Chapter 2

Introduction to Engineering Calculations

2.1 Unit conversion

(a)

From Table A.9 (Appendix A): 1 cP = 10^{-3} kg m^{-1} s^{-1}

1 m = 100 cm

Therefore:

$$1.5\times10^{-6}\text{ cP} = 1.5\times10^{-6}\text{ cP}\,.\left|\frac{10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}}{1\text{ cP}}\right|.\left|\frac{1\text{ m}}{100\text{ cm}}\right| = 1.5\times10^{-11}\text{ kg s}^{-1}\text{ cm}^{-1}$$

Answer: 1.5 × 10^{-11} kg s^{-1} cm^{-1}

(b)

From Table A.8 (Appendix A): 1 hp (British) = 42.41 Btu min^{-1}

Therefore:

$$0.122\text{ hp} = 0.122\text{ hp}\,.\left|\frac{42.41\text{ Btu min}^{-1}}{1\text{ hp}}\right| = 5.17\text{ Btu min}^{-1}$$

Answer: 5.17 Btu min^{-1}

(c)

1 min = 60 s

rpm means revolutions per minute. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rpm are min^{-1}. Therefore:

$$10{,}000\text{ min}^{-1} = 10{,}000\text{ min}^{-1}\,.\left|\frac{1\text{ min}}{60\text{ s}}\right| = 167\text{ s}^{-1}$$

Answer: 167 s^{-1}

(d)

From Table A.8 (Appendix A): 1 W = 1 J s^{-1}

From Table A.7 (Appendix A): 1 J = 9.869 × 10^{-3} l atm

From Table A.1 (Appendix A): 1 m = 3.281 ft

1 min = 60 s

As explained in Section 2.4.6, °C^{-1} is the same as K^{-1}. Therefore:

$$4335\text{ W m}^{-2}\,{}^\circ\text{C}^{-1} = 4335\text{ W m}^{-2}\,{}^\circ\text{C}^{-1}.\left|\frac{1\text{ J s}^{-1}}{1\text{ W}}\right|.\left|\frac{9.869\times10^{-3}\text{ l atm}}{1\text{ J}}\right|.\left|\frac{60\text{ s}}{1\text{ min}}\right|.\left|\frac{1\text{ m}}{3.281\text{ ft}}\right|^2$$

$$= 238.45\text{ l atm min}^{-1}\text{ft}^{-2}\text{ K}^{-1}$$

Answer: 238 l atm min^{-1} ft^{-2} K^{-1}

1 kg = 1000 g

Therefore:

$$10^3 \text{ g l}^{-1} = 10^3 \text{ g l}^{-1} \cdot \left|\frac{1 \text{ kg}}{1000 \text{ g}}\right| \cdot \left|\frac{10^3 \text{ l}}{1 \text{ m}^3}\right| = 10^3 \text{ kg m}^{-3}$$

Answer: 10^3 kg m^{-3}

2.3 Unit conversion

(a)

From Table A.2 (Appendix A): 1 m^3 = 10^3 l

1 g = 10^6 μg

1 l = 1000 ml

Therefore:

$$10^6 \text{ μg ml}^{-1} = 10^6 \text{ μg ml}^{-1} \cdot \left|\frac{1 \text{ g}}{10^6 \text{ μg}}\right| \cdot \left|\frac{1000 \text{ ml}}{1 \text{ l}}\right| \cdot \left|\frac{10^3 \text{ l}}{1 \text{ m}^3}\right| = 10^6 \text{ g m}^{-3}$$

Answer: 10^6 g m^{-3}

(b)

From Table A.9 (Appendix A): 1 cP = 10^{-3} Pa s

1 Pa s = 1000 mPa s

Therefore:

$$3.2 \text{ cP} = 3.2 \text{ cP} \cdot \left|\frac{10^{-3} \text{ Pa s}}{1 \text{ cP}}\right| \cdot \left|\frac{1000 \text{ mPa s}}{1 \text{ Pa s}}\right| = 3.2 \text{ mPa s}$$

Answer: 3.2 mPa s

(c)

