# SOLUTIONS MANUAL

Second Edition

# ADVANCED MECHANICS OF MATERIALS

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4.7-18(a) $v_C = 0.149PR^3/EI$ (b) $\Delta_{BD} = -0.137PR^3/EI$

(c) $M_C = 0$, $w_C = (\pi R^2 M_o/4)(1/GK + 1/EI)$ (d) $T_C = M_o/\pi$

(e) $M_C = 2PR/\pi[1 + (GK/EI)]$ (f) $M_B = 0.429\ qR^2$

(g) $v_C = -\rho R^5\omega^2/6EI$

4.7-19 $M_A = (2T_o a/b)[(bEI + aGK)/(bEI+ 2aGK)]$

4.7-20 $M_C = qL^2/8$, $Q = 5qL/8$

4.8-1 $w = 4PR^3n/Gc^4$; lateral displacement increments cancel

4.10-1 $F_1 = -0.667P$, $F_2 = 0.0833P$, $F_3 = 0.750P$

4.10-2 $P = kL$; $d^2\Pi/d\theta^2 < 0$ if $P > kL$

4.10-3 $d^2\Pi/d\theta^2 > 0$ if $h < 2R$

4.10-4 $u_D = 2.083L\alpha\Delta T$, $v_D = 0$. Bar stresses are zero

4.11-1 $v_L = -qL^4/8EI$, $M_o = -qL^2/2$

4.11-2(a) $v_L = -FL^3/4EI$, $M_o = -FL/2$ (b) $v_L = -FL^3/3EI$, $M_o = -FL$

(c) $v_L = -FL^3/12EI$, $M_o = 0$ (d) $v_L = -0.328FL^3/EI$, $M_o = -0.813FL$

4.11-3 $\theta_L = M_L L/3EI$

4.11-4(a) $v_C = -FL^3/64EI$, $M_C = FL/8$

(b) $v_C = -qL^4/96EI$, $M_C = qL^2/12$

(c) $v_C = -q_L L^4/192EI$, $M_C = q_L L^2/24$

4.11-5 $v_L = -0.308F/k$

4.11-6(a) $u = qLx/2EA$, $\sigma = qL/2A$

(b) $u = (q/EA)(Lx - x^2/2)$, $\sigma = q(L - x)/A$

4.11-7(a) $v = ax^2(L - x)$ (b) $v_W = -0.00549WL^3/EI$

(b) Stable if $EI > \gamma bL^4/420$

4.11-8 $P = 0.7222EA_0u_L/L$ (0.12% high)

4.12-1 $v_o = 0.142FR^3/EI$, $M_o = 4FR/3\pi$

4.12-2 $v_o = \rho R^5\omega^2/18EI$, $M_o = \rho R^3\omega^2/9$

4.12-3 First term: $v_L$ errs by -4.5%; $M_o$ errs by -41%

4.12-4(a) First term: $v_{L/2}$ errs by -1.45%; $M_{L/2}$ errs by -18.9%

(b) First term: $v_{L/2}$ errs by +0.38%; $M_{L/2}$ errs by +3.2%

(c) First term: $v_{L/2}$ errs by +0.38%; $M_{L/2}$ errs by +3.2%

4.12-5 First term: $u_L$ errs by +3.2%; $\sigma_o$ errs by -18.9%

4.13-1 All results are exact

5.2-1 Deformation due to weight displaces equal weight of water.

5.2-2 $k = \rho g/\ell$

5.2-3 [Proof required]

5.2-4 $w = w_o e^{-\beta x}$, where $\beta^2 = k/T$

5.2-5(a) $\theta = -(T_o\lambda/k)e^{-\lambda x}$, $T = T_o e^{-\lambda x}$, where $\lambda^2 = k/GJ$

(b) Replace $T$, $\theta$, $G$, $J$ by $P$, $u$, $E$, $A$ respectively. $\lambda^2 = k/EA$

5.3-1(a,b) [Proof required]

5.3-2(a) $P_o = kw_o/\beta + k\theta_o/2\beta^2$, $M_o = kw_o/2\beta^2 + k\theta_o/2\beta^3$

(b) $\sigma_e = 140 = \frac{1}{\sqrt{2}}\left[(\sigma-20)^2+(20+90)^2 + (-90-\sigma)^2\right]^{1/2}$

yields $\sigma^2+70\sigma-9300=0$

Roots are 67.6 and −137.6

Range: −137.6 to 67.6 MPa

**3.3-6** (a) Eq. 3.3-1: $\frac{200}{\sigma_Y/2}=1$

$\sigma_Y = 400$ MPa ($\tau_{max}$ theory)

