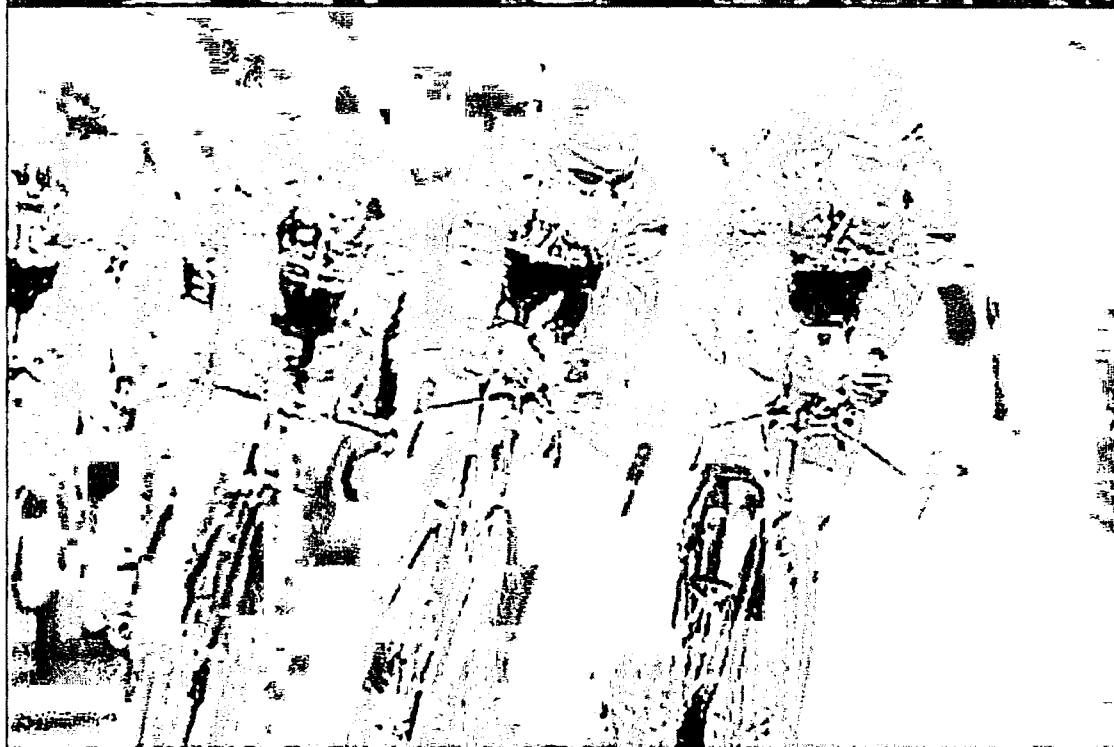


INSTRUCTOR'S SOLUTIONS MANUAL

SEVENTH EDITION

ANALYTICAL MECHANICS



FOWLES & CASSIDAY

$$\begin{aligned}
 1.24 \quad \frac{d}{dt} [\vec{r} \cdot (\vec{v} \times \vec{a})] &= \frac{d\vec{r}}{dt} \cdot (\vec{v} \times \vec{a}) + \vec{r} \cdot \frac{d}{dt} (\vec{v} \times \vec{a}) \\
 &= \vec{v} \cdot (\vec{v} \times \vec{a}) + \vec{r} \cdot \left[\left(\frac{d\vec{v}}{dt} \times \vec{a} \right) + \left(\vec{v} \times \frac{d\vec{a}}{dt} \right) \right] \\
 &= 0 + \vec{r} \cdot [0 + (\vec{v} \times \dot{\vec{a}})] \\
 \frac{d}{dt} [\vec{r} \cdot (\vec{v} \times \vec{a})] &= \vec{r} \cdot (\vec{v} \times \dot{\vec{a}})
 \end{aligned}$$

$$1.25 \quad \vec{v} = v\hat{r} \text{ and } \vec{a} = a_r\hat{r} + a_n\hat{n}$$

$$\vec{v} \cdot \vec{a} = va_r, \text{ so } a_r = \frac{\vec{v} \cdot \vec{a}}{v}$$

