# Solutions Manual to accompany AN INTRODUCTION TO MECHANICS 2nd edition 

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## KLEPPNER / KOLENKOW

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case 1:
horizontal equation of motion:

$$
\frac{M v^{2}}{R}=N \sin \theta-f \cos \theta
$$

The maximum friction force is $\mu N$.

$$
\begin{align*}
\frac{M v^{2}}{R} & \geq N(\sin \theta-\mu \cos \theta) \\
\frac{M v 2_{\min }}{R} & =N(\sin \theta-\mu \cos \theta) \tag{1}
\end{align*}
$$

There is no vertical acceleration if the car is not sliding, so the vertical equation of motion is $N \cos \theta+f \sin \theta-M g=0$. In the limit where $f=\mu N$

$$
\begin{equation*}
M g=N(\cos \theta+\mu \sin \theta) \tag{2}
\end{equation*}
$$

Dividing Eq. (1) by Eq. (2),

$$
\frac{v_{\min }^{2}}{R g}=\frac{\sin \theta-\mu \cos \theta}{\cos \theta+\mu \sin \theta} \Longrightarrow v_{\min }=\sqrt{R g\left(\frac{\sin \theta-\mu \cos \theta}{\cos \theta+\mu \sin \theta}\right)}
$$

case 2:
Proceeding as before,

$$
\begin{gather*}
M \frac{v^{2}}{R} \leq N \sin \theta+f \cos \theta \\
M \frac{v_{\max }^{2}}{R}=N(\sin \theta+\mu \cos \theta) \tag{3}
\end{gather*}
$$

vertical equation of motion:

$$
\begin{equation*}
0=N \cos \theta-f \sin \theta-M g=N(\cos \theta-\mu \sin \theta) \tag{4}
\end{equation*}
$$

Dividing Eq. (3) by Eq. (4) leads to

$$
v_{\max }=\sqrt{R g\left(\frac{\sin \theta+\mu \cos \theta}{\cos \theta-\mu \sin \theta}\right)}
$$

TOPICS IN DYNAMICS

### 6.7 Proton collision

The proton has mass $m$, and the unknown particle has mass $M$. The upper sketch is before the collision, and the lower sketch is after the collision. Both momentum $P$ and mechanical energy
 (kinetic energy $K$ ) are conserved in the elastic collision.

$$
\begin{equation*}
P_{f}=M V-m v^{\prime}=P_{i}=m v_{0} \Longrightarrow v_{0}=\frac{M}{m} V-v^{\prime} \tag{1}
\end{equation*}
$$

$K_{f}=\frac{1}{2} M V^{2}+\frac{1}{2} m v^{\prime 2}=K_{i}=\frac{1}{2} m v_{0}^{2} \Longrightarrow v_{0}^{2}=\frac{M}{m} V^{2}+v^{\prime 2}$
$E_{f}=\frac{1}{2} m v^{\prime 2}=\frac{4}{9}\left(\frac{1}{2} m v_{0}^{2}\right) \Longrightarrow v^{\prime}=\frac{2}{3} v_{0}$
Using Eqs. (1) and (3),

$$
\begin{equation*}
V=\frac{5}{3} \frac{m}{M} v_{0} \tag{4}
\end{equation*}
$$

Using Eqs. (3) and (4) in Eq. (2),

$$
v_{0}^{2}=\frac{M}{m} \frac{25}{9}\left(\frac{m}{M}\right)^{2} v_{0}^{2}+\frac{4}{9} v_{0}^{2} \Longrightarrow \frac{5}{9}=\frac{25}{9} \frac{m}{M} \Longrightarrow M=5 m
$$

### 6.8 Collision of $m$ and $M$

The upper sketch shows the system before the collision, and the lower sketch after the collision. Both momentum $\mathbf{P}$ and mechanical energy (kinetic energy $K$ ) are conserved in the elastic collision. $\mathbf{P}$ has both $x$ and $y$ components.

$$
\begin{align*}
P_{f x} & =\frac{M V^{\prime}}{\sqrt{2}}=P_{i x}=m v_{0}-M V \\
P_{f y} & =\frac{M V^{\prime}}{\sqrt{2}}-\frac{m v_{0}}{2}=P_{i y}=0 \\
m v_{0}-M V & =\frac{M V^{\prime}}{\sqrt{2}} \quad \text { (1) }  \tag{1}\\
0 & =\frac{M V^{\prime}}{\sqrt{2}}-\frac{m v_{0}}{2} \Longrightarrow V^{\prime}=\frac{1}{\sqrt{2}} \frac{m}{M} v_{0} \tag{2}
\end{align*}
$$

