1.1

1.1 The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

$$F = C_D \rho V^2 A/2$$

or

$$C_D = 2F/\rho V^2 A, \quad \text{where} \quad F \doteq MLT^{-2}$$

$$\rho \doteq ML^{-3}$$

$$V \doteq LT^{-1}$$

$$A \doteq L^2$$

Thus,

$$C_D \doteq (MLT^{-2})/[(ML^{-3})(LT^{-1})^2(L^2)] = M^0 L^0 T^0$$

Hence, C_D is dimensionless.

1.10

1.10 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$\left[\frac{FL}{F}\right] \doteq \left[0.04 \text{ to } 0.09\right]\left[\frac{L^4}{L^4}\right]\left[\frac{1}{2}\right]\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right]$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

1.11

1.11 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

$$\Delta p = K_v \frac{\mu V}{D} + K_u\left[\frac{A_0}{A_1} - 1\right]^2 \rho V^2$$

$$\left[FL^{-2}\right] \doteq \left[K_v\right]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + \left[K_u\right]\left[\frac{(L^2)}{(L^2)} - 1\right]^2\left[\frac{FT^2}{L^4}\right]\left[\frac{L}{T}\right]^2$$

$$\left[FL^{-2}\right] \doteq \left[K_v\right]\left[FL^{-2}\right] + \left[K_u\right]\left[FL^{-2}\right]$$

Since each term must have the same dimensions, K_v and K_u are dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent system of units. Yes.

2.8

2.8 Sometimes when riding an elevator or driving up or down a hilly road a person's ears "pop" as the pressure difference between the inside and outside of the ear is equalized. Determine the pressure difference (in psi) associated with this phenomenon if it occurs during a 150 ft elevation change.

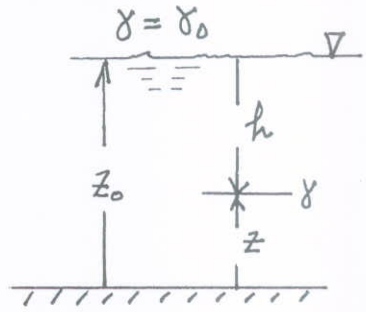
$$\Delta p = \gamma \Delta h = 0.0765 \frac{lb}{ft^3}(150 ft)$$

$$= 11.5 \frac{lb}{ft^2}\left(\frac{1 ft^2}{144 in.^2}\right)$$

$$= \underline{\underline{0.0797 \ psi}}$$

2.9

2.9 Develop an expression for the pressure variation in a liquid in which the specific weight increases with depth, h, as $\gamma = Kh + \gamma_0$, where K is a constant and γ_0 is the specific weight at the free surface.



$$\frac{dp}{dz} = -\gamma \qquad (Eq.\ 2.4)$$

Let $h = z_0 - z$

so that $dh = -dz$

Thus,

$$dp = \gamma \, dh$$

and

$$\int_0^p dp = \int_0^h \gamma \, dh$$

For $\gamma = Kh + \gamma_0$,

$$\int_0^p dp = \int_0^h (Kh + \gamma_0)\, dh$$

and

$$\underline{\underline{p = \frac{Kh^2}{2} + \gamma_0 h}}$$

2.106

2.106 A 2-ft-thick block constructed of wood ($SG = 0.6$) is submerged in oil ($SG = 0.8$). and has a 2-ft-thick aluminum (specific weight = 168 lb/ft³) plate attached to the bottom as indicated in Fig. P2.106. Determine completely the force required to hold the block in the position shown. Locate the force with respect to point A.

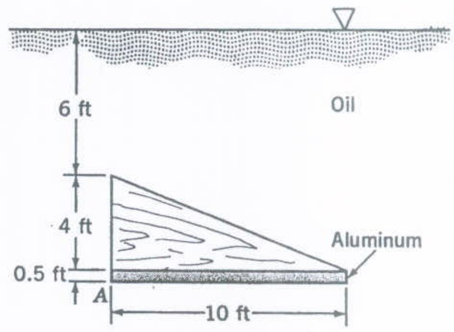
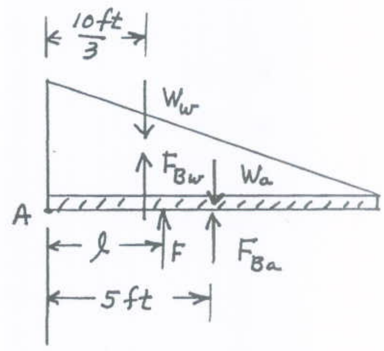


