INSTRUCTOR'S SOLUTIONS MANUAL

to accompany

CALCULUS

ONE AND SEVERAL VARIABLES

EIGHTH EDITION

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$$\frac{d}{dx}\left(f(y) = f'(y)g'(x) = 1 \quad \Longrightarrow \quad f'(y) = \frac{1}{g'(x)} \text{ provided } g'(x) \neq 0\right.$$

73. $\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt} = (3x^2 - 3)(4t - 1)$

At $t = 2$, $x(2) = 8$ and $\dfrac{dy}{dt} = [3(8)^2 - 3][4(2) - 1] = 1323.$

74. $A = \dfrac{\sqrt{3}}{4}\,x^2$, where $x = \dfrac{2\sqrt{3}}{3}\,h$. Now

$$\frac{dA}{dh} = \frac{dA}{dx}\frac{dx}{dh} = \frac{\sqrt{3}}{2}\,x \cdot \frac{2\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}\,h; \quad \frac{dA}{dh} = 4 \text{ when } h = 2\sqrt{3}$$

75. $V = \frac{4}{3}\pi r^3$ and $\dfrac{dr}{dt} = 2$ cm/sec. By the chain rule, $\dfrac{dV}{dt} = \dfrac{dV}{dr}\dfrac{dr}{dt} = 4\pi r^2 \dfrac{dr}{dt} = 8\pi r^2.$

At the instant the radius is 10 centimeters, the volume is increasing at the rate

$\dfrac{dV}{dt} = 8\pi(10)^2 = 800\pi \ \text{cm}^3/\text{sec}.$

76. $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$, and $\dfrac{dV}{dt} = 200.$

$$\begin{aligned}\frac{dS}{dt} &= \frac{dS}{dr} \cdot \frac{dr}{dV} \cdot \frac{dV}{dt} \\ &= 8\pi r \cdot \frac{1}{4\pi r^2} \cdot 200 \\ &= \frac{400}{r} \\ &= 80 \quad \text{when } r = 5\end{aligned}$$

The surface area is increasing 80 cm^2/sec. at the instant the radius is 5 centimeters.

77. $KE = \frac{1}{2}mv^2$; $\dfrac{d(KE)}{dt} = \dfrac{d(KE)}{dv} \cdot \dfrac{dv}{dt} = mv\,\dfrac{dv}{dt}.$

78. (a) $\dfrac{dF}{dt} = \dfrac{dF}{dr} \cdot \dfrac{dr}{dt} = -\dfrac{2k}{r^3} \cdot (49 - 9.8t) - \dfrac{2k}{(49t - 4.9t^2)^3}(49 - 9.8t),\ 0 \leq t \leq 10$

(b) $\dfrac{dF}{dt}(3) = -\dfrac{2k}{(102.9)^3}(19.6)$; $\quad \dfrac{dF}{dt}(7) = \dfrac{2k}{(102.9)^3}(19.6).$

SECTION 3.6

1. $\dfrac{dy}{dx} = -3\sin x - 4\sec x \tan x$

2. $\dfrac{dy}{dx} = 2x\,\sec x + x^2 \sec x\,\tan x$

3. $\dfrac{dy}{dx} = 3x^2 \csc x - x^3 \csc x \cot x$

4. $\dfrac{dy}{dx} = 2\,\sin x\,\cos x$

5. $\dfrac{dy}{dt} = -2\cos t \sin t$

6. $\dfrac{dy}{dt} = 6t\,\tan t + 3t^2 \sec^2 t$

7. $\dfrac{dy}{du} = 4\sin^3\sqrt{u}\,\dfrac{d}{du}(\sin\sqrt{u}) = 4\sin^3\sqrt{u}\,\cos\sqrt{u}\,\dfrac{d}{du}(\sqrt{u}) = 2u^{-1/2}\sin^3\sqrt{u}\,\cos\sqrt{u}$