From Eqs. (1) and (2)

$$
\begin{equation*}
V=\frac{1}{2} \frac{m}{M} v_{0} \tag{3}
\end{equation*}
$$

### 8.12 Euler's disk

The contact point moves on the surface in a circle of radius $R \cos \alpha$, with speed $V=(R \cos \alpha)\left(\Omega_{p}\right.$. The disk is assumed to roll without slipping, so $R \omega_{s}=V=(R \cos \alpha) \Omega_{p}$. equations of motion:
$0=N-M g \Longrightarrow N=M g$
$f=\frac{M V^{2}}{R \cos \alpha}=\frac{M(R \cos \alpha)^{2} \Omega_{p}^{2}}{R \cos \alpha}=M R \cos \alpha \Omega_{p}^{2}$
The total angular velocity is $\boldsymbol{\Omega}_{p}+\boldsymbol{\omega}_{s}=\boldsymbol{\omega}_{r}$. As shown in the sketches, $\omega_{r}$ lies along the axis from the contact point to the center of mass. The moment of inertia along this axis is
$I_{\perp}=\frac{1}{2} I_{0}=\frac{1}{4} M R^{2}$
The spin angular momentum is
$L_{s}=I_{\perp} \omega_{r}=\frac{1}{4} M R^{2} \Omega_{p} \sin \alpha$


The horizontal component of the spin angular momentum is
$L_{h}=L_{s} \cos \alpha=\frac{1}{4} M R^{2} \cos \alpha \sin \alpha \Omega_{p}$
torque about the cm (positive is into the paper):

$$
\begin{aligned}
\tau_{c m} & =N R \cos \alpha-f R \sin \alpha=M g R \cos \alpha-M R^{2} \cos \alpha \sin \alpha \Omega_{p}^{2} \\
& =M R \cos \alpha\left(g-R \sin \alpha \Omega_{p}^{2}\right)
\end{aligned}
$$


force diagram

## RELATIVISTIC DYNAMICS

### 13.1 Energetic proton

(a) In a frame moving with the proton, the galaxy is approaching at speed $v$ and has thickness $D=D_{0} / \gamma$. The proton has such high energy that $v$ is very nearly $c$, to the accuracy of this solution.
The time $T$ to traverse the galaxy is

$$
\begin{aligned}
T & =\frac{D}{v}=\frac{D_{0}}{\gamma v} \simeq \frac{D_{0}}{\gamma c} \\
E & =\gamma m_{0} c^{2} \Longrightarrow \gamma=\frac{E}{m_{0} c^{2}} \\
m_{0} c^{2} & =\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(3 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)^{2}\left(\frac{1 \mathrm{eV}}{1.6 \times 10^{-19} \mathrm{~J}}\right)=9.4 \times 10^{8} \mathrm{eV} \\
\gamma & =\frac{3 \times 10^{20} \mathrm{eV}}{9.4 \times 10^{8} \mathrm{eV}}=3.2 \times 10^{11} \\
D_{0} & =\left(10^{5} \text { light years }\right)\left(3 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)\left(\frac{3 \times 10^{7} \mathrm{~s}}{1 \text { year }}\right)=9 \times 10^{20} \mathrm{~m} \\
T & =\frac{9 \times 10^{20} \mathrm{~m}}{\left(3 \times 10^{11}\right)\left(3 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)}=10 \mathrm{~s}
\end{aligned}
$$

The photon is traveling at the speed of light, so $\gamma \rightarrow \infty$, and $T_{\text {photon }}=0$.
(b)

$$
\begin{aligned}
E_{\text {baseball }} & =\frac{1}{2} M v^{2}=\frac{1}{2}(0.145 \mathrm{~kg})\left[\left(\frac{100 \text { miles }}{1 \text { hour }}\right)\left(\frac{1610 \mathrm{~m}}{1 \text { mile }}\right)\left(\frac{1 \text { hour }}{3600 \mathrm{~s}}\right)\right]^{2}=145 \mathrm{~J} \\
E_{\text {proton }} & =\left(3 \times 10^{20} \mathrm{eV}\right)\left(\frac{1.6 \times 10^{-19} \mathrm{~J}}{1 \mathrm{eV}}\right)=48 \mathrm{~J}
\end{aligned}
$$