From Table A.7 (Appendix A): 1 Btu = 1.055 × 10^3 J

From Table A.8 (Appendix A): 1 W = 1 J s^{-1}

From Table A.1 (Appendix A): 1 ft = 0.3048 m

1 h = 3600 s

From Section 2.4.6, a temperature difference of 1 K corresponds to a temperature difference of 1.8 °F. Therefore:

$$150 \text{ Btu h}^{-1} \text{ ft}^{-2} (\text{°F ft}^{-1})^{-1} = 150 \text{ Btu h}^{-1} \text{ ft}^{-1} \text{ °F}^{-1} \cdot \left|\frac{1.055 \times 10^3 \text{ J}}{1 \text{ Btu}}\right| \cdot \left|\frac{1 \text{ h}}{3600 \text{ s}}\right| \cdot \left|\frac{1 \text{ ft}}{0.3048 \text{ m}}\right| \cdot \left|\frac{1.8 \text{ °F}}{1 \text{ K}}\right| \cdot \left|\frac{1 \text{ W}}{1 \text{ J s}^{-1}}\right|$$

$$= 259.6 \text{ W m}^{-1} \text{ K}^{-1}$$

Answer: 260 W m^{-1} K^{-1}

$$Gr = \frac{(2\text{ mm})^3\,(1.30\times10^{-3}\text{ g cm}^{-3})(0.9962652 - 1.30\times10^{-3})\text{ g cm}^{-3}\,(980.66\text{ cm s}^{-2})\,.\left|\frac{1\text{ cm}}{10\text{ mm}}\right|^3}{(0.87\text{ cP})^2\,.\left|\frac{10^{-2}\text{ g cm}^{-1}\text{ s}^{-1}}{1\text{ cP}}\right|^2}$$

$$= 134$$

Similarly for the Schmidt number:

$$Sc = \frac{0.87\text{ cP}\,.\left|\frac{10^{-2}\text{ g cm}^{-1}\text{ s}^{-1}}{1\text{ cP}}\right|}{(0.9962652\text{ g cm}^{-3})\,(2.5\times10^{-5}\text{ cm}^2\text{ s}^{-1})} = 349$$

Therefore:

$$Sh = 0.31\,(134)^{1/3}\,(349)^{1/3} = 11.2$$

From the equation for *Sh*:

$$k_L = \frac{Sh\,\mathscr{D}}{D_b} = \frac{(11.2)(2.5\times10^{-5}\text{ cm}^2\text{ s}^{-1})}{2\text{ mm}\,.\left|\frac{1\text{ cm}}{10\text{ mm}}\right|} = 1.40\times10^{-3}\text{ cm s}^{-1}$$

Answer: 1.40×10^{-3} cm s^{-1}

2.8 Dimensionless numbers and dimensional homogeneity

First, evaluate the units of the groups ($C_p\mu/k$) and (DG/μ):

$$\text{Units of}\left(\frac{C_p\mu}{k}\right) = \frac{(\text{Btu lb}^{-1}\,°\text{F}^{-1})\text{ lb h}^{-1}\text{ ft}^{-1}}{\text{Btu h}^{-1}\text{ ft}^{-2}\,(°\text{F ft}^{-1})^{-1}} = 1$$

$$\text{Units of}\left(\frac{DG}{\mu}\right) = \frac{(\text{ft})\text{ lb h}^{-1}\text{ ft}^{-2}}{\text{lb h}^{-1}\text{ ft}^{-1}} = 1$$

Therefore, these groups are dimensionless. For the equation to be dimensionally homogeneous, (h/C_pG) must also be dimensionless; the units of h must therefore cancel the units of C_pG.

$$\text{Units of } h = \text{units of } C_pG = (\text{Btu lb}^{-1}\,°\text{F}^{-1})\,(\text{lb h}^{-1}\text{ ft}^{-2}) = \text{Btu }°\text{F}^{-1}\text{ h}^{-1}\text{ ft}^{-2}$$

The dimensions of h can be deduced from its units. From Table A.7 (Appendix A), Btu is a unit of energy with dimensions = L^2MT^{-2}. °F is a unit of temperature which, from Table 2.1, has the dimensional symbol Θ. h (hour) is a unit of time with dimension T; ft is a unit of length with dimension L. Therefore:

$$\text{Dimensions of } h = L^2MT^{-2}\Theta^{-1}T^{-1}L^{-2} = MT^{-3}\Theta^{-1}$$

Answer: Units = Btu °F^{-1} h^{-1} ft^{-2}; dimensions = $MT^{-3}\Theta^{-1}$

2.9 Dimensional homogeneity

λ has dimensions L. ε has units W kg^{-1}; therefore, from Tables A.8 and A.3 in Appendix A, the dimensions of ε are $L^2MT^{-3}M^{-1} = L^2T^{-3}$. Substituting this information into the equation for λ, for dimensional homogeneity:

$$L = \left(\frac{(\text{dimensions of }\nu)^3}{L^2T^{-3}}\right)^{1/4} = \frac{(\text{dimensions of }\nu)^{3/4}}{L^{1/2}T^{-3/4}}$$

The stoichiometric coefficient f_1 is determined from the ethanol yield $Y_{PS1} = 0.21$ g g^{-1} and Eq. (4.17):

$$f_1 = \frac{Y_{PS1}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.21\text{ g g}^{-1}\,(180)}{46} = 0.82$$