8. $\dfrac{dy}{du} = \csc u^2 - 2u^2 \csc u^2 \cot u^2$

9. $\dfrac{dy}{dx} = \sec^2 x^2\,\dfrac{d}{dx}(x^2) = 2x\sec^2 x^2$

(b) $f''(x) = -\frac{2}{9}x^{-4/3} + \frac{2}{9}x^{-5/3} = \dfrac{2(1-x^{1/3})}{9x^{5/3}}$

concave up on $(0,1)$

concave down on $(-\infty,0)\cup(1,\infty)$

vertical tangent at $(0,0)$

41. $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

(a) $f'(x) = x^{2/3} - 2x^{-1/3} = \dfrac{x-2}{x^{1/3}}$

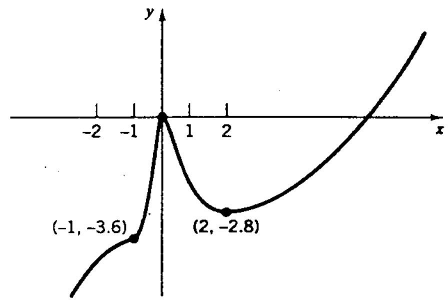
f is increasing on $(-\infty,0]\cup[2,\infty)$

f is decreasing on $[0,2]$

(b) $f''(x) = \frac{2}{3}x^{-1/3} + \frac{2}{3}x^{-4/3} = \dfrac{2x+2}{3x^{4/3}}$

concave up on $(-1,\infty)$; concave down on $(-\infty,-1)$

vertical cusp at $(0,0)$



42. $f(x) = \sqrt{|x|}$

$$= \begin{cases} x^{1/2}, & x \ge 0 \\ (-x)^{1/2}, & x < 0. \end{cases}$$

(a) $f'(x) = \begin{cases} \frac{1}{2}x^{-1/2}, & x > 0 \\ -\frac{1}{2}(-x)^{-1/2}, & x < 0; \end{cases}$

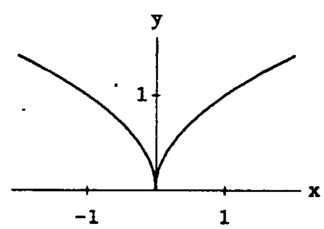
f is increasing on $[0,\infty)$

f is decreasing on $(-\infty,0]$

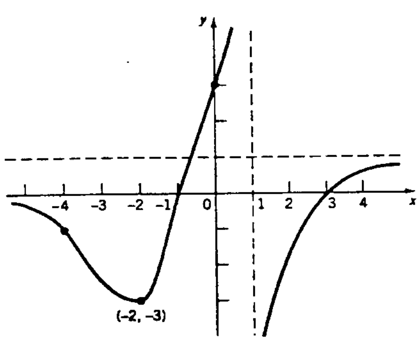
(b) $f''(x) = \begin{cases} -\frac{1}{4}x^{-3/2}, & x > 0 \\ -\frac{1}{4}(-x)^{-3/2}, & x < 0; \end{cases}$

concave down on $(-\infty,0)\cup(0,\infty)$

vertical cusp at $(0,0)$



43.

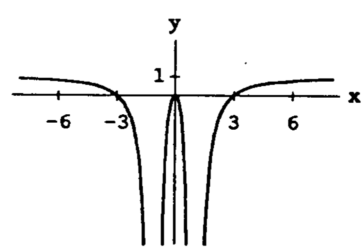


vertical asymptote: $x = 1$

horizontal asymptotes: $y = 0$, $y = 2$

no vertical tangents or cusps

44.