Eq. 3.3-2: $\frac{\frac{1}{\sqrt{2}}\left[6(200^2)\right]^{1/2}}{\sigma_Y}=1$

$\sigma_Y = 346$ MPa (von M. theory)

(b) Axial stress $= \frac{P}{A} = \frac{480{,}000}{\pi 20^2} = 382$ MPa

Eq. 3.3-1: $\frac{[382-(-160)]/2}{\sigma_Y/2}=1$

$\sigma_Y = 542$ ($\tau_{max}$ theory)

Eq. 3.3-2:

$\sigma_e = \frac{1}{\sqrt{2}}\left[(382+160)^2+(-160+160)^2+(-160-382)^2\right]^{1/2}$

$\frac{\sigma_e}{\sigma_Y}=1$ gives $\sigma_Y = 542$ (von M. theory)

**3.3-7** At the fixed support,

$M = 50P$, $\sigma = \frac{Mc}{I} = \frac{(50P)3}{\pi 3^4/4} = 2.36P$

$T = 80P$, $\tau = \frac{Tc}{J} = \frac{(80P)3}{\pi 3^4/2} = 1.89P$

$\tau_{max} = \sqrt{\left(\frac{2.36P}{2}\right)^2+(1.89P)^2} = 2.23P$

$\sigma_1 = \frac{2.36P}{2} + \tau_{max} = 3.41P$

$\sigma_2 = 0$

$\sigma_3 = \frac{2.36P}{2} - \tau_{max} = -1.05P$

Eq. 3.3-1 ($\tau_{max}$ theory):

$\frac{[3.41P-(-1.05P)]/2}{280/2} = 1$, $P = 62.8$ N

Eq. 3.3-2 (von Mises theory):

$\frac{\frac{P}{\sqrt{2}}\left[3.41^2+1.05^2+(3.42+1.05)^2\right]^{1/2}}{280} = 1$,

$P = 69.2$ N

**3.3-8** $\sigma = \frac{Mc}{I} = \frac{8(10^6)70}{\pi 70^4/4} = 29.7$ MPa

$\tau = \frac{Tc}{J} = \frac{12(10^6)70}{\pi 70^4/2} = 22.3$ MPa

$\tau_{max} = \sqrt{\left(\frac{29.7}{2}\right)^2+22.3^2} = 26.8$ MPa

$\sigma_1 = \frac{29.7}{2} + \tau_{max} = 41.6$ MPa

$\sigma_2 = 0$

$\sigma_3 = \frac{29.7}{2} - \tau_{max} = -11.9$ MPa

(a) $\tau_{max}$ theory: $\frac{(SF)26.8}{200/2} = 1$, $SF = 3.73$

(b) von Mises theory:

$\frac{\frac{(SF)}{\sqrt{2}}\left[41.6^2+(-11.9)^2+(41.6+11.9)^2\right]^{1/2}}{200} = 1$

$SF = 4.11$

**3.3-9** (a) $\tau_Y = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(400{,}000)}{\pi 10^3}$

$\tau_Y = 254.6$ MPa, $\sigma_Y = 509$ MPa

(according to max. $\tau$ theory)

In the tank, $\sigma_1 = \frac{pr}{t}$, $\sigma_2 = \frac{pr}{2t}$, $\sigma_3 \approx 0$

$\frac{\sigma_Y}{2} = \frac{1}{2}\left[\frac{(SF)pr}{t} - 0\right]$, $t = \frac{(SF)pr}{\sigma_Y}$

$t = \frac{2(3)500}{509} = 5.89$ mm ($t \ll r$; $\frac{pr}{t}$ is ok)

(b) In cylinder, from above, $\sigma_1 = 2\sigma_2$

For von Mises theory, equate $\sigma_e$ values in shaft and cylinder:

$\frac{1}{\sqrt{2}}\left[6\tau_Y^2\right]^{1/2} = \frac{1}{\sqrt{2}}\left[(2\sigma_2-\sigma_2)^2+\sigma_2^2+(2\sigma_2)^2\right]^{1/2}$