FIGURE P2.106



For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F = W_w - F_{Bw} + W_a - F_{Ba}$$

where :

$w \sim$ wood
$a \sim$ aluminum
$F \sim$ force to hold block

$$W_w = (SG_w)(\gamma_{H_2O})\forall_w$$

$$= (0.6)\left(62.4\,\tfrac{lb}{ft^3}\right)\left(\tfrac{1}{2}\right)(10ft \times 4ft \times 2ft) = 1500\ lb$$

$$W_a = \left(168\,\tfrac{lb}{ft^3}\right)(0.5\,ft \times 10\,ft \times 2ft) = 1680\ lb$$

$$F_{Bw} = (SG_{oil})(\gamma_{H_2O})\forall_w = (0.8)\left(62.4\,\tfrac{lb}{ft^3}\right)\left(\tfrac{1}{2}\right)(10ft \times 4ft \times 2\,ft) = 2000\ lb$$

$$F_{Ba} = (SG_{oil})(\gamma_{H_2O})\forall_a = (0.8)\left(62.4\,\tfrac{lb}{ft^3}\right)(0.5ft \times 10ft \times 2ft) = 499\ lb$$

Thus,

$$F = 1500\ lb - 2000\ lb + 1680\ lb - 499\ lb = \underline{\underline{681\ lb\ \text{upward}}}$$

Also,

$$\sum M_A = 0$$

So that

$$\ell F = \left(\tfrac{10}{3}ft\right)(W_w - F_{Bw}) + (5ft)(W_a - F_{Ba})$$

or

$$\ell\,(681\ lb) = \left(\tfrac{10}{3}\,ft\right)(1500\ lb - 2000\ lb) + (5\,ft)(1680\ lb - 499\ lb)$$

and

$$\underline{\underline{\ell = 6.22\ ft\ \text{to right of point } A}}$$

3.76

3.76 Water flows into the sink shown in Fig. P3.76 and **Video V5.1** at a rate of 2 gal/min. If the drain is closed, the water will eventually flow through the overflow drain holes rather than over the edge of the sink. How many 0.4-in.-diameter drain holes are needed to ensure that the water does not overflow the sink? Neglect viscous effects.

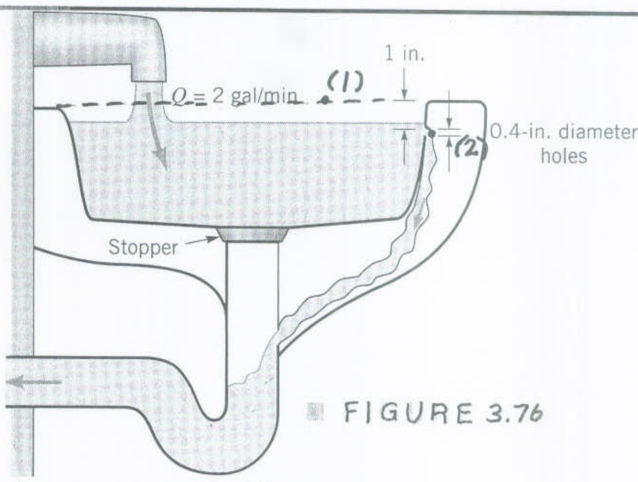


FIGURE 3.76

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{ , where } p_1 = 0,\ V_1 = 0, \text{ and } z_2 = 0,\ p_2 = 0$$

Thus,

$$z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2gz_1} = \left[2\left(32.2\,\frac{ft}{s}\right)\left(\frac{1+0.2}{12}\,ft\right)\right]^{1/2} = 2.54\,\frac{ft}{s}$$

Also,

$$Q = n A_2 V_2 = n\, C_c \frac{\pi}{4} d_2^2 V_2 \text{ , where } n = \text{number of holes required,}$$

$d_2 = 0.4$ in., and C_c = contraction coef. $= 0.61$ (see Fig. 3.14)