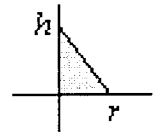


vertical asymptotes: $x = 1$, $x = -1$

horizontal asymptote: $y = 1$

no vertical tangents or cusps

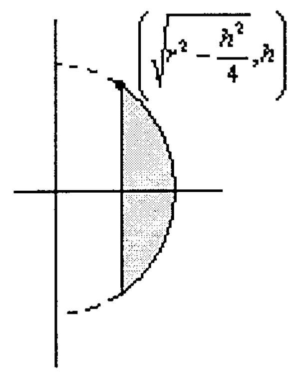
45. $V=\int_0^r 2\pi x(h-\frac{h}{r}x)\,dx=2\pi h\left[\frac{x^2}{2}-\frac{x^3}{3r}\right]_0^r=\frac{\pi r^2 h}{3}.$



46. $V=2\int_{\sqrt{r^2-h^2/4}}^{r} 2\pi x\sqrt{r^2-x^2}\,dx$

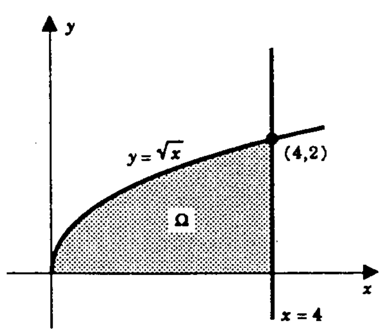
$=-2\pi\int_{h^2/4}^{0} u^{\frac{1}{2}}\,du$ (where $u=r^2-x^2$)

$=2\pi\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^{h^2/4}=\frac{\pi h^3}{6}.$



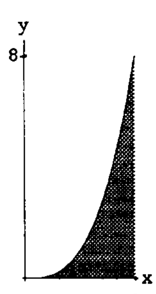
SECTION 6.4

1.



$$\begin{aligned}
A &= \int_0^4 \sqrt{x}\,dx=\frac{16}{3}\\
\overline{x}A &= \int_0^4 x\sqrt{x}\,dx=\frac{64}{5},\quad \overline{x}=\frac{12}{5}\\
\overline{y}A &= \int_0^4 \frac{1}{2}\left(\sqrt{x}\right)^2dx=4,\quad \overline{y}=\frac{3}{4}\\
V_x &= 2\pi\overline{y}A=8\pi,\quad V_y=2\pi\overline{x}A=\frac{128}{5}\pi
\end{aligned}$$

2.



$$\begin{aligned}
A &= \int_0^2 x^3\,dx=4\\
\overline{x}A &= \int_0^2 x\,x^3\,dx=\frac{32}{5},\quad \overline{x}=\frac{8}{5}\\
\overline{y}A &= \int_0^2 \frac{1}{2}(x^3)^2\,dx=\frac{64}{7},\quad \overline{y}=\frac{16}{7}\\
V_x &= 2\pi\overline{y}A=\frac{128}{7}\pi,\quad V_y=2\pi\overline{x}A=\frac{64}{5}\pi
\end{aligned}$$

35. (a) $y^2 = x^2\left(\dfrac{a-x}{a+x}\right)$

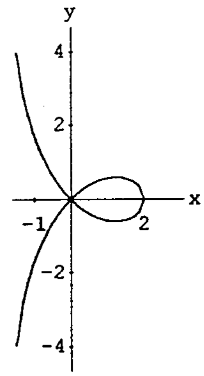
$$r^2\sin^2\theta = r^2\cos^2\theta\left(\frac{a-r\cos\theta}{a+r\cos\theta}\right)$$

$$\sin^2\theta(a+r\cos\theta) = \cos^2\theta(a-r\cos\theta)$$

$$r\cos\theta = a\cos 2\theta$$

$$r = a\cos 2\theta\sec\theta$$

(b) Let $a = 2$



a=2

(c) $A = \displaystyle\int_{3\pi/4}^{5\pi/4} \frac{1}{2}a^2\cos^2 2\theta\sec^2\theta\,d\theta$

$$= 2\int_{3\pi/4}^{5\pi/4} \cos^2 2\theta\sec^2\theta\,d\theta \qquad \text{(a=2)}$$

$$= 2\int_{3\pi/4}^{5\pi/4} \frac{\left(2\cos^2\theta-1\right)^2}{\cos^2\theta}\,d\theta$$