Hence $\tau_Y = \sigma_2$. Insert formulas and safety factor: $\frac{Tc}{J} = \frac{(SF)pr}{2t}$, where $\frac{Tc}{J} = \tau_Y$

Thus $t = \frac{(SF)pr}{2\tau_Y} = \frac{2(3)500}{2(254.6)} = 5.89$ mm

6.4-1 (continued)

$$V\left(\frac{Q_1}{A}+\frac{Rr_n}{Ae}A_1-\frac{R}{Ae}Q_1\right)=\tau_1 t_1 r_1^2$$

$$\tau_1=\frac{V}{Aet_1r_1^2}\left(eQ_1+Rr_nA_1-RQ_1\right)=\frac{Vr_n}{Aet_1r_1^2}\left(RA_1-Q_1\right) \quad (6.4\text{-}7)$$

(d) Substitute Eq. 6.4-7 and the expression for $d\sigma_\phi/d\phi$ from part (c) into Eq. 6.4-5 (divided by $d\phi$):

$$\int_b^{r_1}\left(\frac{V}{A}+VR\frac{r_n-r}{Aer}\right)dA-\int_b^{r_1}\tau\,dA-\frac{Vr_n}{Aer_1}\left(RA_1-Q_1\right)=0$$

$$\int_b^{r_1}\tau\,dA=\frac{Vr_n}{Ae}\left(\frac{e}{r_n}A_1+R\int_b^{r_1}\frac{dA}{r}-\frac{RA_1}{r_n}-\frac{RA_1}{r_1}+\frac{Q_1}{r_1}\right)$$

Substitute $R-e=r_n$. Also differentiate w.r.t. $\phi$ and substitute $dV/d\phi=-N$ from Eqs. 6.4-3.

$$\int_b^{r_1}\frac{d\tau}{d\phi}dA=\frac{Nr_n}{Ae}\left(A_1-R\int_b^{r_1}\frac{dA}{r}+\frac{RA_1}{r_1}+\frac{Q_1}{r_1}\right)$$

Substitute this and Eq. 6.2-10 into Eq. 6.4-4 (divided by $d\phi$). Thus

$$\sigma_r t_1 r_1=\frac{Mr_n}{Ae}\left(\int_b^{r_1}\frac{dA}{r}-\frac{A_1}{r_n}\right)-\frac{NRr_n}{Ae}\left(\int_b^{r_1}\frac{dA}{r}-\frac{A_1}{R}-\frac{A_1e}{Rr_n}\right)+\frac{Nr_n}{Aer_1}\left(A_1R-Q_1\right)$$

But $\frac{A_1}{R}\left(1+\frac{e}{r_n}\right)=\frac{A_1}{R}\left(\frac{r_n+e}{r_n}\right)=\frac{A_1}{r_n}$, so

$$\sigma_r t_1 r_1=\left(\frac{Mr_n}{Ae}-\frac{NRr_n}{Ae}\right)\left(\int_b^{r_1}\frac{dA}{r}-\frac{A_1}{r_n}\right)+\frac{Nr_n}{Aer_1}\left(A_1R-Q_1\right)$$

from which Eq. 6.4-9 follows immediately.

6.4-2 (a) Radial force from radial comp. of $\sigma_\phi$ greatest for $r_1=r_n$. But must divide this force by $t_1r_1\,d\phi$ to get $\sigma_r$ at radius $r_1$. Hence $\sigma_r$ may be max. at some other radius.

(b) For $M=NR$, Eq. 6.4-9 becomes

$$\sigma_r=\frac{Nr_n}{Aet_1r_1^2}\left[RA_1-Q_1\right]$$

For rectangular x-sec., $A_1=t_1(r_1-b)$, and $Q=A_1\frac{r_1+b}{2}=\frac{t_1}{2}(r_1^2-b^2)$. Thus

$$\sigma_r=\frac{Nr_n}{2Ae}\left[\frac{2R(r_1-b)-r_1^2+b^2}{r_1}\right]$$

Solve $\frac{d\sigma_r}{dr_1}=0$ to get the $r_1$ that maximizes $\sigma_r$. Thus

$$r_1=\frac{b(2R-b)}{R}=\frac{b(a+b-b)}{(a+b)/2}=\frac{2ab}{a+b}$$

(c) For $N=0$ and rect. x-sec., Eq. 6.4-9 is

$$\sigma_r=\frac{Mr_n}{Aet_1}\left[\frac{1}{r_1}\left(\int_b^{r_1}\frac{dA}{r}-\frac{A_1}{r_n}\right)\right]=\frac{Mr_n}{Ae}\left[\frac{1}{r_1}\ln\frac{r_1}{b}-\frac{r_1-b}{r_nr_1}\right]$$