Thus, with

$$Q = 2\,\frac{gal}{min}\left(\frac{1\ min}{60\,s}\right)\left(\frac{231\ in.^3}{1\ gal}\right)\left(\frac{1\ ft^3}{1728\ in.^3}\right) = 4.46\times10^{-3}\,\frac{ft^3}{s} ,$$

$$n = \frac{4Q}{\pi C_c d_2^2 V_2} = \frac{4\,(4.46\times10^{-3}\,ft^3/s)}{\pi(0.61)\left(\frac{0.4}{12}\right)^2 ft^2\,(2.54\,ft/s)} = 3.30$$

Thus, $\underline{\underline{4}}$ holes are needed.

4.51

4.51 Air flows from a pipe into the region between two parallel circular disks as shown in Fig. P4.51. The fluid velocity in the gap between the disks is closely approximated by $V = V_0R/r$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for r = 1, 2, or 3 ft if V_0 = 5 ft/s and R = 3 ft.

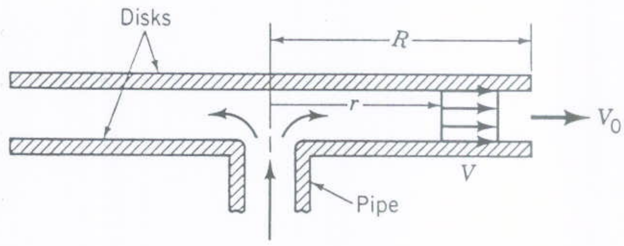


FIGURE P4.51

$\vec{a} = a_n\hat{n} + a_s\hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial r}$, where $V = \frac{V_0R}{r}$

Since $V_0 = 5\frac{ft}{s}$ and $R = 3\,ft$, $V = \frac{15}{r}\frac{ft}{s}$, where $r \sim ft$

Thus,

$$a_s = \left(\frac{V_0R}{r}\right)\left(-\frac{V_0R}{r^2}\right) = -\frac{V_0^2R^2}{r^3} = -\frac{(5\frac{ft}{s})^2(3\,ft)^2}{r^3\,ft^3} = -\frac{225}{r^3}\frac{ft}{s^2}, \quad r \sim ft$$

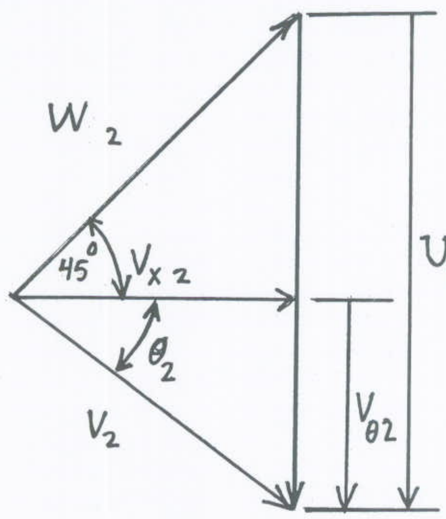
At $r = 1\,ft$, $a_s = \underline{\underline{-225\,\frac{ft}{s^2}}}$

At $r = 2\,ft$, $a_s = \underline{\underline{-28.1\,\frac{ft}{s^2}}}$

At $r = 3\,ft$, $a_s = \underline{\underline{-8.33\,\frac{ft}{s^2}}}$

5.83 (con't)

The velocity triangle for flow just downstream of the rotor is sketched below for the arithmetic mean radius. For incompressible flow $V_{x2} = V_1$. For mean radius flow $U_2 = U_1$. Thus for relative flow tangent to the blade we obtain the velocity triangle sketched below.



With the triangle we conclude that

$$V_{\theta 2} = U_2 - W_{\theta 2} = U_2 - V_{x2}\tan 45^\circ = 5.2\,\frac{m}{s} - \left(3\,\frac{m}{s}\right)\tan 45^\circ = 2.2\,\frac{m}{s}$$