$$a^2 = a_r^2 + a_n^2, \text{ so } a_n = (a^2 - a_r^2)^{\frac{1}{2}}$$

$$1.26 \quad \text{For 1.14, } a_r = \frac{-b^2\omega^3 \cos \omega t \cdot \sin \omega t + b^2\omega^3 \sin \omega t \cdot \cos \omega t + 4c^2t}{(b^2\omega^2 \cos^2 \omega t + b^2\omega^2 \sin^2 \omega t + 4c^2t^2)^{\frac{1}{2}}}$$

$$a_r = \frac{4c^2t}{(b^2\omega^2 + 4c^2t^2)^{\frac{1}{2}}}$$

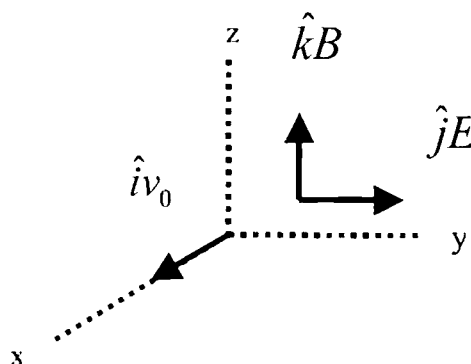
$$a_n = \left(b^2\omega^2 + 4c^2 - \frac{16c^4t^2}{b^2\omega^2 + 4c^2t^2} \right)^{\frac{1}{2}}$$

$$\text{For 1.15, } a_r = \frac{b^2k(k^2 - c^2)e^{2kt} + 2b^2c^2ke^{2kt}}{be^{kt}(k^2 + c^2)^{\frac{1}{2}}} = bke^{kt}(k^2 + c^2)^{\frac{1}{2}}$$

$$a_n = \left[b^2e^{2kt}(k^2 + c^2)^2 - b^2k^2e^{2kt}(k^2 + c^2) \right]^{\frac{1}{2}} = bce^{kt}(k^2 + c^2)^{\frac{1}{2}}$$

$$1.27 \quad \vec{v} = v\hat{r}, \quad \vec{a} = \dot{v}\hat{r} + \frac{v^2}{\rho}\hat{n}$$

$$|\vec{v} \times \vec{a}| = v \cdot a_n = v \frac{v^2}{\rho} = \frac{v^3}{\rho}$$



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}) \times \hat{k}B = \dot{y}B\hat{i} - \dot{x}B\hat{j}$$

$$\vec{F} = \hat{i}q\dot{y}B + \hat{j}q(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{qB}{m}y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m}y\right)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}_0}{m} - \left(\frac{qB}{m}\right)^2 y = -\frac{eE}{m} + \frac{eB\dot{x}_0}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega\dot{x}_0, \quad \omega = \frac{eB}{m}$$

$$y = \frac{1}{\omega^2} \left(-\frac{eE}{m} + \omega\dot{x}_0 \right) + A \cos(\omega t + \theta_0)$$

$$\dot{y} = -A\omega \sin(\omega t + \theta_0)$$

$$\dot{y}_0 = 0, \text{ so } \theta_0 = 0$$

$$y_0 = 0, \text{ so } A = -\frac{1}{\omega^2} \left(-\frac{eE}{m} + \omega\dot{x}_0 \right)$$

$$y = a(1 - \cos \omega t), \quad a = \frac{1}{\omega^2} \left(-\frac{eE}{m} + \omega\dot{x}_0 \right)$$

$$\dot{x} = \dot{x}_0 + \frac{qB}{m}y = \dot{x}_0 - \omega y = \dot{x}_0 - \omega a(1 - \cos \omega t)$$

$$\dot{x} = (\dot{x}_0 - \omega a) + \omega a \cos \omega t$$

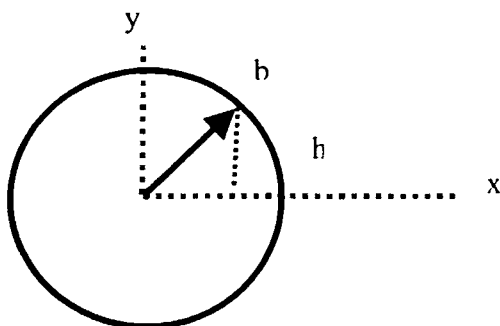
$$x = (\dot{x}_0 - \omega a)t + a \sin \omega t$$