Solve $\frac{d\sigma_r}{dr_1}=0$ to get the $r_1$ that maximizes $\sigma_r$. Thus

$$0=-\ln\frac{r_1}{b}+1-\frac{b}{r_n}, \quad r_1=b\exp\left(1-\frac{b}{r_n}\right)$$

6.4-3 For $N=0$, Eq. 6.4-9 is

$$\sigma_r=\frac{Mr_n}{Aet_1r_1}\left(\int_b^{r_1}\frac{dA}{r}-\frac{A_1}{r_n}\right)$$

Here $r_n=18.20$ mm, $R=20.00$ mm $\}$ $e=R-r_n=1.80$ mm

$t_1=12$ mm, $A=240$ mm$^2$

$r_1=15$ mm, $A_1=12(15-10)=60$ mm$^2$

$\int_{10}^{15}\frac{dA}{r}=12\ln 1.5=4.865$ mm

$A_1/r_n=3.297$ mm

$M=2000(20)=40{,}000$ N·mm

$$\sigma_r=\frac{40{,}000(18.2)}{240(1.80)(12)(15)}(4.865-3.297)=14.7 \text{ MPa}$$

6.4-4 First use Eq. 6.4-7.

$A_1=40(90)+(111.82-100)30=3954$ mm$^2$

$Q_1=40(90)(80)+(111.82-100)(30)\frac{100+111.82}{2}=325{,}600$ mm$^3$

$$\tau_1=\frac{80{,}000(98.85)}{6600(13.0)(30)(111.82)^2}\left[111.82(3954)-325{,}600\right]$$

$\tau_1=28.6$ MPa

Next use $\tau=VQ/It$.

$V=80{,}000$ N

$Q=30(200-111.82)\frac{200-111.82}{2}=116{,}600$ mm$^3$

$$I=\frac{1}{12}30(100)^3+30(100)(150-111.82)^2+\frac{1}{12}90(40)^3+90(40)(111.82-80)^2=11.0(10^6)\text{ mm}^4$$

$$\tau=\frac{80{,}000(116{,}600)}{11.0(10^6)(30)}=28.3 \text{ MPa}$$

OK along axes of straight parts, & slightly into curved part at radius $r=R$.

Away from the equator, assume

$$(M_\phi)_{max} = 0.3224\frac{Q_o}{\lambda} = 0.3224\,\alpha TRD\lambda^2$$

For $\nu = 0.3$, $\lambda^2 = \frac{\sqrt{3(1-\nu^2)}}{Rt} = \frac{1.652}{Rt}$

$$D = \frac{Et^3}{12(1-\nu^2)} = \frac{Et^3}{10.92}$$

$$(M_\phi)_{max} = 0.0488\,\alpha TEt^2$$

$$(\sigma_\phi)_{max} = \frac{6}{t^2}(M_\phi)_{max} = 0.293\,\alpha TE = 35.1\text{ MPa}$$

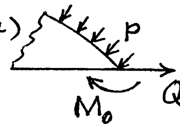
**13.8-9** $\lambda = \left[\frac{3(0.91)}{10^6(15)^2}\right]^{1/4} = 0.01050/\text{mm}$

$\cot\phi_o = 1$, $2\lambda R = 2(0.01050)1000 = 21.0$

$$C_1 = 1 - \frac{1-0.6}{21.0}(1) = 0.981$$

$$C_2 = 1 - \frac{1+0.6}{21.0}(1) = 0.924$$

$$D = \frac{200{,}000(15)^3}{12(0.91)} = 61.8(10^6)\text{ N·mm}$$

(a) 