Also

$$\theta_2 = \tan^{-1}\left(\frac{V_{\theta 2}}{V_{x2}}\right) = \tan^{-1}\left[\frac{\left(2.2\,\frac{m}{s}\right)}{\left(3\,\frac{m}{s}\right)}\right] = 36.2^\circ$$

$$W_2 = \frac{V_{x2}}{\cos 45^\circ} = \frac{\left(3\,\frac{m}{s}\right)}{\cos 45^\circ} = 4.24\,\frac{m}{s}$$

$$V_2 = \frac{V_{x2}}{\cos\theta_2} = \frac{\left(3\,\frac{m}{s}\right)}{\cos 36.2^\circ} = 3.72\,\frac{m}{s}$$

Using the stationary and non-deforming control volume shown above in the first sketch of this solution and Eq. 5.54 we can calculate the energy added to each kg of gasoline.

$$w_{shaft} = U_2 V_{\theta 2} = \left(5.2\,\frac{m}{s}\right)\left(2.2\,\frac{m}{s}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) = \underline{\underline{11.4}}\,\frac{N\cdot m}{kg}$$

This is the actual amount of energy delivered to the gasoline. However, not all of it is absorbed by the gasoline, some is lost.

6.28

6.28 Consider the incompressible, two-dimensional flow of a nonviscous fluid between the boundaries shown in Fig. P6.28. The velocity potential for this flow field is

$$\phi = x^2 - y^2$$

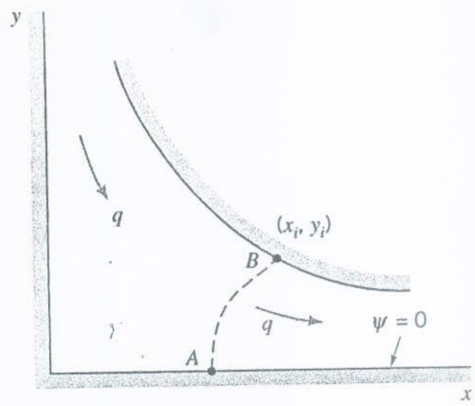


FIGURE P6.28

(a) $$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 2x$$

To determine ψ integrate with respect to y to obtain

$$\int d\psi = \int 2x\, dy$$

or

$$\psi = 2xy + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -2y$$

so that

$$\int d\psi = \int 2y\, dx$$

or

$$\psi = 2xy + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2)

$$\psi = 2xy + C$$

where C is an arbitrary constant. Since $\psi = 0$ along $y = 0$, $C = 0$ and

$$\underline{\underline{\psi = 2xy}} \qquad (3)$$

(b) The discharge, q, passing through any surface connecting the two walls, such as AB (see figure), is

$$q = \psi_B - \psi_A$$

From Eq. (3), $\psi_A = 0$ and $\psi_B = 2x_i y_i$. It follows that

$$q = \underline{\underline{2x_i y_i}}$$

7.10

7.10 The excess pressure inside a bubble (discussed in Chapter 1) is known to be dependent on bubble radius and surface tension. After finding the pi terms, determine the variation in excess pressure if we (a) double the radius and (b) double the surface tension.

Given $\Delta p = f(R, \sigma)$, where $\Delta p \doteq \frac{F}{L^2} = \frac{M}{LT^2}$, $R \doteq L$, and $\sigma \doteq \frac{F}{L} = \frac{M}{T^2}$

Consider the (MLT) units so that

$k-r = 3-3=0$ since there 3 variables and 3 dimensions.

According to this, there should be $k-r=0$ pi terms!?

However, if we consider the (FLT) units we see that it takes only F and L, T is not needed, so that $r=2$.

Hence, $k-r = 3-2 = 1$, so only 1 pi term is needed.

That is, $\Pi_1 = \text{constant}$

To determine Π_1, consider

$\Pi_1 = \Delta p \, R^a \sigma^b$ or

$\Delta p \, R^a \sigma^b \doteq \frac{F}{L^2} L^a \left(\frac{F}{L}\right)^b = F^{1+b} \; L^{-2+a-b}$

Thus:

F: $1+b=0$

L: $a-b-2=0$

or $b=-1$ and $a=b+2=1$

Hence $\underline{\underline{\Pi_1 = \frac{\Delta p \, R}{\sigma}}}$ or $\frac{\Delta p \, R}{\sigma} = C$, where $C = \text{constant}$.

or

$$\Delta p = \frac{C\sigma}{R} \qquad (1)$$

(a) If R is doubled, <u>Δp is reduced by half.</u> (See Eq. (1))