$$x = a \sin \omega t + bt, \quad b = \dot{x}_0 - \omega a$$

$$m\ddot{z} = F_z = 0$$

$$z = \dot{z}_0 t + z_0 = 0$$

4.21



$$\frac{1}{2}mv^2 + mgh = mg\frac{b}{2}$$

$$v^2 = g(b - 2h)$$

$$F_r = -\frac{mv^2}{b} = -mg \cos \theta + R$$

For $r = a\theta$, and the force to be central, try $\theta = bt^n$

$$f(\theta) = m[2ab^2n^2t^{2n-2} + ab^2n(n-1)t^{2n-2}]$$

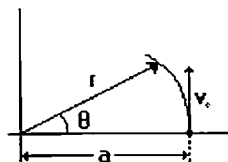
For a central field ... $f(\theta) = 0$

$$2n + (n-1) = 0$$

$$n = \frac{1}{3}$$

$$\theta = bt^{\frac{1}{3}}$$

6.14



(a)

Calculating the potential energy

$$-\frac{dv}{dr} = f(r) = -k\left(\frac{4}{r^3} + \frac{a^2}{r^5}\right)$$

Thus, $V = -k\left(\frac{2}{r^2} + \frac{a^2}{4r^4}\right)$

The total energy is ...

$$E = T_v + V_v = \frac{1}{2}v_v^2 - k\left(\frac{2}{a^2} + \frac{1}{4a^2}\right) = \frac{1}{2}\left(\frac{9k}{2a^2}\right) - \frac{9k}{4a^2} = 0$$

Its angular momentum is ...

$$l^2 = a^2v_v^2 = \frac{9k}{2} = \text{constant} = r^4\dot{\theta}^2$$

Its KE is ...

$$T = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) = \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\dot{\theta}^2 = \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\frac{l^2}{r^4}$$

The energy equation of the orbit is ...

$$\begin{aligned} T + V = 0 &= \frac{1}{2}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\frac{l^2}{r^4} - k\left(\frac{2}{r^2} + \frac{a^2}{4r^4}\right) \\ &= \left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]\frac{9k}{4r^4} - k\left(\frac{2}{r^2} + \frac{a^2}{4r^4}\right) \end{aligned}$$

or $\left(\frac{dr}{d\theta}\right)^2 = \frac{1}{9}(a^2 - r^2)$

Letting $r = a \cos \phi$ then $\frac{dr}{d\theta} = -a \sin \phi \frac{d\phi}{d\theta}$

So $\left(\frac{d\phi}{d\theta}\right)^2 = \frac{1}{9} \therefore \phi = \frac{1}{3}\theta$

Chapter 7

Dynamics of Systems of Particles

7.1 From eqn. 7.1.1, $\vec{r}_{cm} = \frac{1}{m} \sum_i m_i \vec{r}_i$

$$\vec{r}_{cm} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) = \frac{1}{3}(\hat{i} + \hat{j} + \hat{j} + \hat{k} + \hat{k})$$

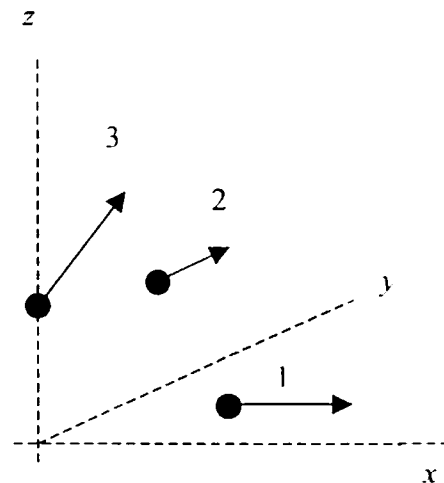
$$\vec{r}_{cm} = \frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{v}_{cm} = \frac{d}{dt} \vec{r}_{cm} = \frac{1}{3}(\vec{v}_1 + \vec{v}_2 + \vec{v}_3) = \frac{1}{3}(2\hat{i} + \hat{j} + \hat{i} + \hat{j} + \hat{k})$$