Similarly for f_2 using the glycerol yield $Y_{PS2} = 0.07$ g g^{-1}:

$$f_2 = \frac{Y_{PS2}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.07\text{ g g}^{-1}\,(180)}{92} = 0.14$$

Because fructose has the same molecular formula as glucose, from Table C.2 (Appendix C), we can say that the degree of reduction of fructose relative to NH_3 is $\gamma_S = 4.00$. Also from Table C.2, the degrees of reduction of ethanol and glycerol relative to NH_3 are $\gamma_{P1} = 6.00$ and $\gamma_{P2} = 4.67$, respectively. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.8\times 1 - 0.5\times 2 - 0.2\times 3}{1} = 4.20$$

(This value for γ_B is also given in Table C.2.) The oxygen demand is calculated using a modified form of Eq. (4.20) to account for transfer of electrons to two separate products, with $w = 6$ for fructose, $j_1 = 2$ for ethanol and $j_2 = 3$ for glycerol:

$$\begin{aligned} a &= \frac{1}{4}(w\gamma_S - c\gamma_B - f_1 j_1 \gamma_{P1} - f_2 j_2 \gamma_{P2}) \\ &= \frac{1}{4}(6\times 4.00 - 0.18\times 4.20 - 0.82\times 2\times 6.00 - 0.14\times 3\times 4.67) \\ &= 2.86 \end{aligned}$$

Therefore, 2.86 gmol of oxygen are required per gmol of fructose consumed. Converting the rate of fructose consumption to gmol h^{-1}:

$$190\text{ g fructose h}^{-1} = 190\text{ g h}^{-1}\cdot\left|\frac{1\text{ gmol}}{180\text{ g}}\right| = 1.056\text{ gmol h}^{-1}$$

From the result for a, the oxygen requirement is 2.86 × 1.056 gmol h^{-1} = 3.02 gmol h^{-1}. Converting this to a mass basis using the molecular weight of O_2 = 32 (Table C.1, Appendix C):

$$3.02\text{ gmol O}_2\text{ h}^{-1} = 3.02\text{ gmol h}^{-1}\cdot\left|\frac{32\text{ g}}{1\text{ gmol}}\right| = 96.6\text{ g h}^{-1}$$

Answer: 97 g h^{-1}

(b)

Calculation of RQ using Eq. (4.9) requires the stoichiometric coefficient d, which can be obtained from an elemental balance on C.

C balance: $6 = c + d + 2f_1 + 3f_2$

Substituting values for c, f_1 and f_2 from **(a)**:

$$d = 6 - 0.18 - 2\times 0.82 - 3\times 0.14 = 3.76$$

Applying Eq. (4.9) with the value of a from **(a)**:

$$RQ = \frac{3.76}{2.86} = 1.31$$

Answer: 1.3

$$\ln C_A = \frac{-k_1}{u} z + K$$

Applying the initial condition from **(b)** at $z = 0$, $\ln C_{Ai} = K$. Substituting this value of K into the equation:

$$\ln C_A = \frac{-k_1}{u} z + \ln C_{Ai}$$

$$\ln \frac{C_A}{C_{Ai}} = \frac{-k_1}{u} z$$

$$C_A = C_{Ai}\, e^{(-k_1/u)z}$$

Answer: $C_A = C_{Ai}\, e^{(-k_1/u)z}$

(d)

The equation derived in **(c)** is directly analogous to the equation for the reactant concentration in a batch reactor as a function of time. As $z = ut$ where t is the time taken for the fluid to travel distance z, the above equation can be written as:

$$C_A = C_{Ai}\, e^{-k_1 t}$$

which is the same as the equation for reactant concentration in a batch reactor where C_{Ai} is the concentration at time zero.

Answer: Essentially identical

6.11 Sequential batch reactors

The seed and production fermenters are operated as separate batch systems. The general unsteady-state mass balance equation for each fermenter is Eq. (6.5). For a batch culture, $\hat{M}_i = \hat{M}_o = 0$. For a mass balance on cells, assuming that there is no loss of cells from the system such as by lysis, $R_C = 0$. From the equation provided, the rate of generation of cells $R_G = r_X V = kxV$ where k is the rate constant, x is the cell concentration and V is the culture volume. The total mass of cells in the fermenter M is equal to the culture volume V multiplied by the cell concentration x: $M = Vx$. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(Vx)}{\mathrm{d}t} = kxV$$

Assuming that V is constant for each batch fermenter, it can be taken outside of the differential and cancelled:

$$V \frac{\mathrm{d}x}{\mathrm{d}t} = kxV$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = kx$$

