m\sin 3\theta]}{(1-2m\cos 2\theta + m^2)^2}$$

Note : $\overline{\sigma}_\theta(\pi/2) = -1/(1+m)$

MATLAB Plots for $m = 0, \pm 1/2, \pm 1$: