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# ANTENNA THEORY

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CONSTANTINE A. BALANIS

SOLUTIONS MANUAL  
TO ACCOMPANY

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ANTENNA THEORY  
ANALYSIS AND DESIGN  
THIRD EDITION

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## Preface

This Solutions Manual consists of solutions for all the problems found in *Antenna Theory: Analysis and Design* (3<sup>rd</sup> edition, 2005) at the end of Chapters 2–16. There are 596 problems, most of them with multiple parts. The degree of difficulty and length varies. While certain solutions need special functions, found in tabular and graphical forms in the appendices, others require the use of the computer program. These computer programs are contained in a CD, which is included with the book. All of the computer programs, especially those at the end of Chapters 6, 11, 13 and 14 have been developed to design, respectively, uniform and nonuniform arrays, log-periodic arrays, horns and microstrip patch antennas. In some cases, the computer programs also perform analysis on the designs. The programs at the end of Chapters 2, 4, 5, 7, 8, 10 and 16 are primarily developed for analysis. The problems have been designed to test the student's grasp of this text's material and to apply the concepts to the analysis and design of many practical radiators. In this third edition, more emphasis has been placed on design. To accomplish this, equations, procedures, examples, graphs, end-of-the-chapter problems, and computer programs have been developed.

This manual has been prepared to assist the instructor in making homework and test assignments, and to provide one set of solutions for all of the problems. There are undoubtedly errors which have been overlooked. In addition, the solutions contained in this manual are not necessarily the simplest and/or the best. The author would, therefore, appreciate having errors brought to his attention and solicits alternate solutions to the problems.

This Solutions Manual for the third edition has been prepared from the manuals of the first and second editions and many other new problems provided by the author.

$$2-85. \frac{P_r}{P_t} = |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left( \frac{\lambda}{4\pi R} \right)^2 G_{0t} G_{0r}$$

$$G_{0t} = 20 \text{ dB} \Rightarrow G_{0t}(\text{power ratio}) = 10^2 = 100$$

$$G_{0r} = 15 \text{ dB} \Rightarrow G_{0r}(\text{power ratio}) = 10^{1.5} = 31.623$$

$$f = 1 \text{ GHz} \Rightarrow \lambda = 0.3 \text{ meters}$$

$$R = 1 \times 10^3 \text{ meters}$$

$$a. \text{ For } |\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 1$$

$$P_r = \left( \frac{0.3}{4\pi \times 10^3} \right)^2 (100)(31.623) (150 \times 10^{-3}) = 270.344 \mu\text{watts}$$

b. When transmitting antennas is circularly polarized and receiving antenna is linearly polarized, the PLF is equal to

$$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = \left| \left( \frac{\hat{a}_x \pm j\hat{a}_y}{\sqrt{2}} \right) \cdot \hat{a}_x \right|^2 = \frac{1}{2}$$

Thus

$$P_r = \frac{1}{2} (270.344 \times 10^{-6}) = 135.172 \times 10^{-6} = 135.172 \mu\text{watts}$$

$$2-86. \text{ Lossless: } e_{cd} = 1, \text{ polarization matched: } |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1, \text{ line matched: } (1 - |\Gamma|^2) = 1$$

$$D_0 = 20 \text{ dB} = 10^2 = 100 = D_{0r} = D_{0t}$$

$$P_r = P_t \left( \frac{\lambda}{4\pi R} \right)^2 D_{0t} D_{0r} = 10 \left( \frac{\lambda}{4\pi \cdot 50\lambda} \right)^2 (100)(100) = 0.253 \text{ watts}$$

$$P_r = 0.253 \text{ watts}$$

$$2-87. \text{ Lossless: } e_{cd} = 1, \text{ PLF} = 1. \text{ Line matched: } (1 - |\Gamma|^2) = 1.$$

$$D_0 = 30 \text{ dB} = 10^3 = 1000 = D_{0r} = D_{0t}$$

$$P_r = P_t \left( \frac{\lambda}{4\pi \cdot 100\lambda} \right)^2 \cdot (1000)^2 = 20 \cdot \left( \frac{1}{4\pi} \right)^2 \cdot 100 = 12.665 \text{ watts}$$

$$2-88. G_{0r} = 20 \text{ dB} = 100, G_{0t} = 25 \text{ dB} = 316.23, \lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m.}$$

$$= P_t \cdot |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \cdot \left( \frac{\lambda}{4\pi R} \right)^2 \cdot G_{0r} \cdot G_{0t}$$

$$= 100 \cdot (1) \cdot \left( \frac{0.1}{4\pi \times 500} \right)^2 (100)(316.23)$$

$$P_r = 8 \times 10^{-4} \text{ watts}$$

If  $z = klv$ ,

$$I_{\text{int}} = \int_0^{kl} \frac{1 + \cos(kl) - \cos(v)}{v} dv$$

$$+ \frac{1}{2} \int_0^{2kl} \frac{-\cos(kl) + \cos(v) \cos(kl) + \sin(v) \sin(kl)}{v} dv$$