(b) If σ is doubled, <u>Δp is doubled.</u> (See Eq. (1))

8.9

8.9 The pressure distribution measured along a straight, horizontal portion of a 50-mm-diameter pipe attached to a tank is shown in the table below. Approximately how long is the entrance length? In the fully developed portion of the flow, what is the value of the wall shear stress?

x (m) (± 0.01 m)	p (mm H_2O) (± 5 mm)
0 (tank exit)	520
0.5	427
1.0	351
1.5	288
2.0	236
2.5	188
3.0	145
3.5	109
4.0	73
4.5	36
5.0 (pipe exit)	0

The entrance length extends to the fully developed portion in which $\frac{\partial p}{\partial x} = \text{constant}$. Approximate $\frac{\partial p}{\partial x} \approx \frac{\delta p}{\delta x}$ to obtain the following:

From $x=$	to $x=(\ )m$	δp, mm H_2O	δx	$\frac{\partial p}{\partial x}$, $\frac{mm\,H_2O}{m}$
0	0.5	−93	0.5	−186
0.5	1.0	−76	0.5	−152
1.0	1.5	−63	0.5	−126
1.5	2.0	−52	0.5	−104
2.0	2.5	−48	0.5	−96
2.5	3.0	−43	0.5	−86
3.0	3.5	−36	0.5	−72
3.5	4.0	−36	0.5	−72
4.0	4.5	−37	0.5	−74
4.5	5.0	−36	0.5	−72

Within the error on δp, the pressure gradient is constant for $x \gtrsim 3$, m Thus, $\underline{\underline{\ell_e \approx 3\,m}}$.

For $x > 3\,m$, $\frac{\Delta p}{\ell} = 72\,\frac{mm\,H_2O}{m}$ Since $1\,mm\,H_2O \times \gamma_{H_2O} = (1 \times 10^{-3}\,m)(9800\,\frac{N}{m^3}) = 9.80\,\frac{N}{m^2}$, then

$$\frac{\Delta p}{\ell} = 72\,\frac{mm\,H_2O}{m}\left(\frac{9.80\,\frac{N}{m^2}}{mm\,H_2O}\right) = 706\,\frac{N}{m^3}$$

Since $\Delta p = \frac{4\tau_w \ell}{D}$ it follows that

$$\tau_w = \frac{D}{4}\frac{\Delta p}{\ell} = \frac{0.050\,m}{4}\left(706\,\frac{N}{m^3}\right) = \underline{\underline{8.83\,\frac{N}{m^2}}}$$

12.44

12.44 Do the head-flowrate data shown in Fig. 12.12 appear to follow the similarity laws as expressed by Eqs. 12.39 and 12.40? Explain.

The data in Fig. 12.12 show the effect of changing impeller diameter on head-flowrate characteristics. According to the similarity laws expressed by Eq. 12.39 and Eq. 12.40

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad (Eq.\ 12.39)$$

$$\frac{h_{a_1}}{h_{a_2}} = \frac{D_1^2}{D_2^2} \qquad (Eq.\ 12.40)$$

Thus, as the diameter is increased from 6 in. to 7 in. to 8 in. the flowrate increases according to Eq. 12.39 as

(from 6 in. to 7 in.) $Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{7\ in.}{6\ in.}\right)^3 Q_1 = 1.59\, Q_1$

and

(from 6 in. to 8 in.) $Q_2 = \left(\frac{8\ in.}{6\ in.}\right)^3 Q_1 = 2.37\, Q_1$

Similarly, from Eq. 12.40

(from 6 in. to 7 in.) $h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{7\ in.}{6\ in.}\right)^2 h_{a_1} = 1.36\, h_{a_1}$

and

(from 6 in. to 8 in.) $h_{a_2} = \left(\frac{8\ in.}{6\ in.}\right)^2 h_{a_1} = 1.78\, h_{a_1}$

Thus, for any given point, such as (A) where $Q = 120$ gpm and $h_a = 250$ ft (see Fig. 12.12 on following page) for the 6-in. diameter impeller, the corresponding predicted point would be at (B) where

$$Q_2 = (1.59)(120\ gpm) = 191\ gpm$$

$$h_{a_2} = (1.36)(250\ ft) = 340\ ft$$

(con't)