$$\vec{v}_{cm} = \frac{1}{3}(3\hat{i} + 2\hat{j} + \hat{k})$$

From eqn 7.1.3, $\vec{p} = \sum_i m_i \vec{v}_i = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$

$$\vec{p} = 3\hat{i} + 2\hat{j} + \hat{k}$$



7.2 (a) From eqn. 7.2.15, $T = \sum_i \frac{1}{2} m_i v_i^2$

$$T = \frac{1}{2} [2^2 + 1^2 + (1^2 + 1^2 + 1^2)] = 4$$

(b) From Prob. 7.1, $\vec{v}_{cm} = \frac{1}{3}(3\hat{i} + 2\hat{j} + \hat{k})$

$$\frac{1}{2} m v_{cm}^2 = \frac{1}{2} \times 3 \times \frac{1}{9} (3^2 + 2^2 + 1^2) = 2 \frac{1}{3}$$

(c) From eqn. 7.2.8, $\vec{L} = \sum_i \vec{r}_i \times m \vec{v}_i$

$$\vec{L} = [(\hat{i} + \hat{j}) \times 2\hat{i}] + [(\hat{j} + \hat{k}) \times \hat{j}] + [\hat{k} \times (\hat{i} + \hat{j} + \hat{k})]$$

$$\vec{L} = (-2\hat{k}) + (-\hat{i}) + (\hat{j} - \hat{i}) = -2\hat{i} + \hat{j} - 2\hat{k}$$

7.3 $\vec{v}_c = \vec{v}_b - \vec{v}_g$

Since momentum is conserved and the bullet and gun were initially at rest:

$$0 = m\vec{v}_b + M\vec{v}_g$$

$$\vec{v}_g = -\gamma \vec{v}_b, \quad \gamma = \frac{m}{M}$$

$$\vec{v}_c = (1 + \gamma) \vec{v}_b$$

$$\vec{v}_b = \frac{\vec{v}_c}{1 + \gamma}$$

$$\begin{aligned}
 &= C - \frac{1}{2}gt^2 - \frac{gt}{k} \ln(1-kt) - \frac{g}{k} \int \left(-1 + \frac{1}{1-kt}\right) dt \\
 &= C - \frac{1}{2}gt^2 - \frac{gt}{k} \ln(1-kt) + \frac{gt}{k} + \frac{g}{k^2} \ln(1-kt) \\
 &= C - \frac{1}{2}gt^2 + \frac{g}{k^2}(1-kt) \ln(1-kt) + \frac{gt}{k}
 \end{aligned}$$

but $y = 0$ at $t = 0$ so $C = 0$

$$y = \frac{gt}{k} - \frac{1}{2}gt^2 + \frac{g}{k^2}(1-kt) \ln(1-kt)$$

and at $t = t_*$

$$(a) \quad H = \frac{gt_*^2}{2m_*^2} \left[(2M + m_*)m_* + 2M(M + m_*) \ln\left(\frac{M}{M + m_*}\right) \right]$$

$$(b) \quad v = \frac{gt_*}{m_*} \left[(M + m_*) \ln\left(\frac{M + m_*}{M}\right) - m_* \right]$$

(c) letting $\varepsilon = \frac{m_*}{M} \ll 1$ we have

$$\begin{aligned}
 H &= \frac{gt_*^2}{2\varepsilon^2} \left[(2 + \varepsilon)\varepsilon - 2(1 + \varepsilon) \ln(1 + \varepsilon) \right] \\
 &= \frac{gt_*^2}{2\varepsilon^2} \left[2\varepsilon + \varepsilon^2 - 2(1 + \varepsilon) \left(\varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \dots \right) \right] \\
 H &\cong \frac{gt_*^2}{6} \varepsilon
 \end{aligned}$$