$$= 2\int_{3\pi/4}^{5\pi/4} \left(4\cos^2\theta-4+\sec^2\theta\right)\,d\theta$$

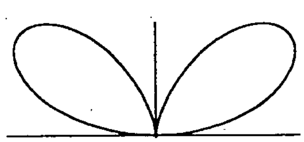
$$= 2\int_{3\pi/4}^{5\pi/4} \left(-2+2\cos 2\theta+\sec^2\theta\right)\,d\theta$$

$$= 2\left[-2\theta+\sin 2\theta+\tan\theta\right]_{3\pi/4}^{5\pi/4} = 8-2\pi$$

36. (a) $(x^2+y^2)^2 = ax^2y \implies r^4 = ar^2\cos^2\theta r\sin\theta \implies r = a\sin\theta\cos^2\theta$

(b) same for all values of a,

with different scale



(c) $A = \displaystyle\int_0^{\pi/2} \frac{1}{2}a^2\sin^2\theta\cos^4\theta\,d\theta$

$$= \frac{a^2}{2}\int_0^{\pi/2}(\cos^4\theta-\cos^6\theta)\,d\theta$$

$$= \frac{a^2}{2}\cdot\frac{\pi}{32} = \frac{\pi a^2}{64}$$

SECTION 9.6

1. $4x = (y-1)^2$

2. $2x+3y = 13$

3. $y = 4x^2+1, \quad x \geq 0$

4. $y = (x+1)^3-5$

5. $9x^2+4y^2 = 36$

6. $x = (y-2)^2+1$

37. $V = \int_0^a \int_0^{\phi(x)} \int_0^{\psi(x,y)} dz\,dy\,dx = \frac{1}{6}abc$ with $\phi(x) = b\left(1 - \frac{x}{a}\right)$, $\psi(x,y) = c\left(1 - \frac{x}{a} - \frac{y}{b}\right)$

$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{1}{4}a, \frac{1}{4}b, \frac{1}{4}c\right)$

38. $I_z = \iiint\limits_T \frac{M}{V}(x^2 + y^2)\,dx\,dy\,dz = \frac{1}{30}\left(\frac{M}{V}\right) = \frac{1}{5}M$

39. $\Pi: 0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c$

(a) $I_z = \int_0^a \int_0^b \int_0^c \frac{M}{abc}\left(x^2 + y^2\right)\,dz\,dy\,dx = \frac{1}{3}M\left(a^2 + b^2\right)$

(b) $I_M = I_z - d^2M = \frac{1}{3}M\left(a^2 + b^2\right) - \frac{1}{4}\left(a^2 + b^2\right)M = \frac{1}{12}M\left(a^2 + b^2\right)$

parallel axis theorem (16.5.7)

(c) $I = I_M + d^2M = \frac{1}{12}M\left(a^2 + b^2\right) + \frac{1}{4}a^2M = \frac{1}{3}Ma^2 + \frac{1}{12}Mb^2$

parallel axis theorem (16.5.7)

40. $V = \int_1^2 \int_1^2 \int_{-2}^{1+x+y} dz\,dy\,dx = \int_1^2 \int_1^2 (3 + x + y)\,dy\,dx = 6$

$\bar{x}V = \int_1^2 \int_1^2 \int_{-2}^{1+x+y} x\,dz\,dy\,dx = \frac{109}{12} \implies \bar{x} = \frac{109}{72} = \bar{y}$ by symmetry

$\bar{z}V = \int_1^2 \int_1^2 \int_{-2}^{1+x+y} z\,dz\,dy\,dx = \frac{73}{12} \implies \bar{z} = \frac{73}{72}.$

41.
$$M = \int_0^1 \int_0^1 \int_0^y k\left(x^2 + y^2 + z^2\right)\,dz\,dy\,dx = \int_0^1 \int_0^1 k\left(x^2y + y^3 + \frac{1}{3}y^3\right)dy\,dx$$
$$= \int_0^1 k\left(\frac{1}{2}x^2 + \frac{1}{3}\right)dx = \frac{1}{2}k$$