$$- \int_0^{kl} \frac{\cos(kl) \cos(z) + \sin(kl) \sin(z)}{z} dz$$

$$I_{\text{int}} = [1 + \cos(kl)] \int_0^{kl} \frac{1 - \cos v}{v} dv - 2 \int_0^{kl} \frac{\sin(v) \sin(kl)}{v} dv$$

$$+ \sin(kl) \int_0^{2kl} \frac{\sin v}{v} dv - \cos(kl) \int_0^{2kl} \frac{1 - \cos v}{v} dv$$

which reduces to

$$I_{\text{int}} = \left\{ C + \ln(kl) - C_i(kl) + \frac{1}{2} \sin(kl) [S_i(2kl) - 2S_i(kl)] \right.$$

$$\left. + \frac{1}{2} \cos(kl) \left[ C + \ln\left(\frac{kl}{2}\right) + C_i(2kl) - 2C_i(kl) \right] \right\}$$

where  $C = 0.5772$

and

$$P_{\text{rad}} = \eta \frac{|I_0|^2}{4\pi} I_{\text{int}} \text{ is identical to (4-68)}$$

From (4-88)

$$P_{\text{rad}} = \eta \frac{|I_0|^2}{4\pi} \int_0^\pi \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} d\theta$$

Letting

$$\left. \begin{aligned} u &= \cos \theta \\ du &= -\sin \theta d\theta \end{aligned} \right\} \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta = 1 - u^2.$$

We can write

$$P_{\text{rad}} = -\eta \frac{|I_0|^2}{2\pi} \int_1^0 \frac{\cos^2\left(\frac{\pi}{2} u\right)}{1 - u^2} du = \eta \frac{|I_0|^2}{2\pi} \int_0^1 \frac{\cos^2\left(\frac{\pi}{2} u\right)}{1 - u^2} du$$

which can also be written as

$$P_{\text{rad}} = \eta \frac{|I_0|^2}{4\pi} \left[ \int_0^1 \frac{\cos^2\left(\frac{\pi}{2} u\right)}{1 - u} du + \int_0^1 \frac{\cos^2\left(\frac{\pi}{2} u\right)}{1 + u} du \right]$$

Making another change of variable of the form

$$\left. \begin{aligned} v &= 1 - u \\ dv &= -du \end{aligned} \right\} \text{ for the first integral, } \left. \begin{aligned} v &= 1 + u \\ dv &= du \end{aligned} \right\} \text{ for the second integral}$$

If you check closely, it also leads to a maximum at  $\theta = 0^\circ$ .

So you cannot only have one maximum at  $\theta = 60^\circ$

$$4-53. E_\theta \sim C_1 \cdot \sin \theta \cdot \cos(kh \cos \theta) \Big|_{\theta=80^\circ} = 0$$

$$\cos(kh \cos \theta) \Big|_{\theta=80^\circ} = 0, kh \cos \theta \Big|_{\theta=80^\circ} = \frac{\pi}{2}, \frac{2\pi}{\lambda} h \cos' \theta \Big|_{\theta=80^\circ} = \frac{\pi}{2}$$

$$h = \frac{\lambda}{4 \cos \theta} \Big|_{\theta=80^\circ} = \frac{\lambda}{4(0.1736)} = \frac{\lambda}{0.6946} = 1.4397 \cdot \lambda$$

$$h = 1.4397 \lambda, \lambda = \frac{3 \times 10^8}{50 \times 10^6} = \frac{30 \times 10^7}{5 \times 10^7} = 6 \text{ meters}$$

$$h = 1.4397 \cdot \lambda = 1.4397 \cdot (6) = 8.6382 \text{ m}$$

$$h = 8.6382 \text{ meters}$$

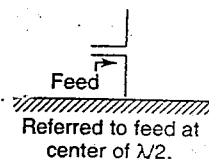
$$4-54. \text{ a. } Z_{im}(l = \lambda/2) \Big|_{\text{above ground plane}} = \frac{1}{2} Z_{im}(l = \lambda) \Big|_{\text{free space}} \simeq \frac{1}{2} (R_{im} + jX_{im}) \Big|_{l=\lambda}$$

$$\text{From Problem 4-23} \Rightarrow R_{im} = R_r = 199.099$$

$$\text{From Figure 4.20} \Rightarrow X_{im} \left( l = \frac{\lambda}{2} \right) \Big|_{\text{above ground plane}} \simeq 62.5$$

Therefore

$$Z_{im}(l = \lambda/2) \Big|_{\text{above ground plane}} = \frac{199.099}{2} + j62.5 = 99.5495 + j62.5$$



$$\text{b. } Z_{in} = \frac{Z_{im}}{\sin^2 \left( \frac{kl}{2} \right)} = \frac{99.5495 + j62.5}{\sin^2(\pi)} = \infty$$

$$\text{c. } \Gamma = \frac{Z_{in} - Z_c}{Z_{in} + Z_c} = \frac{\infty - 50}{\infty + 50} = \frac{1 - 50/\infty}{1 + 50/\infty} = 1$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1}{1 - 1} = \infty$$