## DIGITAL COMMUNICATION - Second EditionSOLUTIONS MANUAL



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5-18. The IF noise source can be referenced to the input to the RF amplifier with noise temperature $\frac{T_{I F}}{G_{R F}}$. Hence, the total noise referenced to the input has temperature

$$
\begin{equation*}
T_{i n}+T_{R F}+\frac{T_{I F}}{G_{R F}} \tag{5.146}
\end{equation*}
$$

Note that any noise in the IF amplifier is inherently less significant than noise introduced at the input or RF amplifier.
5-19. The excess distance for the reflected path is approximately

$$
\begin{equation*}
\frac{2 h^{2}}{d}=1.66 \text { meters } \tag{5.147}
\end{equation*}
$$

and the excess delay (and hence the delay spread) is therefore about 5.5 nsec . The reciprocal of the delay spread is 180 MHz , and assuming the narrowband model is valid over $1 \%$ of this bandwidth, that would be 1.8 MHz .

5-20.
(a) The spectrum will become asymmetric about the carrier, with more power concentrated at frequencies near $\left(\omega_{c}+k v\right)$
(b) The spectrum will fill out in the middle and get smaller near ( $\omega_{c} \pm k \nu$ ), but still be close to symmetric about the carrier frequency.
5-21. There are two oscillators, the worst case $\left|f_{c}-f_{d}\right|<3 \mathrm{~Hz}$. This means that each oscillator should not deviate more than 1.5 Hz from the nominal frequency 1 MHz , implying 1.5 parts per million accuracy.
5-22.

$$
\begin{equation*}
y(t)=\operatorname{Re}\left[s(t) e^{j\left[\omega_{1} t+\omega_{0}-a \cos \left(\omega_{t} t\right)\right]}\right] \tag{5.148}
\end{equation*}
$$

## CHAPTER 6: SOLUTIONS TO PROBLEMS

6-1. Using (6.4), we can write

$$
\begin{equation*}
H(j \omega)=G(j \omega) B(j \omega) . \tag{6.280}
\end{equation*}
$$

Using the result of appendix 3-A, we can write the power spectrum of the received signal as

$$
\begin{equation*}
S_{R}(j \omega)=\frac{1}{T}|H(j \omega)|^{2} S_{\Lambda}\left(e^{j \omega T}\right)=\frac{\sigma_{A}^{2}}{T}|G(j \omega) B(j \omega)|^{2}=\frac{\sigma_{A}^{2}}{T}|G(j \omega)|^{2} B(j \omega) . \tag{6.281}
\end{equation*}
$$

Using appendix 2-A

$$
\begin{equation*}
G(j \omega)=e^{-j \omega T / 2} T \frac{\sin (\omega T / 2)}{\omega T / 2} \tag{6.282}
\end{equation*}
$$

so

$$
\begin{equation*}
|G(j \omega)|^{2}=\frac{\sin ^{2}(\omega \pi / W)}{\omega^{2} / 4} \tag{6.283}
\end{equation*}
$$

The power spectrum $S_{R}(j \omega)$ is sketched below:

## 8-8.

(a) The decision regions are bounded by the planes formed by pairs of axes.
(b) The probability of error is

$$
\begin{equation*}
\operatorname{Pr}[\text { symbol error }]=1-(1-Q(d / 2 \sigma))^{M} . \tag{8.244}
\end{equation*}
$$

8-9. Since the distance between symbols is $d=a \sqrt{2}$,

$$
\begin{equation*}
\operatorname{Pr}\left[\mathrm{E}_{i j}\right]=Q(a / \sigma \sqrt{2}) \tag{8.245}
\end{equation*}
$$

so

$$
\begin{equation*}
\operatorname{Pr}[\text { symbol error }]=\operatorname{Pr}\left[\text { symbol error } \mid A_{k}=a_{i}\right] \leq(M-1) Q(a / \sigma \sqrt{2}) \tag{8.246}
\end{equation*}
$$

8-10.
(a) There are 4 symbols at distance $d=2 c, 4$ symbols at distance $d=2 \sqrt{2} c, 2$ symbols at distance $d=4 c, 4$ symbols at distance $d=2 \sqrt{5} c$, and 1 symbol at distance $d=4 \sqrt{2} c$, so from the union bound

$$
\begin{align*}
\operatorname{Pr}\left[\text { symbol error } \mid A_{k}=c+j c\right] \leq & 4 Q(c / \sigma)+4 Q(\sqrt{2} c / \sigma)+2 Q(2 c / \sigma) \\
& +4 Q(\sqrt{5} c / \sigma)+Q(2 \sqrt{2} c / \sigma)  \tag{8.247}\\
\approx & 4 Q(c / \sigma) .
\end{align*}
$$