Due to pressure load $p$,

$$w = -\frac{1}{\sqrt{2}}\frac{R}{E}\frac{pR}{2t}(1-\nu)$$

$$w = -\frac{0.8(1000)^2}{2\sqrt{2}(200{,}000)15}\,0.7 = -0.0660\text{ mm}$$

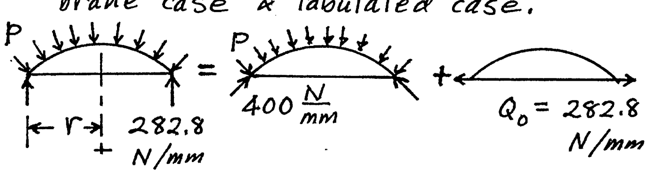
At the base, zero deflection and zero rotation: using Eqs. 13.7-6,

$$\frac{Q_o/2}{4(61.8)10^6(1.158)10^{-6}(0.981)}(1+0.906) - \frac{M_o(0.707)}{2(61.8)10^6(1.103)10^{-4}(0.981)} - 0.0660 = 0$$

$$\frac{Q_o(0.707)}{2(61.8)10^6(1.103)10^{-4}(0.981)} - \frac{M_o}{61.8(10^6)(0.01050)(0.981)} = 0$$

$$\left.\begin{aligned} 3394Q_o - 52.87M_o &= 66{,}000 \\ 52.87Q_o - 1.571M_o &= 0 \end{aligned}\right\} \begin{aligned} Q_o &= 40.9\text{ N/mm} \\ M_o &= 1376\ \tfrac{\text{N·mm}}{\text{mm}} \end{aligned}$$

(b) To obtain given case, superpose membrane case & tabulated case:




The source of these numbers is as follows.

$$282.8 = \frac{p\pi r^2}{2\pi r} = \frac{pr}{2} = \frac{pR}{2\sqrt{2}} = \frac{0.8(1000)}{2\sqrt{2}}$$

$$400 = \frac{pR}{2} = \frac{0.8(1000)}{2}$$

At the base,

$$\sigma_\phi = \frac{Q_o\cos\phi_o}{t} - \frac{pR}{2t} = \frac{282.8(0.707)}{15} - \frac{400}{15}$$

$\sigma_\phi = -13.3$ MPa (no flexural contribution)

$$M_\theta = \frac{Q_o t^2\lambda^2 R\cos\phi_o}{6C_1} = 843\text{ N·mm/mm}$$

Due to $Q_o$,

$$\sigma_\theta = \frac{Q_o\lambda R\sin\phi_o}{2t}\left(\frac{2}{C_1} + C_1 + C_2\right)$$

$$= \frac{282.8(0.01050)1000(0.707)}{2(15)}(3.944)$$

$= 276$ MPa

Net $\sigma_\theta$ at the base is

$$\sigma_\theta = 276 - \frac{pR}{t} \pm \frac{6M_\theta}{t^2}$$

On the inside surface,

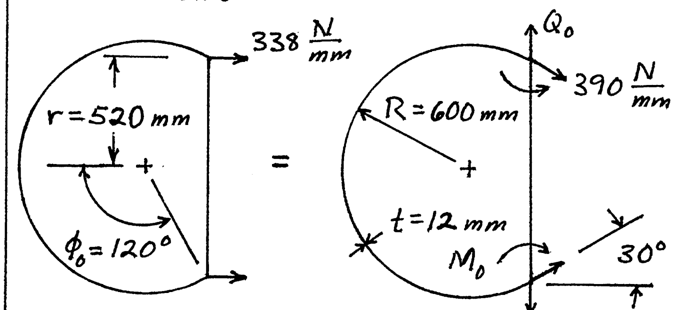
$$\sigma_\theta = 276 - 26.7 + 22.5 = 272\text{ MPa}$$

**13.8-10** $\lambda = \left[\frac{3(0.91)}{600^2\,12^2}\right]^{1/4} = 0.01515/\text{mm}$

$\phi_o = 120°$, $\cot\phi_o = -0.57735$, $2\lambda R = 18.18$

$$C_1 = 1 - \frac{0.6}{18.18}(-0.57735) = 1.019$$

$$C_2 = 1 - \frac{1.6}{18.18}(-0.57735) = 1.051$$




No rotation at $\phi = \phi_o$

$$\frac{pR}{2} = 390\,\frac{\text{N}}{\text{mm}}, \quad \frac{pr}{2} = 338\,\frac{\text{N}}{\text{mm}}$$

$390\cos 30° = 338$ ; OK

$390\sin 30° - Q_o = 0$ ; $Q_o = 195\ \frac{\text{N}}{\text{mm}}$

With $\psi = 0$, Eq. 13.7-6b yields

$$0 = \frac{Q_o\sin\phi_o}{2\lambda} + M_o, \quad \text{so } M_o = -5573\ \frac{\text{N·mm}}{\text{mm}}$$