The differential equation contains only two variables, x and t. Separating variables and integrating:

$$\frac{\mathrm{d}x}{x} = k\,\mathrm{d}t$$

$$\int \frac{\mathrm{d}x}{x} = \int k\,\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln x = kt + K$$

$$1.67\times10^{-3}=e^{-V_D}$$

where V_D has units of m^3. Taking the logarithm of both sides and applying Eq. (E.3) from Appendix E:

$$-6.39=-V_D$$

Therefore:

$$V_D=6.4\ m^3$$

Answer: 6.4 m^3

(b)

In this case, V_0 = 5 m^3. Applying (1):

$$\frac{0.002\%}{1.2\%}=e^{-V_D/(5\ m^3)}$$

$$1.67\times10^{-3}=e^{-0.2V_D}$$

where V_D has units of m^3. Solving this equation gives:

$$-6.39=-0.2V_D$$

$$V_D=32.0\ m^3$$

Answer: 32 m^3

11.28 Scale-up of virus ultrafiltration

For the pilot-scale filtration, u = 0.45 m s^{-1} and J = 27 l m^{-2} h^{-1}. If $J = Cu^{0.66}$ where C is a proportionality constant, when J has units of l m^{-2} h^{-1} and u has units of m s^{-1}:

$$C=\frac{J}{u^{0.66}}=\frac{27}{(0.45)^{0.66}}=45.7$$

For the large-scale filtration, u = 2.2 m s^{-1}. As $0.40 < u < 3.5$ m s^{-1}:

$$J=Cu^{0.66}=45.7\,(2.2)^{0.66}=76.9\ l\ m^{-2}\ h^{-1}$$

F_0 = (100 m^3)/(1 h) = 100 m^3 h^{-1}. From Eq. (11.72), to achieve a concentration factor C_R/C_0 = 2.5 for the virus with R = 1, VCR = 2.5. Therefore, from Eq. (11.100):

$$F_R=\frac{F_0}{VCR}=\frac{100\ m^3\ h^{-1}}{2.5}=40\ m^3\ h^{-1}$$

From Eq. (11.98):

$$F_P=F_0-F_R$$

Therefore:

$$F_P=100\ m^3\ h^{-1}-40\ m^3\ h^{-1}=60\ m^3\ h^{-1}$$

Applying Eq. (11.48):

$$A=\frac{F_P}{J}$$

Substituting values for the large-scale filtration gives:

14.8 Bioreactor design for immobilised enzymes

$s_0 = s_i$ = 10% (w/v) = 10 g per 100 ml = 100 g l^{-1} = 100 kg m^{-3}. $s_f = s = 0.01 \times 100$ kg m^{-3} = 1 kg m^{-3}. Based on the unsteady-state mass balance equation for first-order reaction derived in Example 6.1 (Chapter 6), the equation for the rate of change of substrate concentration in a batch reactor is:

$$\frac{\mathrm{d}(Vs)}{\mathrm{d}t} = -k_1 s V$$

where V is the reaction volume and k_1 is the reaction rate constant. As V can be considered constant in a batch reactor, this term can be taken outside of the differential and cancelled from both sides of the equation:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -k_1 s$$

The differential equation contains only two variables, s and t. Separating variables and integrating:

$$\frac{\mathrm{d}s}{s} = -k_1\,\mathrm{d}t$$

$$\int \frac{\mathrm{d}s}{s} = \int -k_1 \mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln s = -k_1 t + K$$

The initial condition is: at $t = 0$, $s = s_0$. Therefore, from the equation, $\ln s_0 = K$. Substituting this expression for K gives:

$$\ln s = -k_1 t + \ln s_0$$

$$\ln \frac{s}{s_0} = -k_1 t$$

$$t = \frac{-\ln \dfrac{s}{s_0}}{k_1}$$

The batch culture time t_b is the time required for the substrate concentration to reach s_f:

$$t_b = \frac{-\ln \dfrac{s_f}{s_0}}{k_1} = \frac{-\ln \left(\dfrac{1\ \text{kg m}^{-3}}{100\ \text{kg m}^{-3}} \right)}{0.8 \times 10^{-4}\ \text{s}^{-1} \cdot \left| \dfrac{3600\ \text{s}}{1\ \text{h}} \right|} = 16.0\ \text{h}$$

If the downtime between batches t_{dn} is 20 h, from Eq. (14.33):

$$t_T = 16.0\ \text{h} + 20\ \text{h} = 36\ \text{h}$$

Therefore, in one year or 365 days, the number of batches carried out is:

$$\text{Number of batches} = \frac{365\ \text{days} \cdot \left| \dfrac{24\ \text{h}}{1\ \text{day}} \right|}{36\ \text{h per batch}} = 243$$