(b) There are 2 symbols at distance $d=2,2$ symbols at distance $d=\sqrt{5}$, 2 symbols at distance $d=2 \sqrt{2}, 4$ symbols at distance $d=\sqrt{17}, 2$ symbols at distance $d=2 \sqrt{5}$, 1 symbol at distance $d=4 \sqrt{2}$, and 2 symbol at distance $d=\sqrt{37}$, so from the union bound

$$
\begin{align*}
\operatorname{Pr}\left[\text { symbol error } \mid A_{k}=1+j\right] \leq 2 Q & (1 / \sigma)+2 Q(\sqrt{5} / 2 \sigma)+2 Q(\sqrt{2} / \sigma)+4 Q(\sqrt{17} / 2 \sigma) \\
& +2 Q(\sqrt{5} / \sigma)+Q(2 \sqrt{2} / \sigma)+2 Q(\sqrt{37} / 2 \sigma)  \tag{8.248}\\
& \approx 2 Q(1 / \sigma) .
\end{align*}
$$

(c) The noise components in the directions of the nearest neighbors are not orthogonal, and hence not independent.
(d) The average power in the 16 -QAM constellation is $10 c^{2}$ and in the V. 29 constellation it is 13.5 , so $c=\sqrt{1.35} \approx 1.16$. The approximate probabilities of error are $4 Q(1.16 / \sigma)$ for $16-$ QAM and $2 Q(1 / \sigma)$ for V.29, so assuming the SNR is high enough that the constant multipliers are not important, 16-QAM is about 1.3 dB better. There are good reasons, nonetheless, for using the V. 29 constellation. In particular, it is less sensitive to phase jitter.
8-11. Taking a second pulse of the form of (8.113),

$$
\begin{equation*}
g(t)=\sum_{m=0}^{N-1} y_{k} h_{c}\left(t-m T_{c}\right) . \tag{8.249}
\end{equation*}
$$

then the inner product of $h(t)$ and $g(t)$ is

$$
\begin{equation*}
\int_{-\infty}^{\infty} h(t) g^{*}(t) d t=\sigma_{c}^{2} \sum_{m=0}^{N-1} x_{m} y_{m}^{*} . \tag{8.250}
\end{equation*}
$$

Considering $\left\{x_{m}, 0 \leq m \leq N-1\right.$ ) and $\left\{x_{m}, 0 \leq m \leq N-1\right.$ ) as vectors x and y in $N$-dimensional Euclidean space, then the pulses $h(t 0$ and $g(t)$ will be orthogonal when Euclidean vectors $\mathbf{x}$ and y are orthogonal. The number of pulses specified in this fashion that can be mutually orthogonal is $N=2 B T$, the dimensionality of the Euclidean space.
geometry can be shown to be $\sqrt{2-\sqrt{2}} \approx 0.77$. The minimum distance of all the error events shown is the square root of the sums of the squares of these stage distances, or

$$
\begin{equation*}
\sqrt{2+2-\sqrt{2}+2}=\sqrt{6-\sqrt{2}} \approx 2.14 \tag{14.96}
\end{equation*}
$$

It is easy to see that any error event with length greater than three stages will have a distance greater than these error events, so 2.14 is the distance of the second closest error event.
14-14.
(a) The partition is shown in figure 14-39, along with the minimum distances. Notice that at the final partitioning stage (into 16 subsets) there is no improvement in minimum distance for some of the subsets.
(b) The average power of the 16-QAM constellation has been computed elsewhere and is 10 . The 32 -cross constellation has all the same points, plus 16 additional points with average power

$$
\begin{equation*}
\frac{8}{16}\left(5^{2}+3^{2}\right)+\frac{8}{16}\left(5^{2}+1^{2}\right)=30 \tag{14.97}
\end{equation*}
$$

so the overall average power is

$$
\begin{equation*}
1 / 2(10+30)=20 . \tag{14.98}
\end{equation*}
$$

This is $10 \log (2)=3 d B$ more power.
(c) It should be adequate to use the subsets in the third row of figure 14-39. The minimum distance between parallel transitions is 4 . There are 4 such subsets, so $\bar{m}=1$. Compared to the 16 -QAM constellation in


Figure 14-39. Set partitioning for a 32 -cross constellation. The minimum distances between points in the subset are shown.