$(x_M, y_M, z_M) = \left(\frac{7}{12}, \frac{34}{45}, \frac{37}{90}\right)$

42. T is symmetric (a) about the yz-plane, (b) about the xz-plane, (c) about the xy-plane, (d) about the origin.

43. (a) 0 by symmetry

Therefore,

$$y = e^{2x}\left(-\tfrac{1}{3}x\right) + xe^{2x}\left(\tfrac{1}{3}\ln|x|\right) = -\tfrac{1}{3}xe^{2x} + \tfrac{1}{3}x\ln|x|\,e^{2x}.$$

Note: Since $u = -\tfrac{1}{3}xe^{2x}$ is a solution of the reduced equation,

$$y = \tfrac{1}{3}x\ln|x|\,e^{2x}$$

is also a particular solution of the given equation.

32. $r^2 + 4 = 0 \implies r = \pm 2i.$ Fundamental solutions: $u_1 = \cos 2x, u_2 = \sin 2x.$

Wronskian: $W = u_1u_2' - u_1'u_2 = 2\cos^2 2x + 2\sin^2 2x = 2;$ $\phi(x) = \sec^2 2x$

$$z_1 = -\int \frac{u_2\phi}{W}\,dx = -\int \frac{\sin 2x}{2\cos^2 2x}\,dx = \sec 2x,$$

$$z_2 = \int \frac{u_1\phi}{W}\,dx = \int \frac{\cos 2x}{2\cos^2 2x}\,dx = \frac{1}{2}\int \sec 2x\,dx = \ln|\sec 2x + \tan 2x|.$$

Therefore $y_p = z_1u_1 + z_2u_2 = \sec 2x\cos 2x + \ln|\sec 2x + \tan 2x|\sin 2x = 1 + \ln|\sec 2x + \tan 2x|\sin 2x.$

33. First consider the reduced equation $y'' + 4y' + 4y = 0.$ The characteristic equation is:

$$r^2 + 4r + 4 = (r+2)^2 = 0$$

and $u_1(x) = e^{-2x},$ $u_2(x) = xe^{-2x}$ are fundamental solutions. Their Wronskian is given by

$$W = u_1u_2' - u_2u_1' = e^{-2x}\left(e^{-2x} - 2xe^{2x}\right) - xe^{-2x}(-2e^{-2x}) = e^{-4x}.$$

Using variation of parameters, a particular solution of the given equation will have the form

$$y = u_1z_1 + u_2z_2,$$

where

$$z_1 = -\int \frac{xe^{-2x}\left(x^{-2}e^{-2x}\right)}{e^{-4x}}\,dx = -\int \frac{1}{x}\,dx = -\ln|x|$$

$$z_2 = \int \frac{e^{-2x}\left(x^{-2}e^{-2x}\right)}{e^{-4x}}\,dx = \int \frac{1}{x^2}\,dx = -\frac{1}{x}$$

Therefore,

$$y = e^{-2x}\left(-\ln|x|\right) + xe^{-2x}\left(-\frac{1}{x}\right) = -e^{-2x}\ln|x| - e^{-2x}.$$

Note: Since $u = -e^{-2x}$ is a solution of the reduced equation, we can take

$$y = \tfrac{1}{3}x\ln|x|e^{2x}$$

34. $r^2 + 2r + 1 = 0 \implies r = -1.$ Fundamental solutions: $u_1 = e^{-x},\ u_2 = xe^{-x}.$

Wronskian: $W = u_1u_2' - u_1'u_2 = (1-x)e^{-2x} + xe^{-2x} = e^{-2x};$ $\phi(x) = e^{-x}\ln x.$

$$z_1 = -\int \frac{u_2\phi}{W}\,dx = -\int \frac{xe^{-x}e^{-x}\ln x}{e^{-2x}}\,dx = -\int x\ln x\,dx = -\frac{1}{2}x^2\ln x + \frac{x^2}{4},$$