Similarly:

$$\begin{aligned}
 v &= \frac{gt_*}{\varepsilon} \left[(1 + \varepsilon) \ln(1 + \varepsilon) - \varepsilon \right] \\
 &= \frac{gt_*}{\varepsilon} \left[(1 + \varepsilon) \left(\varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \dots \right) - \varepsilon \right] \\
 &\approx \frac{1}{2}gt_*\varepsilon
 \end{aligned}$$

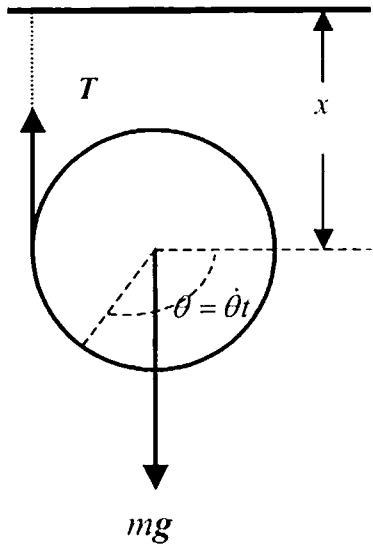
$$(d) \quad H = 327m; \quad v = 9.8ms^{-1}$$

$$7.26 \quad \dot{m} = -k \text{ or } m = m_* - kt$$

$$\text{Burn-out occurs at time } T = \frac{\varepsilon m_*}{k}$$

So – the rocket equation (7.7.7) becomes

$$m \frac{dv}{dt} = -V\dot{m} \quad (-) \text{ since } V \text{ is oppositely directed to } \dot{v}$$



$$L = T - V = \frac{1}{2} m \dot{x}^2 + \frac{1}{5} m a^2 \dot{\theta}^2 + mgx$$

The equation of constraint is ...

$$f(x, \theta) = x - a\theta = 0$$

The 2 Lagrange equations with multipliers are ...

$$\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} + \lambda \frac{\partial f}{\partial x} = 0$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} + \lambda \frac{\partial f}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = m\ddot{x} \quad \frac{\partial L}{\partial x} = mg \quad \lambda \frac{\partial f}{\partial x} = \lambda$$

$$mg - m\ddot{x} + \lambda = 0 \quad \text{and from the } \theta\text{-equation ...}$$

$$-\frac{2}{5} m a^2 \ddot{\theta} - \lambda a = 0$$

Differentiating the equation of constraint ... $\ddot{\theta} = \frac{\ddot{x}}{a}$ and substituting into the above ...

$$\ddot{x} = \frac{5}{7} g \quad \text{and} \quad \lambda = -\frac{2}{7} mg$$

(b) The generalized force that is equivalent to the tension T is ...

$$Q_x = \lambda \frac{\partial f}{\partial x} = \lambda = -\frac{2}{7} mg$$

10.16 For \vec{v}' the velocity of a differential mass element, dm , of the spring at a distance x' below the support ...

$$\vec{v}' = \frac{x'}{x} \vec{v}, \quad dm = \frac{m'}{x} dx'$$

$$T = \frac{1}{2} m v^2 + \int_0^x \frac{1}{2} (v')^2 dm = \frac{1}{2} m \dot{x}^2 + \int_0^x \frac{1}{2} \left(\frac{x'}{x} \dot{x} \right)^2 \frac{m'}{x} dx'$$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \frac{m'}{3} \dot{x}^2$$

$$V = \frac{1}{2} k (x-l)^2 - mgx - \int_0^x gx' dm'$$

$$= \frac{1}{2} k (x-l)^2 - mgx - \int_0^x gx' \frac{m'}{x} dx'$$

$$V = \frac{1}{2} k (x-l)^2 - mgx - \frac{m'}{2} gx$$

$$L = T - V = \frac{1}{2} \left(m + \frac{m'}{3} \right) \dot{x}^2 - \frac{1}{2} k (x-l)^2 + \left(m + \frac{m'}{2} \right) gx$$

$$\frac{\partial L}{\partial \dot{x}} = \left(m + \frac{m'}{3} \right) \dot{x}, \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \left(m + \frac{m'}{3} \right) \ddot{x},$$